Non-Classical Problems of Irreversible Deformation in Terms of the Synthetic Theory

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Abstract: The paper is concerned with the generalization of the synthetic theory to the modeling of both plastic and creep deformation. Non-classical problems such as creep delay, the Bauschinger negative effect and reverse creep have been analytically described; the calculated results show satisfactory agreement with experiments. These problems cannot be modeled in terms of classical creep/plasticity theories. The main peculiarity of the generalized synthetic theory consists in the fact that the macro-deformation is highly associated with processes occurring on the micro-level of material.

Keywords: plastic deformation; primary creep; steady-state creep; the Bauschinger negative effect; creep delay; reverse creep

1 Introduction

The work presented herein regards the generalization of the synthetic theory of plastic deformation [18] to the modeling of not only plastic, but also creep (both primary and steady-state) deformation. This theory, which is concerned with small strains of work-hardening metals, incorporates (synthesizes) the Budiansky slip concept and the plastic flow theory developed by Sanders.

The key points of the generalized synthetic theory are:

I It is of both mathematical and physical nature. As a mathematical (formal) theory, the synthetic theory is in full agreement with the basic laws and principles of plasticity, such as Drucker's postulate, the law of the deviator proportionality, the isotropy postulate, etc. [18]. As a physical model, the synthetic theory allows for real processes occurring at the micro-level of material during loading, and the macro-behavior of material is fully governed by these processes. Therefore, the synthetic theory is a two-level theory.

II Independently of the type of deformation (creep or plastic) to be modeled, a single notion, irreversible (permanent) deformation is introduced, i.e. the deformation is not split into "instantaneous" plastic and viscous parts [17]. The manifestation of the plastic or viscous component and their interrelations depend on the concrete loading/temperature-regime. The correctness to use the notion of irreversible deformation follows from the similarity of the mechanism of time-dependent and plastic deformation. Indeed, this mechanism is slips of the parts of crystal grains relative to each other. These slips are induced mainly by the motions of dislocations which, in turn, are induced/accompanied by other micro-structural imperfections (defects) of the crystalline lattice (vacancies, interstitial atoms, etc.). Undoubtedly, the driving forces and concrete configurations of the defects are different under different conditions. Nevertheless, despite the variety of processes occurring in a body subjected to different loading regimes, numerous experiments systematically record the arising of dislocation gliding for any type of inelastic straining. Other facts justifying the similarity of the nature of plastic and time-dependent deformation are (**i**) hydrostatic stress does not affect creep deformation; (**ii**) the axes of principal stress and creep strain rate coincide; (**iii**) no volume change occurs during creep [3]. These observations are the same as those for plastic deformation [6, 7].

III Following the tendency of unified approaches to the determination of irreversible deformation [4, 5], the system of constitutive equations that governs the whole spectrum of inelastic deformation has been worked out**.** In terms of generalized synthetic theory, the universality of this system is based on:

(**i**) a single equation provides the relation between a) micro-irreversible deformation, b) defects of crystalline structure inducing this deformation and c) time. Further, the procedure of the transition from micro- to macro-level is also uniformed: the sum of irreversible micro-strains determines the magnitude of macro-strain.

ii) the hardening rule is set in such a way that the transformation of loading surface obeys a unique rule. In addition, the kinetics of the loading surface transformation is not set a priori but is fully determined by the loading regime.

The objectives of this papers are to demonstrate how, by utilizing the uniformed method, the generalized synthetic theory is capable of embracing both plastic and creep deformation. In addition, some non-classical problems such as creep delay, the Bauschinger negative effect and reverse creep [12] are considered. The investigation of reverse creep is of great importance due to the fact that this phenomenon contradicts the hypothesis of creep potential [3, 13]. The advantages of synthetic theories above classical theories of creep and plasticity are considered.

2 Fundamentals of the Synthetic Theory of Plastic Deformation

The synthetic theory is based on the Budiansky slip concept [2] and the plastic flow theory developed by Sanders [19]. Below, the basic principles of the synthetic theory [18] are briefly reviewed.

A) The establishment of strain-stress relationships takes place in the Ilyushin stress deviatoric space, $\mathbf{S^5}$, [8]. A load is presented by stress-deviator vector, $\vec{\mathbf{S}}$, whose components are defined as

$$S_1 = \sqrt{3/2}S_{xx}\,,\ S_2 = S_{xx}\big/\sqrt{2} + \sqrt{2}S_{yy}\,,\ S_3 = \sqrt{2}S_{xz}\,,$$

$$S_4 = \sqrt{2}S_{xy}\,,\ S_5 = \sqrt{2}S_{yz}\,, \tag{2.1}$$

where S_{ij} (i, j = x, y, z) are the stress-deviator tensor components; $\left|\vec{\mathbf{S}}\right| = 3\sqrt{2}J_2$, where J_2 is the second invariant of stress deviator tensor [7]. Further throughout we will consider the cases when $\vec{\mathbf{S}} \in \mathbf{S^3}$ ($S_4 = S_5 = 0$).

