Extending the Input and Transformation Space of Different TP Models: An LMI-based Feasibility Analysis

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Abstract: This paper discusses that the selection and modification of the input and the transformation space, of the Tensor Product (TP) model representation, of a given quasi–Linear Parameter Varying (qLPV) state-space model, has an influence on the feasibility regions of the Linear Matrix Inequality (LMI) based control design techniques. Moreover, three factors affect the feasibility regions of the LMI-based control design:
The manipulation of the position of vertices
The number of the inputs of the TS fuzzy model representation
Modifying the transformation space
The proof is based on a complex control design example, where the impact of the above factors can be clearly demonstrated. Furthermore, the paper presents that the maximal and minimal parameter space of the controller depends on factors (i) and (ii). The aim of the feasibility test is to show that there exists a solution for LMIs or not, considering these factors. The example is based on the academic Translational Oscillator with Rotational Actuator (TORA) system. Then, the TP model transformation-based framework is used to vary the input space of the TP model representation. In addition, the paper gives a very decisive conclusion that the design technique may be sensitive for the input space of the TS fuzzy model, hence it is necessary to consider the number of inputs, the transformation space and the gains defined on the inputs when a TP model is generated to achieve the best solution to the control purposes. All in all, this paper investigates the effect of input space

modification of the TP model representation of a given qLPV state-space model on the feasibility regions of LMI-based controller design methods.

Keywords: TP model transformation; LMI-based controller; input space; transformation space; qLPV model

1 Introduction

Three factors influence the feasibility regions of LMIs:

(i) The number and the non-linearity of the inputs

(ii) The manipulation of the position of vertices

(iii) Variations of the transformation space

The proof is based on a state feedback control design example, where these factors can be clearly detected. Accordingly, the TP model transformation is applied to generate various alternative TP models to represent the qLPV models. These alternatives have different number of inputs. Furthermore, different nonlinear gains are defined on these inputs to decrease the rank of the TP model. As a consequence, different controllers are resulted to the given qLPV model using the same Parallel Distributed Compensation (PDC) based LMI design technique, since different TP model representations have been used. This leads to a question that which one of the resulted controllers is the best one. This results in a very significant point that the design technique is sensitive to the TP model input space. Therefore, it is necessary to consider the number of inputs, the transformation space and the gains defined on the inputs when a TP model is generated to achieve the best solution to our purposes. Although it sounds quite obvious, most of the papers about TP model-based control design, do not consider this fact. Without checking the proposed method on various alternative TP models, we cannot conclude if the design method is efficient essentially or not.

Previous investigations on the topic of TP model transformation focuses on internal parameters, such as the number of antecedents and consequent fuzzy sets. On the other hand, the current article focuses on the external parameters, the number of inputs and their non-linearity. The present paper examines that the TP model may have different input space and transformation space. The suggested extension is capable of varying the number of inputs and decreasing the non-linearity and transforming the input dimensions.

The effect of the vertices of the TP model representation was examined in paper [1] via the NATA model of the three degrees of freedom Aeroelastic Wing Section, where the feasibility regions of the LMI-based controller design was influenced by the factors: manipulating of the position of vertices, the size and complexity of the model. Paper [2] examines the statement, that the convex hull of the TP model representation has influence on the feasibility of LMI based control design. Present paper shows the feasibility regions if the transformation space is changed.

1.1 The Novel Contribution of this Paper

In the current paper, the following system is represented by the qLPV form; the academic TORA system. The controller design method includes the TP model transformation-based design framework. This paper investigates a methodical manipulation of the TP model representation complexity via changing the input and the transformation space. Furthermore, it is presented how the feasibility regions of the LMI based design change.

1.2 Preliminaries and Related Literature

The TP model transformation was developed in [3] to derive multi-level TP model structures. In addition, the Singular Value Decomposition (SVD) based method is applied to decrease a given fuzzy rule set in [4] [5]. Further details, adaption, steps and extensions of the TP model transformation can be found in books [6] [7] and papers [8] [9].

Paper [10] presents the TP model transformation of the TORA system in the control of a nonlinear benchmark problem, where the stability analysis is executed via LMI-based controller design method in PDC framework. Thus, the global asymptotic stabilization of the TORA system is executed. Therefore, it can be seen that the control design can easily be implemented. Otherwise, LMIs can give the optimal solution of the global asymptotic stabilization. Furthermore, the convex optimization via LMI-based design method can be solved rapidly.

Paper [11] shows the state-variable feedback control design of the 2DoF model of NATA Wing Section via TP model transformation and LMI based controller design method. The aim is to derive an observer for the Wing Section to estimate the non-measurable state values, then the output feedback control is designed using the LMI-based technique. The TP transformation is resulted in a tight convex hull. In case of observer design, the tight convex hull does not lead to feasible LMI. In report [12], stabilizing the 3DoF model of NATA is presented via asymptotic stability, decay rate and constraint of the control signal. Furthermore, the convex hull manipulation is investigated.

