Experimental Validation of Heat Propagation: Results of the Numerical Modelling for Real Scale Steel Structural Elements and Different Assigned Models, Subjected to Simulated Fire

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Abstract: Currently the fire protection problems for structural elements, is usually solved by means of the prototype-model correlations, established by means of dimensional methods. The authors, based on a reliable state-of-art analysis, established the advantages and limits of the presently-used dimensional methods. Finally, they state that these methods have several serious shortcomings, which entirely disappear by applying the "Modern Dimensional Analysis (MDA)", conceived by Szirtes [65] [66]. The authors have used MDA since 2000, in solving several engineering problems; for instance, heat transfer modelling along steel structural elements, unprotected-, as well as protected, against the fire. They briefly describe both the obtained Model Laws and their experimental validation, based on an original electric-heated test bench. In this paper, we offer the numerical simulation (with Finite Elements Method, FEM) of the experimental-obtained thermal fields for a real structural element, as well as for its reduced scale models. The obtained simulation results are in good agreement with the experimental data and consequently, the proposed numerical model is valid.

Keywords: modern dimensional analysis; heat transfer; finite element methods

1 Introduction

The fire-protection of the structural elements of a given building represents a continuous challenge of the engineers, involved in conceiving and designing as much as possible safe buildings [1] [22-25] [32-35] [39-41] [44] [50] [58] [75] [78-80].

In this process there are involved several heat-protecting procedures, analyzed in detail in literature [2] [5] [6] [29] [52] [70] [75]. One of them, widely applied, is the intumescent paint, which assures an efficient heat-insulation. This type of heat-insulation preserves the structural elements' initial aspect ratio, contributing to a modern design of the building.

One other significant topic nowadays consists in involving the experimental information obtained on reduced-scale *model*s and their transfer to the real-scale structural element, i.e., the so-called *prototype*.

This aspect can be solved by means of the dimensional methods, such as Geometric Analogy (*GA*), Theory of Similarity (*TS*), respectively Dimensional Analysis (*DA*), analyzed in detail in the literature [3] [4] [7-9] [11-14] [16] [17] [19] [20] [30] [31] [36] [38] [43] [45] [46] [48] [49] [51] [53-58] [77].

These methods, which take into consideration the particularities of the heat-transfer along the involved structural elements, manufactured mainly from steel, are analyzed in [10] [15] [18] [25] [37] [42] [53] [67] [69].

We can mention the fact that the model-prototype correlations are applied not only in the fire-protection problem, but also in common load-bearing capacity evaluation (prediction), too [62].

2 Survey on the Involved Approaches

As well known, the problem of heat transfer in structural elements is a complex phenomenon, which can be significantly simplified by a prototype-model approach. If we summarize only those synthesized in the previous works of the authors [27] [28] [61] [63] [68], then the following major aspects can be noted, briefly analyzed below.

The first approaches were related to the use of *GA*. In these cases, by default, the existence of well-defined ratios between dimensions, respectively equality of angles that define the shape and dimensions of the prototype, respectively of the model, is required. Since the number of correlations is limited (usually very small), the relationships useable in prototype-model behavior are also minimal.

TS, in addition to geometric similarity, also include a similarity of all physical quantities, which intervene in the studied phenomenon. Thus, homologous points of the prototype and the model can be defined, where the phenomena will unfold in homologous times, ensuring to each ω physical quantity involved an S_ω constant ratio of the values obtained (called scale factors) on the model ω_2 and on the prototype ω_1 (in this order) [65] [66]: $S_\omega = \omega_2 / \omega_1$ [-]. Obviously, there will be so many S_ω scale factors, as many ω physical quantities (variables) intervene in the description of the respective phenomenon.

From a practical point of view, the mathematical solution of complex equations, which theoretically describe the real phenomenon studied, is replaced by establishing some correlations between the dimensionless quantities obtained from the fundamental relations related to the phenomenon, based on appropriate groupings of the terms, also called "similarity criteria". In thermodynamic phenomena, we have the well-known similarity criteria Nu, Re, St, Pr, etc.

In this sense, in the fundamental relations, the actual physical quantities will be substituted by their related scale factors, amplified with some constants, and through a subsequent favorable grouping, both the actual similarity criteria and the criterion relations will result of these, such as for example in Thermodynamics: Nu=*f*(Re, Pr, Gr,…).

The use of these criterion relations, through the results of experimental measurements, leads to the simplification of the analysis and allows the reduction of the number of effective measurements in order to obtain some important parameters of the analyzed phenomenon. It should be noted that each given phenomenon will correspond to a concrete set of criteria relations, which cannot be applied in the analysis of another phenomenon (for example, those deduced for free convection will not be valid for forced convection).

Two basic aspects of *TS* should be highlighted, namely [15]:

- For two similar phenomena, the homologous dimensionless groups are identical
- The necessary and sufficient conditions for two phenomena to presents a correspondence of similarity (thus to be describable by the same mathematical equations) is that both phenomena:
  - Will be of the same nature
  - Will have the same determining criteria, respectively
  - Will have identical initial and boundary conditions

In the case of complex phenomena, the number of dimensionless parameters, scales of physical variables involved, as well as criterion equations becomes very large, which is why *TS* must be replaced with a more efficient means, which is Dimensional Analysis [3] [4] [7-9] [11-14] [16] [17] [19] [20] [30] [31] [36] [38] [43] [45] [46] [48] [49] [51] [53-58] [77]. Classical Dimensional Analysis (*CDA*) uses the results of experimental investigations carried out exclusively on the model, which through the π_j dimensionless relationships (i.e., dimensionless groups), allow forecasting the behavior of the prototype, obviously in conditions of similarity. Thus, compared to *GA*, but also compared to *TS*, both the volume of experimental investigations and related graphic representations is substantially simplified. *CDA* is not a substitute for experimental measurements, nor does it aim to explain physical phenomena; it only comes to simplify and optimize the strategy of experimental measurements by synthesizing the measurable parameters of a phenomenon in the form of the π_j dimensionless groups, defined by the π_j – theorem (Buckingham's theorem). The two systems (the model and the prototype) by their behavior will fully respect the conditions provided in the respective dimensionless group.