B) ***Yield criterion and yield surface***. One of the key points consists in the construction of planes tangential to the yield surface in $\mathbf{S^5}$ instead of the yield surface itself. The inner-envelope of tangent planes constitutes the yield surface. By making use of this method, a new yield criterion is introduced, which coincides with neither the Tresca nor the von-Mises yield criterion in $\mathbf{S}^5$. At the same time, the new criterion is reduced to the von-Mises yield criterion in $\mathbf{S^3}$ meaning that the trace of the five-dimensional yield surface takes the form of a sphere in $\mathbf{S^3}$ ($S_4 = S_5 = 0$):

$$S_1^2 + S_2^2 + S_3^2 = 2\tau_S^2\,, \tag{2.2}$$

where τ_S is the yield limit of a material in pure shear.

C) ***Loading surface.*** Following Sanders [19], the stress deviator vector shifts planes tangential to the yield surface on its endpoint during loading. The movements of the planes located on the endpoint of stress deviator vector are translational, i.e. without a change in their orientations. Those planes which are not on the endpoint of the stress deviator vector remain unmovable. Despite the fact the $\vec{\mathbf{S}} \in \mathbf{S^3}$, the displacements of planes tangential to the **five-dimensional yield surface** must be considered. On the other hand, the positions of these planes can be set by their traces in $\mathbf{S^3}$. As a result, any plane in $\mathbf{S^3}$ (either tangential to the sphere (2.2) or locating beyond this sphere) is the trace of the plane tangential to the five-dimensional yield surface [18].

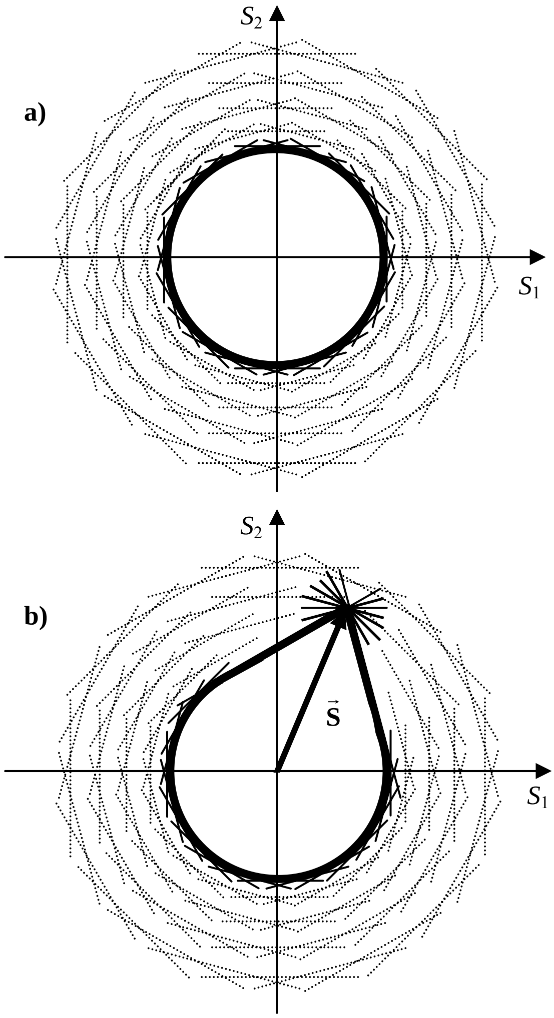

Figure 1
Yield and loading surface in terms of synthetic theory

The loading surface constructed as the inner-envelope of the tangent planes takes the shape fully determined by the current positions of the planes. Therefore, the behavior of the loading surface is not prescribed a priori, but is fully determined by the hodograph of the stress deviator vector.

For simplicity, let S_1-S_2 coordinate-plane play the role of $\mathbf{S}^3$. Then, Fig. 1a illustrates the yield surface (2.2) (circle) in the virgin state of the material. The planes (lines) tangential both to the five-dimensional yield surface and to its trace in $\mathbf{S}^3$ are shown as solid lines. The lines filling up the S_1-S_3 plane beyond the circle (the traces of the planes tangential to only the five-dimensional yield surface) are shown as dotted lines.

Fig. 1b shows loading surface due to the action of vector $\vec{\mathbf{S}} \in \mathbf{S}^{\mathbf{3}}$ which shifts a set of planes. It is easy to see that the corner point arises on the loading surface at the endpoint of $\vec{\mathbf{S}}$ (loading point). This fact is of great importance for the description of the peculiarities of plastic straining at non-smooth (orthogonal) loading trajectories [18] where any theory with regular loading surface has proved to be unsuitable.