Paper [1] presents that the vertices have influence on the feasibility regions of LMI-based control design technique via the 3DoF model of Aeroelastic Wing (NATA). Furthermore, there are two factors influencing the feasibility regions: position of the vertices, size and complexity of the TP model representation. Moreover, the CNO type weighting functions are investigated from the perspective of convex hull manipulation. This has influences on the feasibility of LMI-based solutions. Paper [2] shows that the convex hull of the TP model transformation has effect on the feasibility of LMI based stability representation. The investigation is achieved via different convex hulls.

The novel control approaches of the TP model transformation were published in [14] [25-28]. Varying the input space of the TS fuzzy model is introduced in paper [14].

For additional important applications can be found in [15-38]. Most recent results are published in [8] [13] [14] [22-24] [39-54].

1.3 Structure of the Paper

The paper is structured as follows: Section 2 defines the notation and definitions of the TP model transformation used in this paper. The proposed extension is represented in Section 3 through the statements and proofs. Section 4 presents the applied methods, for instance varying the input space, the transformation space and adapting LMI-based controller design. Furthermore, the TP transformation of the TORA system the applied weighting functions are presented in Section 5. Section 6 deals with the feasibility analysis for TORA system. The globally asymptotic stabilization of this nonlinear system is introduced in Section 7. And finally, in Section 8 conclusions are presented.

2 Notations and Definitions

In this section the notations and definitions are presented that used in current paper. For detailed definitions, see paper [14].

- Indices: i, j, k are the upper bounds of the indices e.g., $i = 1,2,\dots,I$ and $j = 1,2,\dots,J$ and $k = 1,2,\dots,K$ or $i_n = 1,2,\dots,I_n$, where $n = 1,2,\dots N$ and I, J, K are the number of the vertices
- Scalar: $a \in \mathbb{R}$
- Vector: $\mathbf{a} \in \mathbb{R}^I$ contains elements $a_i \in \mathbb{R}$
- Matrix: $\mathbf{a} \in \mathbb{R}^{I\times J}$ contains elements $a_{i,j} \in \mathbb{R}$
- Tensor: $\mathcal{a} \in \mathbb{R}^{I\times J\times K\times \dots}$ has elements $a_{i,j,k,\dots} \in \mathbb{R}$
- Interval: $\omega \subset \mathbb{R}$ is bounded as $\omega = [\omega_{min}, \omega_{max}]$
- $\mathbb{R}^{I^N}$ is brief notation of $\mathbb{R}^{I_1\times I_2\times\dots\times I_N}$. For instance, $\mathbb{R}^{I^N\times O^K}$ denotes $\mathbb{R}^{I_1\times I_2\times\dots\times I_N\times O_1\times O_2\times\dots\times O_K}$
- System matrix $\mathbf{S}(\mathbf{p})$ is determined with parameter $\mathbf{p} = \mathbf{p}(t) \in \Omega$

- Transformation space $\Omega = [\omega_1, \omega_1] \times [\omega_2, \omega_2] \times \ldots \times [\omega_n, \omega_n] \subset \mathbb{R}^n$ denotes a closed hypercube, hence the TP model representation is interpretable only in this space
- Weighting functions $w_n = w_n(\mathbf{p}(t))$
- CNO: Close to Normalized membership function, that means its largest value is 1 or close to 1
- X-type weighting functions: Varying the input space with reduction of non-linearity results less complex weighting functions, see in Fig. 1, where $w_n(\mathbf{p_n}(t))$ are illustrated for n dimensions

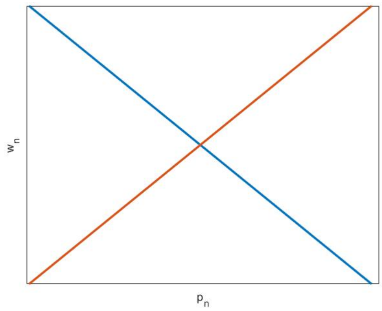

Figure 1
X-type weighting functions

- System matrix $\mathbf{S}(\mathbf{p}(t))$ is the parameter (p) dependent matrix of qLPV representation
- $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector, $\mathbf{u}(t) \in \mathbb{R}^m$ is the input vector, $\mathbf{y}(t) \in \mathbb{R}^q$ is the output vector and $\mathbf{A}_i(t) \in \mathbb{R}^{n\times n}$, $\mathbf{B}_i(t) \in \mathbb{R}^{n\times m}$

Definition 1: TP structure. Tensors can be given as a tensor product such as

$$\mathcal{S} = \mathcal{B} \boxtimes_{n=1}^{N} \mathbf{U}_n \tag{1}$$

where $\mathcal{B} \in \mathbb{R}^{I^N \times O^K}$ termed as core tensor and $\mathbf{U}_n \in \mathbb{R}^{M_n \times I_n}$ are the weighting matrices.