The ways of establishing the π_j dimensionless groups are [3-4] [7-9] [11-14] [16] [17] [19] [20] [30] [31] [36] [38] [43] [45] [46] [48] [49] [51] [53-58] [77]:

- Direct application of Buckingham's theorem
- Applying the method of partial differential equations on the fundamental differential relations related to the phenomenon, when the initial variables are transformed into dimensionless quantities (through a normalization process) and through their appropriate grouping, the desired π_j dimensionless groups will result
- Identifying the complete, but also the simplest form of the equation(s) describing the phenomenon, which we will transform into dimensionless forms, from which the desired π_j dimensionless groups will be identified

Through a careful analysis of these modalities, the main limits of *CDA* can also be defined, namely:

- It is a cumbersome method, which requires deep knowledge in the field of the studied phenomenon
- It is arbitrary, depending on the experience and ingenuity of the one who forms the πj dimensionless groups
- It cannot provide, except in very particular cases, the complete set of the desired πj dimensionless groups, i.e., the law of the actual model, which also results from the limited number of equations, which describe the analyzed phenomenon
- The lack of protocols for establishing the optimal model (so that the method is sufficiently flexible) represents another major disadvantage
- It does not represent a general engineering method (simple, unique and safe), but rather a theoretical one

In addition, by a searching and critical analysis of the afore-mentioned contributions, one can mention not only their advantages, but also several shortcomings, such as the following:

- The applied dimensional methods take into consideration only a given amount of the involved parameters and start always from the geometric analogy of the model with the prototype, which represents a serious disadvantage
- As a result of these approaches the attached model was not enough flexible and diminishes the possible-obtainable useful correlations
- When the authors of the afore-mentioned references applied strictly the *GA*, only a few numbers of useful model-prototype correlations were obtained together with a limited improvements' opportunities of the involved model
- *TS* improves a little bit this situation, but the expected (and obtained) correlations remain less than the real demand
- *CDA* based on the Buckingham's π theorem, also offers a limited number of useful correlations, due to the involved procedure of the wanted (in demand) dimensionless quantities
- In this sense, one has to mention, that in the case of applying the *CDA* one has to start from a limited number of differential equations, from where there are obtained by an adequate grouping the wanted dimensionless quantities
- One other significant shortcoming of the *CDA* consists in impossibility to conceive an adequate flexible model, which can assure an efficient and low-price experimental investigation on this model; this disadvantage disappears at the below-analyzed Modern Dimensional Analysis (*MDA*), widely applied by the authors of the present contribution in their investigations
- One other shortcoming consists in the fact that, all who tries to apply the *CDA,* has to present deeply knowledge both in Thermodynamics as well as in the Fire-protection; otherwise became impossible to performing an adequate (efficient) grouping of the involved parameters of the limited number of the available differential equations and other co-relations between the above-mentioned parameters.

Consequently, in the authors' opinion, the above-mentioned approaches are not very suitable solving the proposed problem, because they will offer only partial solutions, instead of some global, general ones. This means that instead a complete Model Law (*ML*), one will obtain only a partial (incomplete) one.

The authors of the present contribution, during the last 15-20 years, were involved in solving different aspects of the above-mentioned problem. Since 2000', they applied the *MDA* method in different fields of engineering, between others in structural elements load-bearing capacity estimation [62], as well as in the heat-transfer problem, described in the forthcoming chapter.

3 Theoretical Results

Compared to these methods (*GA*, *TS*, and *CDA*), the methodology developed by Szirtes regarding Dimensional Analysis, hereinafter called *Modern Dimensional Analysis* (*MDA*) [65-66] represents an efficient and accessible solution for any engineer or researcher.

In the first instance, based on the Szirtes' Theory [65] [66], as well as the authors' previous results [27] [28] [47] [60-64] [68] [71] [73] [74], let us briefly analyze the *incontestable advantages of the MDA*:

- The method supposes only knowing the set of those variable (of course, together with their dimensions) which can influence in any manner the analyzed phenomenon, but without deeply knowledge in the given phenomenon; the exponents of the dimensions of the selected variables will constitute the below-described Dimensional Set.
- From these variables the user will select a limited number, equal to main dimensions' one, which will constitute the so-called *independent variables*, which can be choose a priori and freely both for prototype and model; they will constitute the matrix A from the *Dimensional Set*; the single condition being that the obtained Matrix A will not be singular, i.e., det|A|≠0
- By means of these independent variables the user will conceive an optimal (the most suitable) model, attached to the analyzed prototype, regarding as both material, shape, type of section, dimensions, test conditions, as well as, but not least, reducing the number of qualified personnel in performing experiments, respectively in interpreting and transferring data from model to prototype; this advantage is not proper to any of the above-mentioned dimensional methods.
- The remaining *variables*, named *dependent* ones, which will constitute the matrix B from the Dimensional Set, can be choose a priori and freely only for the prototype; for the model, they will result strictly by rigorously applying of the obtained *ML.*
- Afterward, the Dimensional Set is completed by means of a matrixes $C = -(A^{-1}\cdot B)^T$ and D; in the matrix C the exponents (-1), and (*T*) represent the inverse matrix, respectively the transposed ones; the matrix D is an adequate unite matrix, i.e.: $D=I_{n\times n}$, where *n* represents the lines' number from matrix C, and corresponds also with the complete number of the requested dimensionless variables π_j, $j=1,\dots,n$ used in the *ML* creation.
- Finally, the Dimensional Set has the following aspect:

The rows correspond to the remaining primary k dimensions after defining matrix *A*	1.	B	A
	2.		
	3.		

	...		
	k.		
The rows correspond to *n* columns (dependent variables) that had matrix *B*; the number of the rows is the same as that of the π_j, resulting in dimensionless quantities	1.	$D=I_{n\times n}$	$C=-(A^{-1}\cdot B)^T$
	2.		
	3.		
	...		
	n		

Here it is worth noting, that each column of these matrices contain the exponents of the primary dimensions, which describe the respective variables.

- Each line, corresponding to one of the π_j dimensionless variable, will offer by a simple calculus one element of the requested *ML*.
- The order of introducing the dependent variables in matrix B and independent variables in matrix *A* and thus, their positioning in the reduced dimensional matrix (B-A) and dimensional set (B-A-D-C), respectively, does not influence the π_j relations and *ML*.

In the authors' previous contribution [27], they illustrated the simply, unique and safety protocol in obtaining the mentioned π_j dimensionless variable, i.e., the complete *ML*.

In the given case for illustration, one will consider 14 variables (H_1, H_2, H_3,…, H_{14}) which intervene in the analyzed phenomenon, variables described by means of 6 dimensions (h_1, h_2, …, h_6). From the variables six are the independent ones (H_9, H_{10}, H_{11},…, H_{14}), forming the matrix A, and the rest of them (H_1, H_2, H_3,…, H_8) form the matrix B; their exponents will be placed in the below (B-A) matrixes of the Dimensional Set. The Dimensional Set is completed by the afore-mentioned matrices D and C (see the table below).

In order to obtain for example, the fifth element of the *ML*, by means of the π_5 dimensionless variable, one can observe that in its line (the fifth one from matrices D-C) there are the exponents of all the independent variables involved (H_9, …, H_{14}), i.e., (a_5, … , f_5) , and the dependent one's (H_5), i.e., "1", located on the main diagonal of the matrix D.

Consequently, based on the unique protocol from [65] [66], the relation can be written:

$$\pi_5 = (H_5)^1 \cdot (H_9)^{a_5} \cdot (H_{10})^{b_5} \cdot (H_{11})^{c_5} \cdot (H_{12})^{d_5} \cdot (H_{13})^{e_5} \cdot (H_{14})^{f_5}.$$

The relation, as shown before, is equal to the unit, and from this equality, the dependent variable is expressed (being here H_5), i.e.:

$$\pi_5 = (H_5)^1 \cdot (H_9)^{a_5} \cdot (H_{10})^{b_5} \cdot (H_{11})^{c_5} \cdot (H_{12})^{d_5} \cdot (H_{13})^{e_5} \cdot (H_{14})^{f_5} = 1 \Rightarrow$$
$$\Rightarrow H_5 = \frac{1}{(H_9)^{a_5} \cdot (H_{10})^{b_5} \cdot (H_{11})^{c_5} \cdot (H_{12})^{d_5} \cdot (H_{13})^{e_5} \cdot (H_{14})^{f_5}} \ .$$

	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}	H_{11}	H_{12}	H_{13}	H_{14}
h_1														
h_2														
h_3				***Matrix B***							***Matrix A***			
h_4														
h_5														
h_6														
π_1	1	0	0	0	0	0	0	0	a_1	b_1	c_1	d_1	e_1	f_1
π_2	0	1	0	0	0	0	0	0						
π_3	0	0	1	0	0	0	0	0						
π_4	0	0	0	1	0	0	0	0						
π_5	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	a_5	b_5	c_5	d_5	e_5	f_5
π_6	0	0	0	0	0	1	0	0						
π_7	0	0	0	0	0	0	1	0						
π_8	0	0	0	0	0	0	0	1	a_8	b_8	c_8	d_8	e_8	f_8

⇧ ***Matrix D*** ⇧ ***Matrix C***

Figure 1
The illustration of how to obtain the elements of the Model Law

Subsequently, the variables involved ($H_5, H_9, \dots, H_{14}$) will be replaced by the related scale factors $\left(S_{H_n}\right)$, ultimately resulting in the desired expression of the fifth element of the *ML*.

Obviously, some of the exponents involved being negative, the relationship obtained will be in the form of an ordinary fraction, where both the numerator and the denominator will have expressions of scale factors at certain powers.

Useful remarks:

- In this case, the ML consists of 8 elements, since 8 dimensional variables (H1, … , H8) resulted from the calculations (π1, … , π8) by means of their corresponding scale factors.
- At the same time, this law includes the complete set of dimensionless variables πj involved in the description of the analyzed physical phenomenon, and the way to obtain these dimensionless variables is the easiest and safest, which cannot be achieved with the rest of the methods mentioned above.
- For simplification, the so-determined πj variables can be further grouped.
- By applying this, very simply, safety and unitary methodology, the unwanted variables will be automatically eliminated; these unwanted

variables are those, whose influence even is insignificant (very small), even had any influence over the analyzed phenomenon.