The condition that the tangent plane is located on the end-point of the stress deviator vector can be expressed as

$$H_N = \vec{\mathbf{S}} \cdot \vec{\mathbf{N}}\,, \tag{2.3}$$

where H_N is the distance between the origin of coordinates and the tangent plane in $\mathbf{S}^{\mathbf{5}}$; $\vec{\mathbf{N}}$ is the unit vector normal to the tangent plane, which defines the orientation of the plane. If the plane is not reached by $\vec{\mathbf{S}}$, $H_N > \vec{\mathbf{S}} \cdot \vec{\mathbf{N}}$. The distance to plane in $\mathbf{S}^{\mathbf{5}}$ can be expressed through that to its trace in $\mathbf{S}^{\mathbf{3}}$, h_m, as

$$H_N = h_m \cos\lambda\,, \tag{2.4}$$

where index *m* indicates the unit vector, $\vec{\mathbf{m}}$, normal to the tangent plane in $\mathbf{S}^{\mathbf{3}}$:

$$\vec{\mathbf{m}}(\cos\alpha\cos\beta, \sin\alpha\cos\beta, \sin\beta)\,, \tag{2.5}$$

In expression (2.4), λ is the angle between the vectors $\vec{\mathbf{m}}$ and $\vec{\mathbf{N}}$. The angles α and β are shown in Fig. 2.

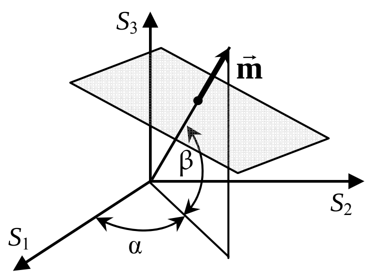

Figure 2
Orientation of normal vector $\vec{\mathbf{m}}$

In addition, the $\vec{\mathbf{N}}$ and $\vec{\mathbf{m}}$ vector components are related to each other as [18]

$$N_k = m_k \cos\lambda\,, \quad k = 1,2,3$$

$$N_1 = \cos\alpha\cos\beta\cos\lambda\,,\quad N_2 = \sin\alpha\cos\beta\cos\lambda\,,\quad N_3 = \sin\beta\cos\lambda\,., \tag{2.6}$$

Therefore, expressions (2.3) and (2.6) give that

$$H_N = \vec{\mathbf{S}}\cdot\vec{\mathbf{m}}\cos\lambda = (S_1 m_1 + S_2 m_2 + S_3 m_3)\cos\lambda\,,\quad \vec{\mathbf{S}} \in \mathbf{S^3}\,. \tag{2.7}$$

As follows from formulae (2.4) and (2.6), if $\lambda = 0$, then $H_N = h_m$ and $N_k = m_k$ ($k = 1,2,3$). This holds true only for the planes which are tangential both to the five-dimensional yield surface and to its trace in $\mathbf{S^3}$. It is these planes with $\lambda = 0$ that govern the transformation of the loading surface in $\mathbf{S^3}$ [18].

D) ***Plastic strain vector components.*** Similarly to the ***Batdorf-Budiansky slip concept***, the synthetic theory is of a two-level nature. Each tangent plane represents an appropriate slip system at a point in a body (microlevel, see Fig. 3), and the plane motion symbolizes an elementary process of plastic deformation within this slip system.

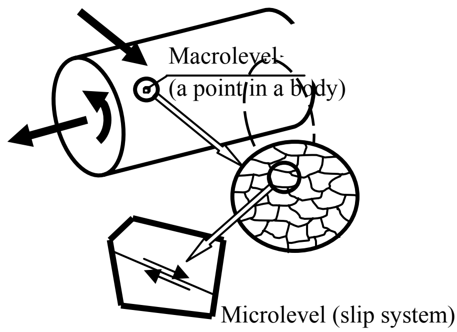

Figure 3
Two levels of the determination of deformation

To define an average, continuous measure of plastic slip within one slip system, we introduce a scalar magnitude, plastic strain intensity (φ_N), is proposed as

$$r\varphi_N = H_N - \sqrt{2}\tau_S = \vec{\mathbf{S}}\cdot\vec{\mathbf{N}} - \sqrt{2}\tau_S\,. \tag{2.8}$$

Formula (2.8) holds true for the planes displaced by the stress deviator vector, i.e. if $H_N = \vec{\mathbf{S}}\cdot\vec{\mathbf{N}}$. If $H_N > \vec{\mathbf{S}}\cdot\vec{\mathbf{N}}$, φ_N is set to be zero. An incremental plastic strain-vector, $d\vec{\mathbf{e}}^S$, (micro plastic deformation on the lower(micro)-level) is assumed to be in the direction of the outer normal to the plane and determined as

$$d\vec{\mathbf{e}}^S = \varphi_N \vec{\mathbf{N}} dV\,. \tag{2.9}$$

In expression (2.9), dV is an elementary volume constituted of the elementary set of planes in $\mathbf{S}^3$ that covered an elementary distance due to an infinitesimal increase in the stress vector [1]:

$$dV = \cos\beta d\alpha d\beta d\lambda . \tag{2.10}$$

The total (macro) plastic strain-vector at a point in a body, $\vec{\mathbf{e}}^S$, is determined as the sum (three-folded integral) of the micro plastic strains 'produced' by movable planes:

$$\vec{\mathbf{e}}^S = \int_V \varphi_N \vec{\mathbf{N}} dV \quad \text{or} \quad \dot{\vec{\mathbf{e}}}^S = \int_V \dot{\varphi}_N \vec{\mathbf{N}} dV \tag{2.11}$$