Definition 2: TP model. The TP model is a continuous variant of the TP structure. Here, instead of weighting vectors the weighting functions are as follow:

$$\mathcal{S}(\mathbf{p}) = \mathcal{B} \boxtimes_{n=1}^{N} [w_{n,1}(p_n) \quad w_{n,2}(p_n) \quad \cdots \quad w_{n,I_n}(p_n)] \tag{2}$$

that is $\mathcal{S}(\mathbf{p}) = \mathcal{B} \boxtimes_{n=1}^{N} \mathbf{w}_n(p_n)$ (3)

where the vector of the weighting function is $\mathbf{w}_n(p_n) = [w_{n,1}(p_n) \quad w_{n,2}(p_n) \quad \cdots \quad w_{n,I_n}(p_n)] \in \mathbb{R}^{I_n}$ and $\mathcal{S}(\mathbf{p}) \in \mathbb{R}^{O^K}$, $\mathbf{p} \in \mathbb{R}^N$ and core tensor $\mathcal{B} \in \mathbb{R}^{I^N \times O^K}$ contains the vertices $\mathcal{s}_{i_1,i_2,\dots,i_N} \in \mathbb{R}^{O^K}$.

To understand the basic steps of varying the input space, it is necessary to view paper [14], that presents the hyper rectangular grid and the difference between TS fuzzy model and TP model in detail.

3 Statements and Proofs

Present section explains the statements about the TP model representation of a given qLPV model and the feasibility regions influencing the LMI-based controller design method. Furthermore, it is shown how the behavior of the LMI-based control is changed via modifying the transformation space Ω to Ω_{min} and Ω_{max} and via selecting the input space. Control design of TP models of the academic TORA system is executed by LMI. The statements are as follows:

Statement 1. Varying the input space and the non-linearity of inputs of the TP model has influence on the feasibility of the LMI-based design. Hence, the complexity of the model is reduced, but the controllers are different in each example. The proof is based on that the position of the vertices is modified (see in paper [14]).

Statement 2. It is possible to modify the transformation space Ω provided that the LMI is still feasible. Thus, it is shown how the behaviour of the LMI-based controller changes. Changing the transformation space does not always result in a feasible solution for LMI.

Statement 3. Decreasing the non-linearity through varying the input space and changing the transformation space results in increasing feasibility regions.

The proofs are based on a complex control design example of nonlinear dynamic qLPV system. The investigations follow these key points:

1) Control design and all investigations are executed on the nonlinear models.

2) Selecting the input space is based on the non-linearity of the model examples. These non-linearities belong to the original state-space representation of qLPV models. The aim is to reduce or remove these non-linearities of the different TP models of TORA system.

3) Modification of the transformation space is based on the fact, that LMI is influenced by many factors. These modifications belong to the original transformation spaces of the TP models.

Hence, the proofs show through the statements, which TP model results the better controller depending on varying the input and transformation space.

4 Applied Methods

In this section, the applied methods are presented. Manipulation of the input space is a new extension of the TP model transformation, that is discussed in paper [14]. The input space can be changed in such a way that the new inputs are functions. Thereby, the suggested method can reduce or remove the non-linearity from the TP models. However, paper [14] does not include the analysis of transformation space, so this is a novel contribution of the current investigations.

4.1 How to Vary the Input Space of the TP Model Transformation

Consider the qLPV state-space representation:

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{y}(t) \end{bmatrix} = \mathbf{S}(\mathbf{p}(t)) \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} \tag{4}$$

where $\mathbf{x}(t)$, $\mathbf{u}(t)$ and $\mathbf{y}(t)$ are the state, input and output vectors, $\mathbf{S}(\mathbf{p}(t))$ is the system matrix respectively. Thus, the aim is to control this system by reducing non-linearity.

The first step is to determine the Higher Order Singular Value Decomposition (HOSVD) based canonical form for the above qLPV model [6]. The model is based on the number of inputs and the position of the vertices. Furthermore, the LMI based control design requires a convex TP model structure of the system matrix $\mathbf{S}(\mathbf{p}(t))$. In the present paper, the examples follow the current TP based model structure:

$$\begin{pmatrix} \dot{\mathbf{x}} \\ \mathbf{y} \end{pmatrix} = \left(\mathcal{S} \boxtimes_{n=1}^{N} \mathbf{w}_n(p_n) \right) \begin{pmatrix} \mathbf{x} \\ \mathbf{u} \end{pmatrix} \tag{5}$$

Paper [14] shows more detailed research about the new extension of the TP model transformation. Therefore, this extension is capable to transform an alternative input space. Hence, consider the following function:

$$\mathcal{S}(\mathbf{p}) \in \mathbb{R}^{O^K}, \ \mathbf{p} \in \Omega^p \subset \mathbb{R}^N \tag{6}$$

then the TP model transformation for all $\mathbf{p}$ is:

$$\mathcal{S}(\mathbf{p}) = \mathcal{S} \boxtimes_{n=1}^{N} \mathbf{w}_n(p_n) \tag{7}$$