- After performing searching experimental investigations (strictly on the attached model), by means of the deduced ML became possible obtaining the foresighted (anticipated) parameters for the prototype.
- The deduced ML represents also a very flexible set of information, because one can ignore (eliminate) some elements, if we are looking for a simplified model-prototype correlation; this also constitutes another unique (distinctive) advantage of the MDA, which is not proper to any of the above-mentioned dimensional methods.
- In addition, depending on the concrete conditions available, the strategy of the experimental investigations can be modified, adapted to the new conditions, only by re-seating and reconsidering the sets of independent variables, respectively of the dependent variables.

Customizing the analysis to the heat transfer problem, the authors, in their previous works [27] [28] [61] [63] [68], were able to establish with the help of *MDA* in a simple manner for bars and reticular bar structures (either solid or tubes) a *ML* with 50 elements, considered to be the complete one. Of these, nine characterize the thermal protective (intumescent) paint layer, and the remaining 41 describe the behaviour of the thermally unprotected structural element (or structure).

The variables that can have an influence on the heat transfer in the beam with rectangular or rectangular-hole section are the following: heat amount Q; heat rate $\dot{Q}$; time τ; density ρ; constant-pressure specific heat of air c_p; specific heat capacity (steel, air) C; thermal conductivity (steel, paint coat), along directions x,y,z: λ_x, λ_y, λ_z; thermal diffusivity of air, along directions x,y,z: a_x, a_y, a_z; velocity w_0; dynamic viscosity of air, along directions x,y,z: η_x, η_y, η_z; kinematic viscosity of air, along directions x,y,z: v_x, v_y, v_z; Prandtl number of air, along directions x,y,z: $\mathrm{Pr_x}$, $\mathrm{Pr_y}$, $\mathrm{Pr_z}$; Reynolds number of air, along directions x,y,z: $\mathrm{Re_x}$, $\mathrm{Re_y}$, $\mathrm{Re_z}$; convection heat transfer coefficient, along directions x,y,z: α_{nx}, α_{ny}, α_{nz}; thickness of the paint coat, along directions y,z δ_y, δ_z; volume of bar or paint coat V; area of the bar cross section A_{tr}; lateral area $(x\text{-}z)$ A'_{lat} ; lateral area $(x\text{-}y)$ A''_{lat} ; bar's dimensions L_x, L_y, L_z; shape factor $\varsigma = A_{lat}/V = P/A_{tr}$; where P is the cross-section's perimeter; gravitational acceleration g; temperature variation ΔT or Δt; coefficient of volume expansion (steel, air) β; Nusselt number, along directions x,y,z $\mathrm{Nu_x}$, $\mathrm{Nu_y}$, $\mathrm{Nu_z}$; Grashof number, along direction x $\mathrm{Gr_x}$; Péclet number, along directions x,y,z $\mathrm{Pe_x}$, $\mathrm{Pe_y}$, $\mathrm{Pe_z}$; Biot number, along directions x,y,z $\mathrm{Bi_x}$, $\mathrm{Bi_y}$, $\mathrm{Bi_z}$; Stanton number, along directions x,y,z $\mathrm{St_x}$, $\mathrm{St_y}$, $\mathrm{St_z}$, as well as Fourier number, along directions x,y,z $\mathrm{Fo_x}$, $\mathrm{Fo_y}$, $\mathrm{Fo_z}$.

It is worth noting that the nine elements related to the thermal protection layer can be easily adapted to other types of thermal protection, which will represent the future research directions of the authors.

Compared to these own results, those contained in a world-reference work on *CDA* by Quintier [53], without any desire to criticize him, give a much smaller number of dimensionless variables (related to heat transfer problem), even though they have all aspects of the combustion process and heat transfer have been taken into account. Thus, the temperature, linear dimensions, time, Re, Nu, Pr, the amounts of heat related to convection and conduction, the thickness of the thermal insulation layer were included. However, the author of [53] states that this would be the complete set of the *ML*, which, compared to those mentioned before, represents only a fairly modest part of the dimensionless variables related to the complete *ML*. At the same time, the methodology presented in [53] does not offer any solution in choosing certain variables as independent ones, with the help of which the model attached to the prototype can become a flexible one, as is done with the help of *MDA*.

In the analyzed cases, the bars, considered either unprotected, or protected with an intumescent paint layer, had a simulated fire at their lower ends, i.e., subjected to controlled heat fluxes at their lower ends.

One can mention that as the independent variables the authors considered two sets of variables, namely:

I. $(Q, L_z, \Delta t, \tau, \lambda_{xsteel}, \varsigma)$

II. $(\dot{Q}, L_z, \Delta t, \tau, \lambda_{xsteel}, \varsigma)$

where Q - the invested heat; $\dot{Q}$ - the heat rate; L_z - the beam dimension along the longitudinal direction z; Δt - the temperature variation; τ - the time; $\lambda_{x\ steel}$ - the thermal conductivity; and ς - the shape factor.

Consequently, they deduced two separate *ML*, with their advantages in the model's testing. Either was controlled the invested heat, or the corresponding heat rate, the tested models were subjected to easy-controllable and repeatable tests and based on this: the obtained experimental results were as much as possible credible.