The strain vector components relate to the strain-deviator tensor components e_{ij} $(i, j = x, y, z)$ as [8]

$$e_1 = \sqrt{3/2} e_{xx}, \quad e_2 = e_{xx} \big/ \sqrt{2} + \sqrt{2} e_{yy}, \quad e_3 = \sqrt{2} e_{xz},$$

$$e_4 = \sqrt{2} e_{xy}, \quad e_5 = \sqrt{2} e_{yz} . \tag{2.12}$$

By using equations (2.6) and (2.10), equation (2.11) becomes

$$e_k^S = \int_\alpha \int_\beta \int_\lambda \varphi_N m_k \cos\lambda \cos\beta d\alpha d\beta d\lambda \quad \text{or}$$

$$\dot{e}_k^S = \int_\alpha \int_\beta \int_\lambda \dot{\varphi}_N m_k \cos\lambda \cos\beta d\alpha d\beta d\lambda , \quad k = 1,2,3 \tag{2.13}$$

The integration in (2.13) must be taken over planes shifted by the stress deviator vector.

3 The Generalization of the Synthetic Theory

To extend the boundaries of the applicability of the synthetic theory, the following is proposed.

I) To reflect the well-known fact that the defects of the crystal structure of metals are the ***carriers*** of irreversible deformation, a new notion, defect intensity (ψ_N), is introduced. ψ_N represents an average continuous measure of the defects (dislocations, vacancies, etc.) generated by irreversible deformation within one slip system.

II) To model the influence of loading rate upon irreversible straining, a new function of time and loading rate, the so called integral of non-homogeneity (I_N), is introduced. By considering the physical nature of irreversible deforming, the formula for I_N will be strictly derived in 3.1.2.

III) Instead of (2.8), the defect intensity is related to H_N and I_N :

$$\psi_N = H_N - I_N - \sqrt{2}\tau_P = \vec{\mathbf{S}} \cdot \vec{\mathbf{N}} - I_N - \sqrt{2}\tau_P , \qquad (3.1)$$

where τ_P is the creep limit of material in pure shear. In terms of the generalized synthetic theory, the yield and creep limits are related to each other by equation derived further (see 4.1)). The establishment of a relationship between ψ_N and H_N is fully logical, because the distance H_N characterizes the degree of work-hardening. Indeed, the greater the plane distance, the greater a stress deviator vector is needed to reach the plane, i.e. to induce irreversible strain.

IV) To establish a relationship between irreversible deformation, defects and time (t), the following equation is proposed

$$d\psi_N = r d\varphi_N - K\psi_N dt , \qquad (3.2)$$

where, r is the model constants and K is a function of homological temperature, Θ, and $\left|\vec{\mathbf{S}}\right|$ (see 5). The units of quantities in (3.2) are $[\psi_N] = \text{Pa}$, $[\varphi_N] = 1$, $[r] = \text{Pa}$ and $[K] = \sec^{-1}$.

In what follows, the parameter of non-homogeneity and the detailed analysis of the proposed generalizations are considered.

3.1 The Integral of Non-Homogeneity

3.1.1 Local Micro-Stresses and the Physics of Primary Creep

As is well known, plastic deformation is accompanied by the formation of dislocation pile-ups, tangles of dislocations, unmovable jogs, grains boundaries, etc (the nucleation of dislocations is also observed at elastic deformation). These defect-formations, being of strongly local character, raise an uneven stress/strain distribution through the microstructure of metal that, in turn, leads to considerable distortions of the crystal lattice where the strain energy is mainly stored.

The considerable non-homogeneity and concentration of micro-strains/stresses of the second and third kind were observed in experiments performed on specimens of pure copper, iron and titanium [9]. The experiments show that both stresses and strains are distributed non-homogeneously within grains (under both elastic and plastic loading). In addition, if the strain is greater than its average value through the grain, then the stress inducing this strain is smaller than average stress and vise verse. At the same time, the total over- and under-loading is equal to zero.

The non-homogeneous stress distribution makes the metal structure more unstable than in an annealed state. Once favorable conditions arise (for example, if the stress stops increasing), the relaxation of crystal lattice distortions is observed. It is the difference between the local and average stresses that is the driving force for the relaxation that occurs mainly due to spontaneous slips in grains induced by the movements of dislocation. Indeed, under thermal fluctuations, locked and tangled dislocations and the obstructions in their way themselves become progressively movable, thereby promoting the development of deformation. Therefore, the time dependent relaxation of the crystal lattice distortions governs the progress of the primary creep deformation.