The alternative input space is $\mathbf{b} \in \Omega^b \subset \mathbb{R}^M$, where the relation between $\mathbf{p}$ and $\mathbf{b}$ is defined. Thus, the expected result for all $\mathbf{p}$ is:

$$\mathcal{S}(\mathbf{p}) = \mathcal{T}(\mathbf{b}) = \mathcal{A} \boxtimes_{m=1}^{M} \mathbf{v}_m(b_m) \tag{8}$$

where the input space $\mathbf{p} \in \Omega^p$ is replaced by $\mathbf{b} \in \Omega^b$, $\mathcal{A}$ is the core tenzor and $\mathbf{v}_m$ is the weighting function. For more detailed presentation see in paper [14].

4.2 Modification of the Transformation Space

In addition to vary the input space, present paper focuses on the investigations of extended transformation space. Ω is determined by a hyper-rectangular space, where the weighting functions are defined. Thus, the TP model representation is interpretable only in this space, see [6]. Moreover, for the examination, the transformation space to Ω_{min} and Ω_{max} is extended. These steps influence the feasibility of the LMI-based controller design method.

Current investigation is based on paper [14]. Thus, the values of the applied transformation spaces Ω for each TP models are the same, as in paper [14]. In the present paper, it is proved that varying the transformation space is possible if the LMI is feasible.

5 TP Model Transformation of TORA

In this section, the academic TORA system is investigated which was a key example to test the first variants of TP model transformation [6].

Previous research through this example [18] presented that TP model transformation and LMI-based control design approach can readily be accomplished independently of the given problem. Therefore, no analytical derivation is required, thus the controller designing is less time consuming. Those analyses focused primarily on the shape of the fuzzy antecedent sets, hence the convex hull defined by the vertices, and the number of fuzzy rules.

In the present section, we focus on the number of inputs as well as in [14]. In this example, the same 137×137 grid density and CNO type weighting functions are used. See the MATLAB code for the CNO type functions in paper [14].

5.1 qLPV Model of the TORA

The applied parameters and equations of motion are given in [6] [14] [18]. Thus, the state-space variables of TORA system are selected as:

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} = \begin{bmatrix} \xi(t) \\ \dot{\xi}(t) \\ \theta(t) \\ \dot{\theta}(t) \end{bmatrix} \tag{9}$$

where, the system matrix $\mathbf{S}(\mathbf{p}(t))$ of the model takes form of:

$$\mathbf{A}(\mathbf{p}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-1}{f(x_3(t))} & 0 & 0 & \frac{\rho x_4(t) sin(x_3(t))}{f(x_3(t))} \\ 0 & 0 & 0 & 1 \\ \frac{\rho cos(x_3(t))}{f(x_3(t))} & 0 & 0 & \frac{-\rho^2 x_4(t) sin(x_3(t)) cos(x_3(t))}{f(x_3(t))} \end{bmatrix} \tag{10}$$

$$\mathbf{B}(\mathbf{p}) = \begin{bmatrix} 0 \\ \frac{-\rho cos(x_3(t))}{f(x_3(t))} \\ 0 \\ \frac{1}{f(x_3(t))} \end{bmatrix} \tag{11}$$

where

$$f(x_3(t)) = 1 - \rho^2 cos^2(x_3(t)) \tag{12}$$

and $p_1(t) = x_3(t)$ and $p_2(t) = x_4(t)$. The system matrix $\mathbf{S}(\mathbf{p}(t))$ is:

$$\mathbf{S}(\mathbf{p}(t)) = [\mathbf{A}(\mathbf{p}(t)) \quad \mathbf{B}(\mathbf{p}(t))] \tag{13}$$

where the elements of matrix $\mathbf{A}(\mathbf{p}(t))$ and $\mathbf{B}(\mathbf{p}(t))$ are defined numerically by HOSVD method and CNO type weighting functions into vertices.

5.2 TP Model 1

This subsection presents the previous version of the TP model transformation (7). Therefore let $p_1(t) = x_3(t)$ and $p_2(t) = x_4(t)$. The transformation space is $\Omega = [-0.8,0.8] \times [-0.8,0.8]$. The resulted TP model representation is:

$$\mathbf{S}(\mathbf{p}(t)) = \mathcal{S} \underset{n=1}{\overset{2}{\boxtimes}} \mathbf{w}_n(p_n(t)) \tag{14}$$

thus, $w_{1,i}(p_1)$ CNO type weighting functions in parameter space Ω are shown in Fig. 2. For parameter $p_2(t)$, the HOSVD results $w_{2,1}$ and $w_{2,2}$ in the X-type weighting function. The number of the resulting linear time invariant (LTI) systems is $5 \times 2 = 10$.