Based on these sets, it is worth noting that the advantage of choosing these two sets of independent variables lies, inter alia, in the following:

- Heating regimes can be chosen independently for prototype and model by:
  - Accepting convenient and well-determined values for the amount of heat introduced into the system (Q or $\dot{Q}$)
  - Setting final temperatures compared to initial ones (Δt)
  - Defining / accepting individual heating times (τ) of the prototype and the model
- Length scales can also be chosen independently (expressed here by L_z, which can be extended to the rest of the dimensions, but it is not mandatory, because the rest of the dimensions are also included in matrix B, which represents a significant reserve for generalizing the model to the prototype)

- The factors ς (shape factor) of the cross sections can be chosen independently in the prototype, respectively in the model
- One can define individual the materials of the prototype and the model (through λ_x), which do not necessarily have to be for both steel, which is also very important to the most favorable experiments (cost price, manufacturing time, test times, etc.)

Afterward, the authors performed, based on searching experimental investigations, a validation of the obtained *ML*s. In this sense, those performed on the rectangular tubular cross-sectional ones [28] [47] [61] [63] [64] [68] [73] [74] are the most significant and are briefly described in the subsequent chapter.

MDA can be used in fire protection and other materials, not only in steel, but also obviously with a suitable λ thermal conductivity. If the results provided by *MDA*, i.e., *ML*, established for a structural element we intend to extend to the real structure, then both the prototype and the model will have corresponding geometries, defining their homologous points.

In this sense, in the initial Dimensional Set, deduced for the singular structural element, the independent variable T_0 or t_0 related to temperature from matrix A remain the same. In addition, in matrix B we have to insert on each direction *x, y* and *z* one new dependent variable T_x, T_y, T_z.

These new variables will allow that at the level of these homologous points of the two structures, based on the corresponding elements of the deduced *ML,* to forecast the thermal variations, for example along the pillar, respectively of the different-oriented crossbars related to an industrial hall.

For the first time, the thermal demands at the level of the fire source (or sources) will be adopted a priori in matrix A, adapting the $T_{0,1}$ and $T_{0,2}$ temperatures, related to prototype, respectively the model, which will define the scale factor of the temperature. Obviously, in this case, both the prototype and the model correspond to a real structure's geometry, e.g., industrial hall with several rooms.

The initial Dimensional Set, deduced for the singular structural element, will be modified/extended by including in matrix B these new T_x, T_y, T_z dependent variables, by means of T^* [°C], related to the homologous points of these new structures.

Following the application of *MDA*, the new *ML* will result, from which, at the level of these T_x, T_y, T_z temperatures, we will have appropriate elements for obtaining the corresponding temperatures from the prototype's homologous points. Finally, the corresponding dimensionless variable will be:

$$\pi_k = (T^*)^1 \cdot (\Delta T)^{-1} = 1 \;\Rightarrow\; T^* = \Delta T \;\Rightarrow\; S_{T^*} = S_{\Delta T}; \text{with } T^* = T_x, T_y, T_z.$$

The proposed set of experimental investigations is carried out exclusively on the model. By substituting, in the corresponding/adequate elements of the *ML*, the

$T_{x,2}$, $T_{y,2}$, $T_{z,2}$ temperatures obtained from the measurements (related to the model), the probable $T_{x,1}$, $T_{y,1}$, $T_{z,1}$ temperatures of the prototype in these areas will result,

i.e.: $\frac{T_2^*}{T_1^*} = S_{\Delta T} \Rightarrow T_1^* = \frac{T_2^*}{S_{\Delta T}}$, for x, y, z.

4 Experimental Results

In order to perform an adequate accuracy and repeatable experimental investigation both on the attached model and the analyzed prototype, the authors conceived an original electric-heated test bench [21] [47] (see Figure 2), having the following main components. By translation, the (***1***) analyzed structural element arrives on the pyramidal trunk shape support (***2***), posed on the rigid frame (***3***) with its legs (***4***). How one can observe in Section A-A, the used (***6***) heating elements, i.e., the twelve Silite rods (connected in series of four items, corresponding to the industrial three phases 380 V power supply), rest on the (*7*) firebricks, respectively on the 0.0254 *m* thick ceramic fiber (***8***) thermal insulation layer. The (***2***) pyramidal trunk is also insulated with the same (***5***) layer. Consequently, the obtained thermal insulation assures a maximum (45…50)°*C* environmental temperature, corresponding to the nominal $t_{0,nom}$=600°*C* heating of the tested element.

In order to perform an adequate validation of the obtained *ML*s, the authors started their experimental investigations based on a real structural element, i.e.: a segment of an industrial hall's column, taken as prototype, and its associated 1:2 and 1:4 reduced scale models.

The temperature fields of these structural elements were monitored by means of several thermo-resistors, type PT 100–402, with 0.150 *m* long terminals and measuring range of (–70…+500°*C*). These thermo-resistors, mounted in the middle of each side of the tested elements, and fixed by means of some *M3* screws, offered, by means of their average indications the measured temperatures at different levels.

Tables 2 and 3, according to Figure 3, present the main dimensions of the tested structural elements, as well as the applied thermo-resistors positioning.

In this process of the higher accuracy validation of the obtained *ML*s, the authors combined these three structural elements, i.e., the 1:1 scale prototype, respectively the 1:2 and 1:4 reduced scales models, considering three sets of prototype-model: (1:1 with 1:2); (1:1 with 1:4), respectively (1:2 with 1:4). The authors performed for all of them (1:1; 1:2 and 1:4 scaled elements) the data acquisition for the involved parameters ($Q, \dot{Q}, t, \tau$, etc.).