The local stresses arising around the lattice distortions we will call local microstresses. These stresses display the following properties: 1) they, being directly correlated with dislocation density, make the material stronger; 2) the greater the loading rate, the greater the local stresses; 3) they are unstable: as soon as favorable conditions arise, they decrease with time. It must be noted that the local microstress relaxation is also observed during slow loading.

Therefore, on the one hand, the local microstresses cause the "rate-hardening" of the material during active loading but, on the other hand, they can relax resulting in the softening of the material. Time-dependent macro-deformation is the result of the concurring processes of the hardening and softening.

3.1.2 The Integral of Non-Homogeneity as the Mathematical Measure of Local Stresses

To establish a relation between the microstress non-homogeneity and elastic strain energy, consider an elementary volume of body (treated as point) consisting of a large number of microparticles. Let $\overline{\sigma}_{kq}^{0}$ denote the average stress deviator tensor components (macrostress) acting at the given point. The microstress non-homogeneity can be expressed through the stress deviator tensor components acting in each microparticle, $\overline{\sigma}_{kq}$, as

$$\overline{\sigma}_{kq} = \overline{\sigma}_{kq}^{0} + \overline{\sigma}_{kq}', \tag{3.3}$$

where $\overline{\sigma}_{kq}'$ are random quantities expressing the over/under-loading in each particle. We set the reaction of $\overline{\sigma}_{kq}'$ on the change in the average stress as

$$d\overline{\sigma}_{kq}' = C_{ijkq} d\overline{\sigma}_{ij}^{0}, \tag{3.4}$$

where C_{ijkl} are random numbers that vary from particle to particle, which are assumed to be independent from $\overline{\sigma}_{ij}^{0}$. Let us suppose that all random numbers C_{ijkl} have an identical distribution function, F, and are independent of each other. Since $d\overline{\sigma}_{ij}^{0}$ are macroscopic (average) stress components, the mathematical expectation of parameters C_{ijkl} is

$$\int_{-\infty}^{\infty} C_{ijkl} F\left(C_{ijkl}\right) dC_{ijkl} = 0 \quad \Sigma \tag{3.5}$$

Formula (3.5) means that the total over/under-loading with respect to the average stress is equal to zero. In addition,

$$\int_{-\infty}^{\infty} F\left(C_{ijkl}\right) dC_{ijkl} = 1 \quad \Sigma, \tag{3.6}$$

As was pointed out earlier, the local stresses are unstable and can relax with time. The equation governing their time-dependent behavior is proposed as

$$d\overline{\sigma}_{ij}' = C_{ijpq} d\overline{\sigma}_{pq}^{0} - p\overline{\sigma}_{ij}' dt \,. \tag{3.7}$$

The first item on the right side in the above formula characterizes the rise of $\overline{\sigma}_{ij}'$ given by (3.4); term $-p\overline{\sigma}_{ij}'dt$ gives the time-dependent decrease of microstresses, which is taken to be proportional to $\overline{\sigma}_{ij}'$. The solution of the differential equation (3.7) for $\overline{\sigma}_{ij}'$ is

$$\overline{\sigma}_{ij}' = C_{ijkq} I_{kq}(t), \quad I_{kq}(t) = \int_{0}^{t} \frac{d\overline{\sigma}_{kq}^{0}}{ds} \exp\left(-p(t-s)\right) ds \,. \tag{3.8}$$

Now, expression (3.3) becomes

$$\overline{\sigma}_{ij} = \overline{\sigma}_{ij}^{0} + C_{ijkq} I_{kq}(t). \tag{3.9}$$

As is well known, elastic strain energy can be expressed as

$$U = \frac{1}{12G}\left[\left(\overline{\sigma}_{xz} - \overline{\sigma}_{yy}\right)^2 + \left(\overline{\sigma}_{yy} - \overline{\sigma}_{zz}\right)^2 + \left(\overline{\sigma}_{zz} - \overline{\sigma}_{xx}\right)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right], \tag{3.10}$$

where G is the elastic shear modulus. By substituting stresses $\overline{\sigma}_{ij}$ from (3.9) into (3.10), we obtain

$$U = \frac{1}{12G}\Big\{\left(\overline{\sigma}_{xx}^{0} + C_{xxkq}I_{kq} - \overline{\sigma}_{yy}^{0} - C_{yykq}I_{kq}\right)^2 + \left(\overline{\sigma}_{yy}^{0} + C_{yykq}I_{kq} - \overline{\sigma}_{zz}^{0} - C_{zzkq}I_{kq}\right)^2 +$$
$$+\left(\overline{\sigma}_{zz}^{0} + C_{zzkq}I_{kq} - \overline{\sigma}_{xx}^{0} - C_{xxkq}I_{kq}\right)^2 + \tag{3.11}$$
$$+6\left[\left(\tau_{xy}^{0} + C_{xykq}I_{kq}\right)^2 + \left(\tau_{yz}^{0} + C_{yzkq}I_{kq}\right)^2 + \left(\tau_{xz}^{0} + C_{xzkq}I_{kq}\right)^2\right]\Big\}$$