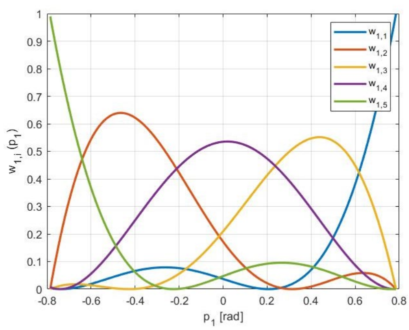

Figure 2
CNO type weighting functions of TP model 1

5.3 TP Model 2

The TP model 1 has minimal number of weighting functions. In order to decrease the non-linearity of the model, the parameter space is defined as $p_1(t) = x_3(t)$, $p_2(t) = x_4(t)$, $p_3(t) = \frac{1}{f(x_3(t))}$, where a new dimension is introduced. Thus, the transformation space is $\Omega = [-0.8,0.8] \times [-0.8,0.8] \times [1,1.05]$. Applying the HOSVD on the qLPV model 2, results in $4 \times 2 \times 2 = 16$ LTI systems. Therefore, the second TP model form is:

$$\mathbf{S}(\mathbf{p}(t)) = \mathbf{T}(\mathbf{p}(t)) = \mathcal{A} \boxtimes_{n=1}^{3} \mathbf{w}_n(p_n(t)) \tag{15}$$

The membership functions of $p_1(t)$ are illustrated in Fig. 3. For $p_2(t)$ and $p_3(t)$, weighting functions $w_{2,1}$,$w_{2,2}$ and $w_{3,1}$, $w_{3,2}$ are depicted on the Fig. 1 X-type. Furthermore, the complexity of the weighting functions is reduced. However, the non-linearity can be further reduced, see in TP model 3.

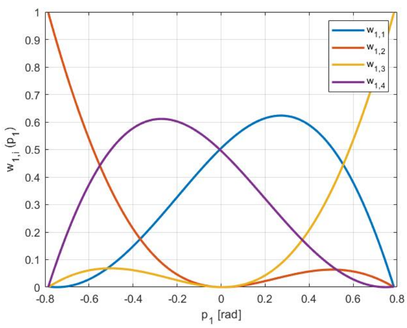

Figure 3
CNO type weighting functions of TP model 2

5.4 TP Model 3

In order to further decrease the complexity of the first dimension, the new input parameters are $p_1(t) = sin(x_3(t))$, $p_2(t) = x_4(t)$ and $p_3(t) = cos(x_3(t))$. Then the space $\Omega = [-0.8,0.8] \times [-0.8,0.8] \times [0,0.8]$ is defined. The resulting number of the LTI systems is $2 \times 2 \times 3 = 12$, thus the TP model structure is:

$$\mathbf{S}(\mathbf{p}(t)) = \mathbf{T}(\mathbf{p}(t)) = \mathcal{A} \boxtimes_{n=1}^{3} \mathbf{w}_n(p_n(t)) \tag{16}$$

The weighting functions of $p_3(t)$ are shown in Fig. 4. For $p_1(t)$ and $p_2(t)$, weighting functions $w_{1,1}$,$w_{1,2}$ and $w_{2,1}$, $w_{2,2}$ are depicted on the Fig. 1 X-type. The rank of the first dimension is minimised, in the third dimension there are 3 weighting functions in contrast to TP model 2. Consequently, the resulted model is less complex than the previous two examples.

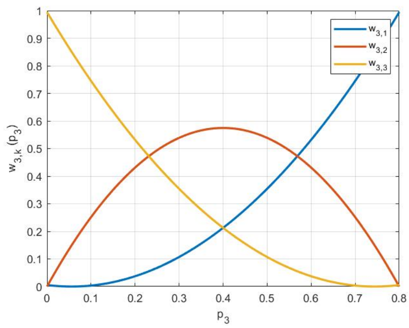

Figure 4
CNO type weighting functions of TP model 3

5.5 TP Model 4

Let is the parameter space $p_1(t) = x_4(t)sin(x_3(t))$, $p_2(t) = \frac{1}{f(x_3(t))}$ and $p_3(t) = \rho cos(x_3(t))$, and the transformation space $\Omega = [0,1.6] \times [1,1.05] \times [0,0.2]$. Executing the proposed TP model transformation, there are $2 \times 2 \times 2 = 8$ LTI systems:

$$\mathbf{S}(\mathbf{p}(t)) = \mathbf{T}(\mathbf{p}(t)) = \mathcal{A} \underset{n=1}{\overset{3}{\boxtimes}} \mathbf{w}_n(p_n(t)) \tag{17}$$

The membership functions are depicted in Fig. 1, where the weighting functions $w_{1,1}$, $w_{1,2}$, $w_{2,1}$, $w_{2,2}$ and $w_{3,1}$, $w_{3,2}$ for parameters $p_1(t)$, $p_2(t)$, and $p_3(t)$ are illustrated. Compared to the previous TP model 1,2,3 examples, this is the simplest model.