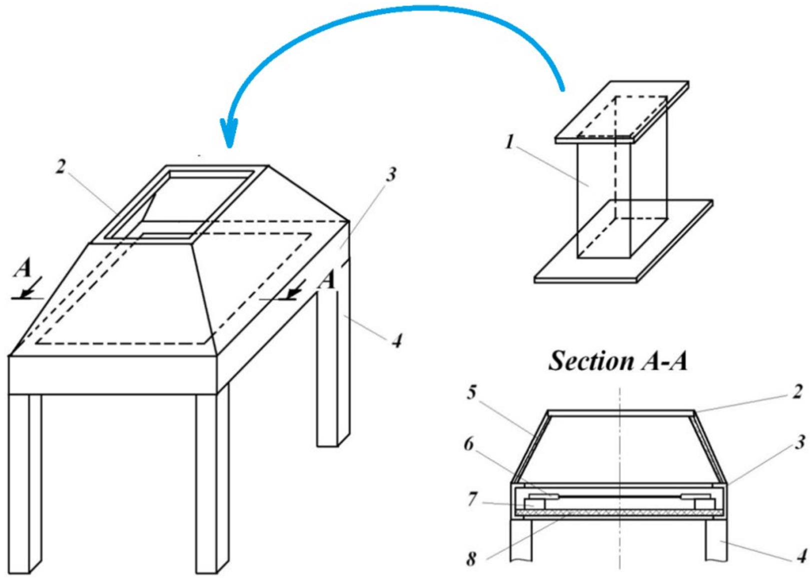


Figure 2
The principia schema of the test bench [21] [47]

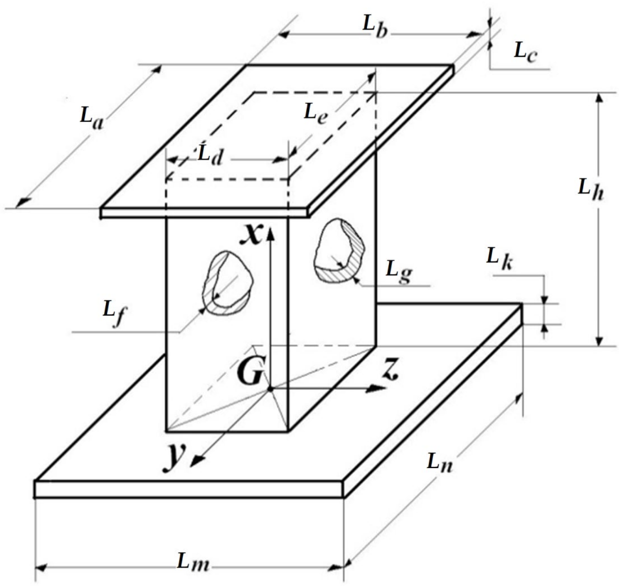


Figure 3
The tested elements dimensions [28] [47] [61] [63] [68]

Table 1
The main dimensions of the tested elements, according to Fig. 3 [28] [47] [61] [63] [68]

	Prototype, at scale 1:1	**Model *I.*, at scale 1:2**	**Model *II.*, at scale 1:4**
Dimensions, in *m*			
L_a	0.370	0.185	0.108
L_b	0.370	0.185	0.108
L_c	0.006	0.003	0.0015
L_d	0.350	0.175	0.0875
L_e	0.350	0.175	0.0875
L_f	0.016	0.008	0.004
L_g	0.016	0.008	0.004
L_h	0.400	0.200	0.100
L_k	0.010	0.005	0.0025
L_m	0.450	0.450	0.450
L_n	0.450	0.450	0.450

Table 2
The co-ordinates of the mounted thermo-resistors, according to Fig. 3 [28] [47] [61] [63] [68]

Prototype, at scale 1:1	**Model *I.*, at scale 1:2**	**Model *II.*, at scale 1:4**
Co-ordinates *x(j)*, in *m*		
0.020	0.020	0.020
0.110	0.060	0.055
0.200	0.105	0.090
0.290	0.150	
0.380	0.190	

The obtained data, depending on the given structural element's role in the analyzed set of prototype-model, were considered as:

- *Directly measured data*, if these variables were elements of matrix A, thus independent variables
- *Calculated data*, if they were elements of matrix B, i.e., dependent variables, and consequently were obtained strictly by calculi with applying of the *ML*, respectively
- *Reference data*, if they serve as a comparative value for the calculated one

In all above-mentioned cases, the authors obtained a very good agreement (correspondence) of the calculated and reference data, detailed in [28] [47] [61] [63] [68]. Consequently, the obtained *ML*s are fully validated and they can be extended for practically any other desired scales.

The heating protocol supposes a systematic temperature growing up to the given nominal temperature $t_{0,nom}$=(100,200,300,400,450,500)°*C*. A *stabilized thermal* *regime* for a given $t_{0,nom}$ nominal temperature was considered, corresponding to the indication of this $t_{0,nom}$ temperature of the last PT 100-402 thermo-element (located at the upper part of the tested structural element), with maximum oscillations (0.2…0.3)°*C* during minimum (120…180)*s*.

5 Numerical Simulation (Finite Elements' Results)

In order to become predictable, the thermal field for a given rectangular-tubular cross-sectional steel structural element, the authors performed a numerical simulation, described hereunder. The Steady-State Thermal Analysis module of ANSYS software accomplished the numerical simulations.