The mean value of U is determined by the following relation

$$\langle U \rangle = \underbrace{\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty}}_{36} UF(C_{xxxx})\dots F(C_{xzxz})dC_{xxxx}\dots dC_{xzxz}\,. \tag{3.12}$$

$\langle U \rangle$ can be decomposed in two parts:

$$\langle U \rangle = J_1 + J_2\,, \tag{3.13}$$

$$J_1 = \underbrace{\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty}}_{36} U_0 F(C_{xxxx})\dots F(C_{xzxz})dC_{xxxx}\dots dC_{xzxz} = U_0\,, \tag{3.14}$$

$$J_2 = \frac{1}{12G}\underbrace{\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty}}_{36}\left(C_{xxxx}^2 I_{xx}^2 + 2C_{xxxx}I_{xx}\overline{\sigma}_{xx}^{0} + \dots\right)F(C_{xxxx})\dots F(C_{xzxz})dC_{xxxx}\dots dC_{xzxz} =$$
$$= \frac{1}{12G}I_{xx}^2\int_{-\infty}^{\infty} C_{xxxx}^2 F(C_{xxxx})dC_{xxxx}\cdot\underbrace{\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty}}_{35} F(C_{xxxy})\dots F(C_{xzxz})dC_{xxxy}\dots dC_{xzxz} + \tag{3.15}$$
$$+\frac{1}{6G}I_{xx}\overline{\sigma}_{xx}^{0}\int_{-\infty}^{\infty} C_{xxxx}F(C_{xxxx})dC_{xxxx}\cdot\underbrace{\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty}}_{35} F(C_{xxxy})\dots F(C_{xzxz})dC_{xxxy}\dots dC_{xzxz} + \dots$$

where U_0 is the strain energy for the case of homogeneous stress distribution determined by formula (3.10) at $\overline{\sigma}_{ij} = \overline{\sigma}_{ij}^{0}$. In arriving at the result (3.14), expression (3.6) has been taken into account. In order to evaluate integral J_2, it is enough to investigate its first two terms. Indeed, formula (3.5) implies that all the integrals in (3.15) containing C_{ijkl} are equal to zero. The integrals containing C_{ijkl}^2 give the variance of random numbers C_{ijkl}, B_1:

$$\int_{-\infty}^{\infty} C_{ijkl}^2 F\left(C_{ijkl}\right) dC_{ijkl} = B_1 \tag{3.16}$$

As a consequence,

$$J_2 = \frac{2B_1}{G}\left(I_{xx}^2 + I_{yy}^2 + I_{zz}^2 + 2I_{xy}^2 + 2I_{yz}^2 + 2I_{zx}^2\right).$$

Finally, expression (3.13) is

$$\langle U \rangle = U_0 + \frac{2B_1}{G}\left(I_{xx}^2 + I_{yy}^2 + I_{zz}^2 + 2I_{xy}^2 + 2I_{yz}^2 + 2I_{zx}^2\right). \tag{3.17}$$

By subtracting from the right-hand side in (3.17) the expression $2B_1/3G\left(I_{xx} + I_{yy} + I_{zz}\right)^2$, which is equal to zero due to $\overline{\sigma}_x^0 + \overline{\sigma}_y^0 + \overline{\sigma}_z^0 = 0$, we obtain

$$\langle U \rangle = U_0 + \\ + \frac{2B_1}{3G}\left[\left(I_{xx} - I_{yy}\right)^2 + \left(I_{yy} - I_{zz}\right)^2 + \left(I_{zz} - I_{xx}\right)^2 + 6\left(I_{xy}^2 + I_{yz}^2 + I_{zx}^2\right)\right]. \tag{3.18}$$

Substituting I_{ij} from (3.8) into (3.18) and converting the variables σ_{ij} to the stress vector components S_n by formula (2.1), the expression for the mathematical expectation of elastic strain energy is obtained as

$$\langle U \rangle = U_0 + \frac{2B_1}{3G}\sum_{n=1}^{5}\left[\int_0^t \frac{dS_n}{ds}\exp\left(-p(t-s)\right)ds\right]^2 \tag{3.19}$$

The value of $\langle U \rangle$ is seen to consist of two parts; the term U_0 corresponds to homogeneous stress distribution and the second term characterizes the time-dependent deviation of stresses from their average value. If a body is ideally homogeneous, the distribution functions of random numbers C_{ijkl} degenerate in the Dirac delta-function and, according to (3.16), we obtain $B_1 = 0$. As seen from formula (3.19), $\langle U \rangle$ depends not only on the rate of stress vector components $\dot{S}_n$ at a given instant, but on its values for the all history of loading as well. For the case $\dot{S}_n = const$,

$$\langle U \rangle = U_0 + \frac{2B_1}{3G}\dot{S}_n\dot{S}_n\left[\int_0^t \exp\left(-p(t-s)\right)ds\right]^2. \tag{3.20}$$