5.6 TP Model 5

Let us define the parameter space as $p_1(t) = \frac{x_4(t)sin(x_3(t))}{f(x_3(t))}$, $p_2(t) = \frac{1}{f(x_3(t))}$ and $p_3(t) = cos(x_3(t))$. and $\Omega = [0,1.6] \times [1,1.05] \times [0,1]$. The number of the LTI systems are $2 \times 2 \times 2 = 8$ again:

$$\mathbf{S}(\mathbf{p}(t)) = \mathbf{T}(\mathbf{p}(t)) = \mathcal{A} \underset{n=1}{\overset{3}{\boxtimes}} \mathbf{w}_n(p_n(t)) \tag{18}$$

The membership functions are given in X-type for all parameters $p(t)$. This is similar to the previous example, but the consequences are different from it.

6 Feasibility Analysis

The feasibility test checks whether there exists a solution for LMI or not. Consider the following solver for LMI feasibility problems: $\mathbf{L}(x) < \mathbf{R}(x)$, where $\mathbf{R}$ is the feasibility radius. The solver minimizes t subject to $\mathbf{L}(x) < \mathbf{R}(x) + t\mathbf{I}$. Thus, the best value of t should be negative for feasibility.

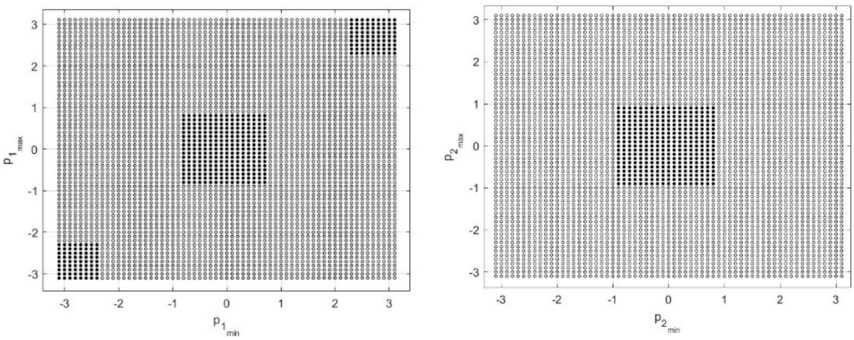

Figure 5

Feasibility regions of TP model 1

The feasibility check for transformation space is investigated only on interval $[-180°, 180°]$ for angles. For all figures; x-axis illustrates p_{min} and y-axis shows p_{max}, illustrations present that LMI is feasible or not in the different dimensions. Therefore, figures show the feasible (bold dotted) and non-feasible regions of the LMI through changing transformation space. In this section the most significant TP models and figures are presented.

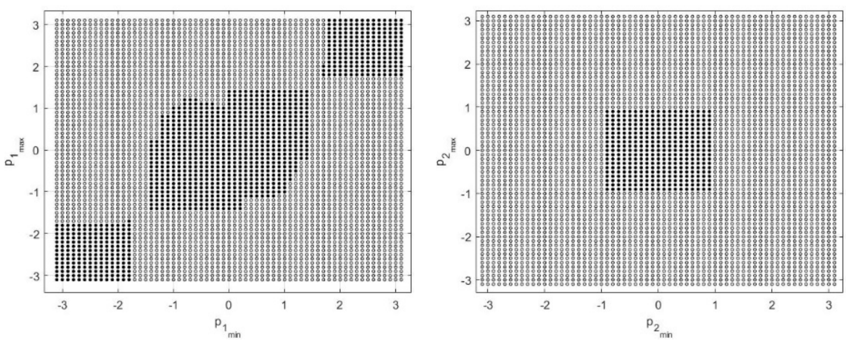

Figure 6

Feasibility regions of TP model 3 for p_1 and p_2

Varying the transformation space Ω has influence on the feasibility regions of LMIs. In the followings the feasibility of LMI based controller for TORA TP models are presented.

In the present example, there are two dimensions in the transformation space $\Omega = [-0.8,0.8] \times [-0.8,0.8]$. The parameters are $p_1(t) = x_3(t)$, $p_2(t) = x_4(t)$. Moreover, Fig. 5 presents two cases. In the first case, the first dimension p_1 of the transformation space is changed, and second dimension p_2 is fixed on interval $[-,-] \times [-0.8,0.8]$. Fig. 5 shows another case, that first dimension p_1 is fixed on $[-0.8,0.8] \times [-,-]$ and second dimension p_2 is changed. Then, it can be seen that LMI is feasible or not, in the dimensions of TORA TP model 1.