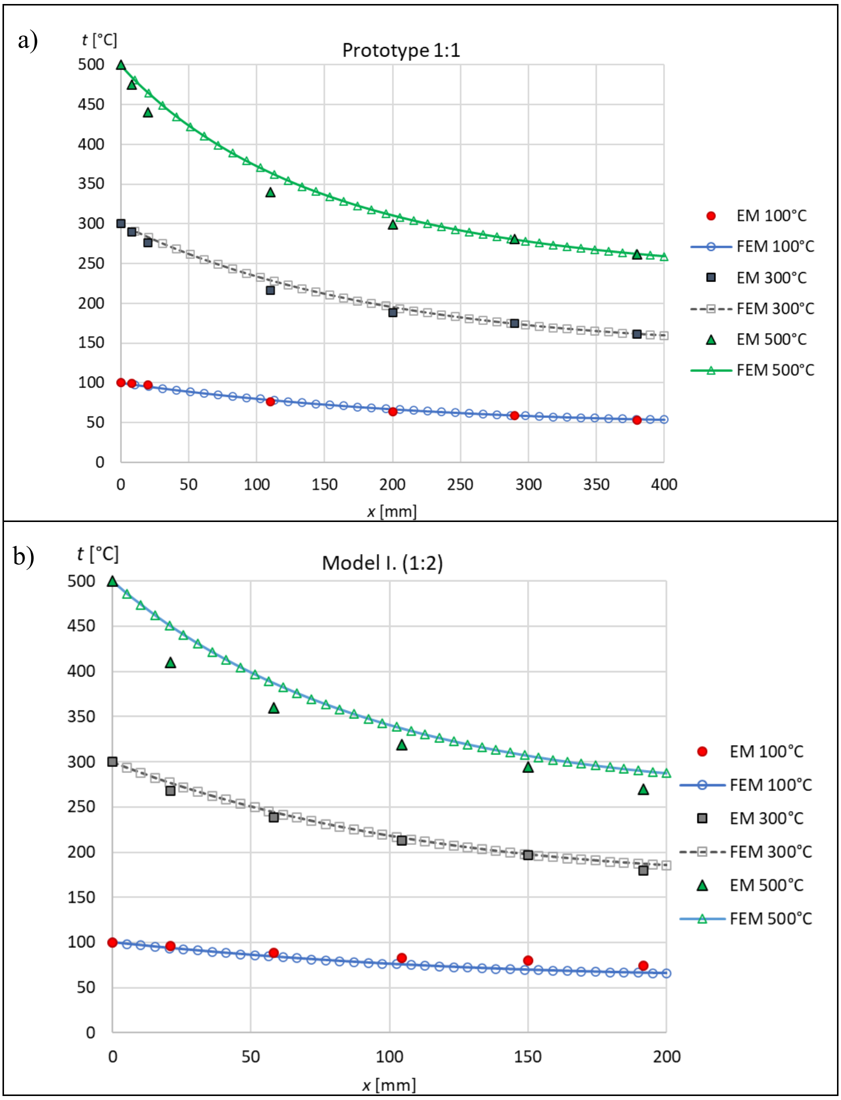
The geometry of the three different models follows the mesh generation. The proper choice of the mesh is very important to obtain realistic results from the numeric simulation. The number of elements of the mesh does not only influence the exact result, but also calculation time. Due to the reduction of grid elements, the fluid domain around the frame structure was not modelled in this study. The prototype (1:1) has an element number of 596,759, while Model I (1:2) consists of 328,160 elements and Model II (1:4) made up of 158,125 cells. In all three cases, hexahedral cells were used.

During the calculations, the value of the heat transfer coefficient (α) between the steel structure wall and the air was determined from measurement, summarized in Table 3. In this table, the *c* index means convection and *r* corresponds to radiation. The top of the structure, closed with a disk ($a \times b$), foreseen with a 5 *mm* hole in the middle, avoids the risk of unwanted deformation during heating. The hole is too small for all the heated air to escape, which means that the temperature of the air inside the structure is higher than that of the outside ambient air, so the heat transfer coefficient is determined separately on the outer and inner surface of the structure.

Table 3
The measured heat transfer coefficient (*c*: convection; *r*: radiation)

T [°C]	α_c [W/(m^{2}K] outside	α_r [W/(m^{2}K] outside	α_c [W/(m^{2}K] inside	α_r [W/(m^{2}K] inside
100	4.182	0.522	3.864	0.377
300	5.441	2.032	4.960	1.574
500	5.992	4.034	5.474	3.274

For different heating conditions, the measurement and calculation results are compared. Figure 4 shows the temperature distribution along the column, where the measurement results are denoted by EM (Experimental Measurement), while the numerical values are denoted by FEM (Finite Element Method). In the case of the prototype, the calculated temperature values are in good agreement with the measured data. Models I and II, where the height and thickness of the column changes, show good agreement with the values measured at 100°C. At 500°C a 16% difference was observed for the (1:4) model (Fig. 4c).



c)