Since $\dot{S}_n\dot{S}_n = \dot{S}^2$ (S denotes the length of stress vector),

$$\langle U\rangle = U_0 + \frac{2B_1}{3G}\left[\int_0^t \frac{dS}{ds}\exp(-p(t-s))ds\right]^2 . \tag{3.21}$$

In the case that the stress deviator vector has only one non-zero component, expressions (3.19) and (3.21) are identical. We take the square root in the right-hand side in relation (3.21) to be the scalar measure of micro-non-homogeneity:

$$I = B\int_0^t \frac{dS}{ds}\exp(-p(t-s))ds\,, \quad B = \sqrt{\frac{2B_1}{3G}} = const\,. \tag{3.22}$$

We will term I as the integral (parameter) of non-homogeneity. To work with the integral of non-homogeneity on the microlevel of material, we replace S in (3.22) by scalar product $\vec{\mathbf{S}}\cdot\vec{\mathbf{N}}$. This replacement reflects the fact that the driving force of plastic flow within a slip system is not the whole macro-stress vector $\vec{\mathbf{S}}$ but only its projection $\vec{\mathbf{S}}\cdot\vec{\mathbf{N}}$ (resolved stress). Thus, finally, the characteristic of local micro-stresses has the form

$$I_N = B\int_0^t \frac{d\vec{\mathbf{S}}}{ds}\cdot\vec{\mathbf{N}}\exp(-p(t-s))ds\,. \tag{3.23}$$

In contrast to (3.22), the adopted integral (3.23) depends on angles α, β, and λ thereby allowing for the orientation of tangent planes in the Ilyushin subspace $\mathbf{S^3}$.

Let us analyze the integral of non-homogeneity for the loading regime shown in Fig. 4 ($\vec{\mathbf{v}} = d\vec{\mathbf{S}}/dt = const$). On the first portion of the loading, formula (3.23) gives

$$I_N(t) = B(\vec{\mathbf{v}}\cdot\vec{\mathbf{N}})\int_0^t \exp(-p(t-s))ds = \frac{B(\vec{\mathbf{v}}\cdot\vec{\mathbf{N}})}{p}[1-\exp(-pt)], \quad t\in[0,t_1]\,. \tag{3.24}$$

As seen from (3.24), $I_N(t)$ grows from the very beginning of loading. If we take the loading rate $v = S/t$ to be infinitely large, we can approximate the function $\exp(-pS/v)$ in (3.24) by the Tailor series, which results in the following relation

$$I_N = B(\vec{\mathbf{S}}\cdot\vec{\mathbf{N}}) \text{ as } v\to\infty\,. \tag{3.25}$$

For the range $t > t_1$ when $\vec{\mathbf{v}} = 0$, let us split the range of integration in formula (3.23) into two parts, $[0,t_1]$ and $[t_1,t)$:

$$I_N(t) = B(\vec{\mathbf{v}} \cdot \vec{\mathbf{N}}) \int_0^{t_1} \exp(-p(t-s))ds = \frac{B(\vec{\mathbf{v}} \cdot \vec{\mathbf{N}})}{p} [\exp(pt_1) - 1] \exp(-pt), \quad t \geq t_1 . \ (3.26)$$

From (3.24) and (3.26) the following properties of the integral of non-homogeneity can be indicated: (i) during loading, it grows proportionally to the loading rate; (ii) it decreases under constant loading. Therefore, the time-dependent behavior of the integral of non-homogeneity correlates with that of local microstresses.

The condition $I_N = 0$ symbolizes the end of transformations occurring in the crystal lattice under primer creep and transition to the steady-state stage of creep.

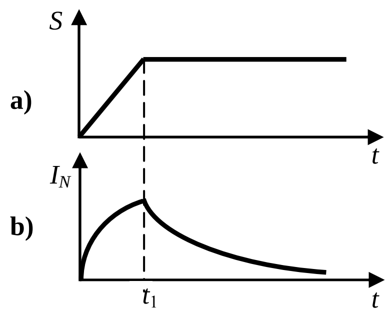

Figure 4
I_N-t diagram

Intermediate discussion. The sum of the two quantities in equation (3.1), $\psi_N + I_N = \vec{\mathbf{S}} \cdot \vec{\mathbf{N}} - \sqrt{2}\tau_P$, characterizes the straining state of the material and determines the stress to induce irreversible deformation. The parameters ψ_N and I_N have a common trait; they can relax in time (see (3.2), and (3.26)). On the other hand, there is an essential difference between these quantities: ψ_N expresses the number of defects that produce irreversible deformation, whereas I_N characterizes the loading-rate-dependent development of these defects. The integral I_N behaves in a different way depending on loading regime: a) under loading, I_N symbolizes the load-rate strengthening of material; b) under constant stress, I_N drops expressing the lattice distortion relaxation that results in time-dependent, progressive deformation. The behavior of ψ_N and I_N is governed by different equations; I_N depends on loading-rate-history, formula (3.23), whereas ψ_N is related to irreversible deformation by (3.2).