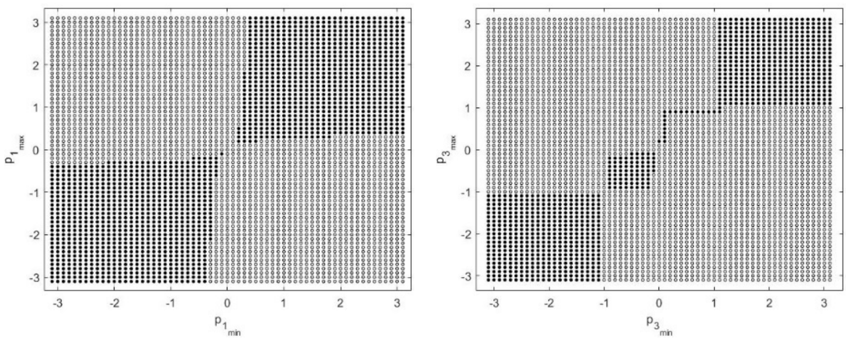

Figure 7

Feasibility regions of TP model 4 for p_1 and p_3

In case of TORA TP model 2, there are three dimensions; $\Omega = [-0.8,0.8] \times [-0.8,0.8] \times [1,1.05]$, where $p_1(t) = x_3(t)$, $p_2(t) = x_4(t)$, $p_3(t) = \frac{1}{f(x_3(t))}$. As before, two cases are considered; each of the two dimensions is fixed separately. Thus, the results for changing dimensions p_1 and p_2 are the same as in Fig. 5.

The feasibility tests for TORA TP model 3 are illustrated on Fig. 6, where the transformation space $\Omega = [-0.8,0.8] \times [-0.8,0.8] \times [0,0.8]$, and $p_1(t) = sin(x_3(t))$, $p_2(t) = x_4(t)$ and $p_3(t) = cos(x_3(t))$. Figures shows that dimension p_1 is changed and, the others are fixed on interval $[-,-] \times [-0.8,0.8] \times [0,0.8]$, then dimension p_2 is changed on interval $[-0.8,0.8] \times [-,-] \times [0,0.8]$. It can be seen, that the feasibility regions are increased compared to TP model 1 and 2.

Fig. 7 present the feasibility regions of TORA TP model 4. In this case, the transformation space is $\Omega = [0,1.6] \times [1,1.05] \times [0,0.2]$, and the parameter space is $p_1(t) = x_4(t)sin(x_3(t))$, $p_2(t) = \frac{1}{f(x_3(t))}$ and $p_3(t) = \rho cos(x_3(t))$. Two cases are examined again. Thus, it can be seen the feasibility regions if dimension p_1 is changed on interval $[-,-] \times [1,1.05] \times [0,0.2]$ and if dimension p_3 is changed on interval $[0,1.6] \times [1,1.05] \times [-,-]$. The feasibility regions are larger, than the previous TP models.

Fig. 8 illustrates the feasibility regions of TORA TP model 5. The transformation space is $\Omega = [0,1.6] \times [1,1.05] \times [0,1]$ and the parameters are $p_1(t) = \frac{x_4(t)sin(x_3(t))}{f(x_3(t))}$, $p_2(t) = \frac{1}{f(x_3(t))}$ and $p_3(t) = cos(x_3(t))$. So, the figures show two cases. In the first case, dimension p_1 is changed on interval $[-,-] \times [1,1.05] \times [0,1]$ and for p_2 and p_3 are fixed. In the second case, dimension p_3 is changed on interval $[0,1.6] \times [1,1.05] \times [-,-]$, and p_1 and p_2 are fixed on this interval. It can be seen that the feasibility regions are larger, than the previous four TP models.

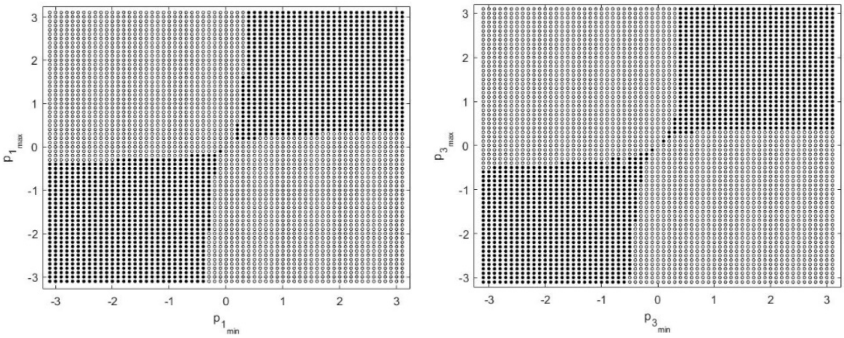

Figure 8

Feasibility regions of TP model 5 for p_1 and p_3

Consequently, the figures present well that if the nonlinear behavior of the TP models are decreased, the LMI feasibility regions will increase.