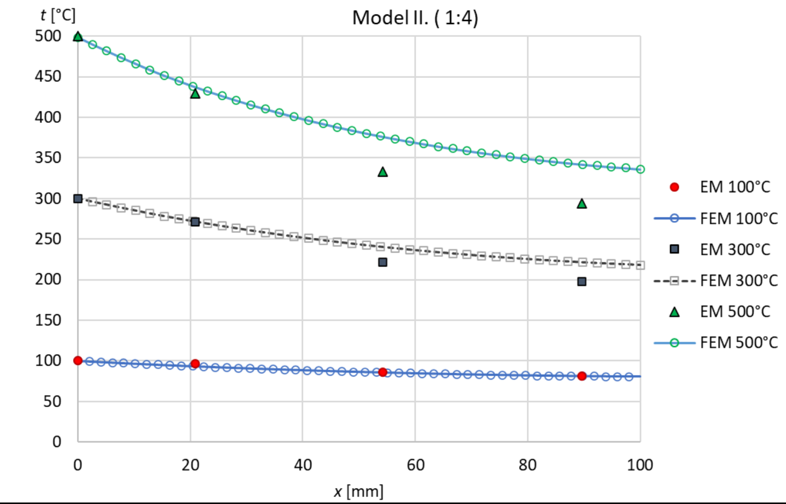


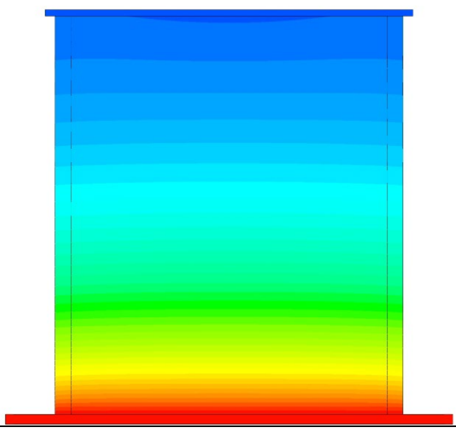
Figure 4

Temperature distribution along the column at three structural elements

Figure 5 shows the temperature distribution on the surface of the three different structural elements at 500°C. Due to comparison, the scale is the same in all three cases. Taking into consideration the rigorous experimental results, the authors were able validating the proposed numerical model, too.

[°C]

a)



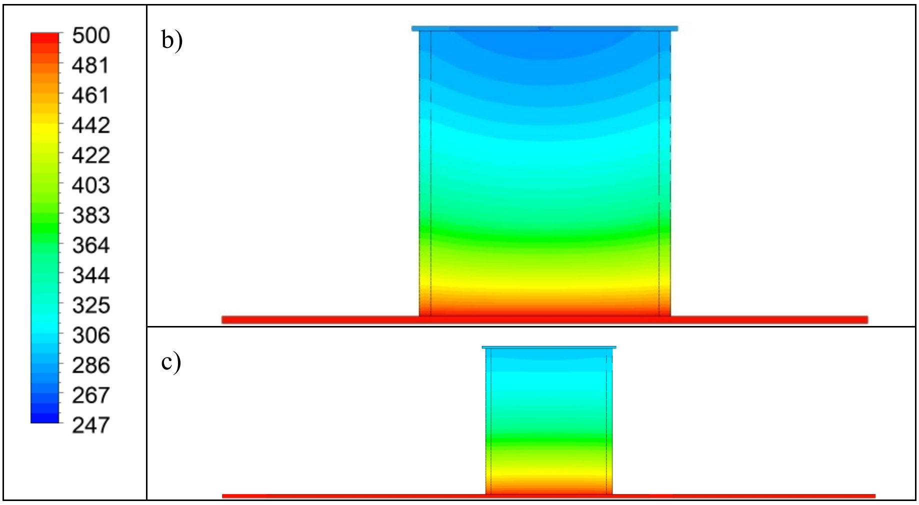


Figure 5
Temperature distribution for (a) prototype (1:1); (b) Model I (1:2); (c) Model II (1:4) at 500°C

Conclusions and Further Goals

The authors present their previous results on fire protection problem, starting from the critical analysis of the involved dimensional methods, widely applied nowadays. Based on their praxis in the field of modelling (in attaching to a given structural element a suitable model), the authors evaluated the advantages and the limits of the applied methods.

They conclude that actually applied dimensional methods, such as Geometric Analogy, Theory of Similarity, respectively Classical Dimensional Analysis, do not satisfy the engineering requirements consisting in flexibility, simplicity, repeatability of the method. One other significant requirement, which is not fulfilled by the analyzed methods, consists in the fact that the applied method has to assure to obtaining the complete Model Law.

Based on the Szirtes' approach, i.e., on the Modern Dimensional Analysis, they deduced and validated for two cases the corresponding complete Model Laws. This experimental-based validation process supposes the conceiving, manufacturing and accurately testing of three structural elements, starting from a real column of an industrial hall. In the present contribution, they initiated to propose a numerical model, validated by means of the previously obtained experimental results.

The numerical modelling was in a good agreement with the accurately performed experimental results. Their further goal consists in extending the obtained results to other significant structural elements subjected to fire.

The comparative FEM-experimental results can serve as a starting point in the real structures' analysis, obtained from elements of the type analyzed before, i.e., solid bars, or tubes/pipes of rectangular sections. As shown in section 3, *MDA* allows the *ML*'s extension to structures made of elements of the type mentioned before, obviously with the appropriate modification of the set of dependent variables in matrix B.

In addition, a generalization of the *MDA* methodology is also possible, to be carried out in the near future by the authors. Similarly, FEM results can also be involved in this extension/generalization process. Additionally, based on the measurements' results, it can be checked whether the numerical model is adequate or needs to be improved for a more accurate description of the heat flow propagation. The authors, will have this aspect in mind and propose the establishment of specific databases, with these theoretical and experimental results, to assist designers and specialists in the field.

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