7 Globally Asymptotic Stabilization

In this section, all of the examined TP models are illustrated. The results of the control design of the TORA system are presented via LMI based stabilization method, through state feedback control. The LMIs are feasible. The LMI solver can find the stabilizing controller using the following LMI conditions [10]:

$$-\mathbf{X}\mathbf{A}_i^T - \mathbf{A}_i\mathbf{X} + \mathbf{M}_i^T\mathbf{B}_i^T + \mathbf{B}_i\mathbf{M}_i \succ 0 \quad (19)$$

$$-\mathbf{X}\mathbf{A}_i^T - \mathbf{A}_i\mathbf{X} - \mathbf{X}\mathbf{A}_j^T - \mathbf{A}_j\mathbf{X} + \mathbf{M}_j^T\mathbf{B}_i^T + \mathbf{B}_i\mathbf{M}_j$$

$$+\mathbf{M}_i^T\mathbf{B}_j^T + \mathbf{B}_j\mathbf{M}_i \succeq 0 \quad (20)$$

$$\mathbf{k}_i = \mathbf{M}_i\mathbf{X}^{-1} \quad (21)$$

where $i = 1, \cdots, I$ and $j = I + 1, \cdots, I$, I is the total number of the LTI vertex systems. Using these LMI conditions, present paper shows a stable controller design task. Matrices $\mathbf{X}$ and $\mathbf{M}$ can be found by convex optimization methods involving LMIs.

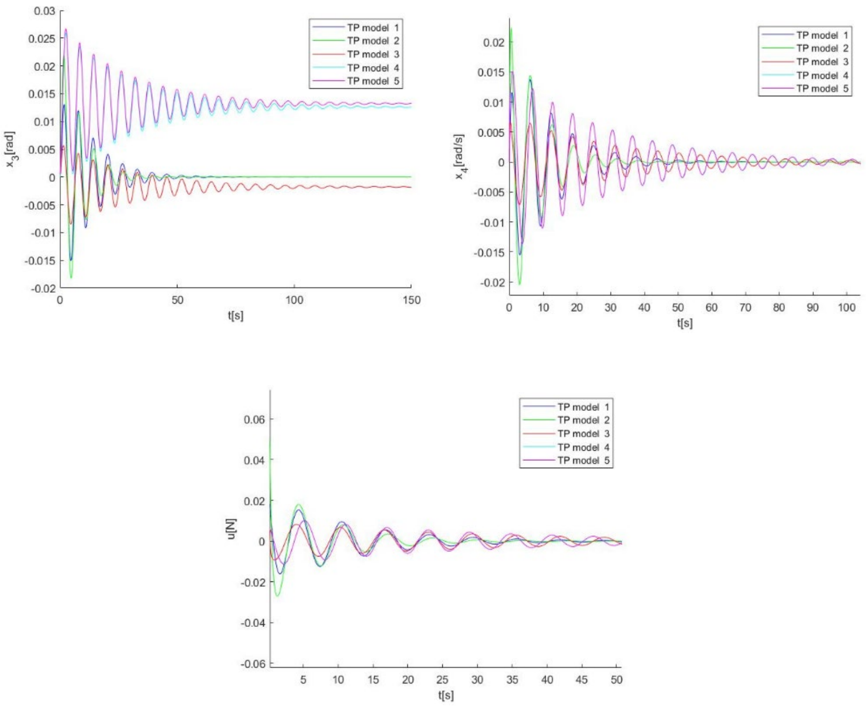

Figure 9

State variables x_3, x_4 and control signal u of TORA

The resulted LTI vertex systems defined by TP transformation are substituted into the above LMI conditions.

The overall TP controller is:

$$\mathbf{u}(t) = -(\sum_{i=1}^{I} w_i(\mathbf{p}(t))\mathbf{k}_i)))\mathbf{x}(t) = -\mathbf{k}^T\mathbf{x}(t) \tag{22}$$

where the same membership functions are applied as in the TP model examples. It can be seen that the various inputs of the TS fuzzy models influence the feasibility of the LMI based controller design.

Angular position θ, and angular speed $\dot{\theta}$ of TP model 1, 2, 3, 4, 5 examples and control signal u are illustrated in Fig. 9, where the overall initial conditions are $[\xi, \dot{\xi}, \theta, \dot{\theta}]^T = [0.023,0,0,0]^T$. Notice that the TP model 2, results in the best controller performance. In example TP model 2, a new dimension is introduced and the number of LTI systems are $4 \times 2 \times 2 = 16$. Although, the complexity of the weighting functions is still exists, decreasing the non-linearity results in a better controller.

It can be seen that control signal u is stabilized after 50 seconds. Therefore, the simulation results show that the system reaches the steady state quicker and the complexity of the TP model 2 is less than the other ones. Furthermore, varying the input space, it results in different TP models, as shown in Fig. 3. Number of the dimensions and complexity of the weighting function have influence on the feasibility of LMI based control design method. Consequently, the TP model 2 is the better model.

Conclusions

This work shows the proposed extension of the TP model representation, where it is capable of generating an alternative input space. This step has influence on non-linearity and complexity of the model. In this paper, the investigation is based on the TORA system. Moreover, modifying the transformation space affects the LMI-based control design methods. Consequently, the controller can be sensitive for varying the input space and transformation space. Decreasing the non-linearity via changing the input space, results in increasing feasibility regions varying the transformation space. Furthermore, the new properties improve the capability of the TP model in achieving the best possible solution of the control design methods.

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