3.2 System of Constitutive Equations

Formulae (3.1), (3.23), (3.2) and (2.13) constitute the base of the generalized synthetic theory:

$\psi_N = H_N - I_N - \sqrt{2}\tau_P,$	**(A)**
$I_N = B\int_0^t \frac{d\vec{\mathbf{S}}}{ds}\cdot\vec{\mathbf{N}}\exp(-p(t-s))ds,$	**(B)**
$d\psi_N = rd\varphi_N - K\psi_N dt,$	**(C)**
$e_k^i = \int_\alpha\int_\beta\int_\lambda \varphi_N m_k \cos\lambda\cos\beta d\alpha d\beta d\lambda$ or $\dot{e}_k^i = \int_\alpha\int_\beta\int_\lambda \dot{\varphi}_N m_k \cos\lambda\cos\beta d\alpha d\beta d\lambda,\quad k = 1,2,3$	**(D)**

The procedure of the calculation of irreversible strain vector components (e_k^i ***) is the following, (i) at a given stress deviator vector and loading rate, the defect intensity is determined by (A) and (B), (ii) the strain intensity can be found by (C) and, finally, (iii) formula (D) gives the values of strain(rate) vector components.***

Expression (C) is one of the most important in terms of the generalized synthetic theory. It reflects the well-known fact that the defect intensity $d\psi_N$ ***grows with the increase in deformation (*** $rd\varphi_N$ ***) and simultaneously decreases (relaxes) with time (*** $-K\psi_N dt$ ***). Owing to (C), one does not need to split a deformation into its "instantaneous" (plastic) and viscous parts; both of them develop simultaneously. The degree of this development depends on concrete loading- and temperature-regimes. That is why, further throughout, we will use a single notion, irreversible deformation, by which we mean the deformation progressing with time (independently of whether we consider very short-termed loadings at plastic deformations or loadings lasting several hours or days as in creep tests).***

The (A)-(D) system governs all types of irreversible deformation for any state of stresses and loading regimes.

Regard must be paid to the integration limits in formula (D). When founding the boundary values of angles α, β and λ, one must follow a single rule – only tangent planes which are on the endpoint of the stress deviator tensor produce irreversible strains. Since the plane distances are related to ψ_N, the limits of integration in (D) are determined from the conditions $\psi_N = 0$,

$$0 \le \lambda \le \lambda_1, \quad \cos\lambda_1(\alpha,\beta) = \frac{\sqrt{2}\tau_P}{\left(\vec{\mathbf{S}} \cdot \vec{\mathbf{m}}\right) - I_N}. \tag{3.27}$$

The condition $\lambda_1 = 0$ gives the equation for the boundary values of angles α and β:

$$\vec{\mathbf{S}} \cdot \vec{\mathbf{m}} - I_N = \sqrt{2}\tau_P. \tag{3.28}$$

4 Irreversible Deformation in Terms of the Generalized Synthetic Theory

4.1 Creep-Yield Limit Relation

Consider the case of arbitrary stress state, and assume that the loading rate is infinitely small so that the parameter of non-homogeneity tends to zero. If an irreversible deformation does not occur, $\psi_N = 0$, formula (A) gives that the tangential planes in $\mathbf{S}^3$ ($\lambda = 0$) are equidistant from the origin of coordinates:

$$h_m(\alpha,\beta) = H_N(\alpha,\beta,\lambda = 0) = \sqrt{2}\tau_P. \tag{4.1}$$

The above formula implies that the creep surface (creep locus in $\mathbf{S}^3$ setting the condition for the onset of first plastic flow at infinitesimal loading rate), being constructed as the inner-envelope of tangential planes, takes the form of the sphere of radius $\sqrt{2}\tau_P$:

$$S_1^2 + S_2^2 + S_3^2 = S_P^2, \quad S_P = \sqrt{2}\tau_P. \tag{4.2}$$

For the case of pure shear, expression (2.1) gives $S_3 = \sqrt{2}\tau_{xz}$ and $S_1 = S_2 = 0$ meaning that the stress vector $\vec{\mathbf{S}}(0,0,S_3)$ acts along S_3-axis. Let τ_P denote the value of shear stress when vector $\vec{\mathbf{S}}\left(0,0,\sqrt{2}\tau_P\right)$ reaches the first tangential plane on the sphere (4.2). Since this plane is perpendicular to S_3-axis ($\beta = \pi/2$ and $\lambda = 0$), formulae (2.5) and (2.7) give $H_N = \sqrt{2}\tau_P$. Therefore, τ_P expresses the creep limit of metal in pure shear.

Now, our goal is to establish the relation between τ_S and τ_P. It is worth starting with the case of pure shear. Let the loading be of constant rate, $v = \dot{S}_3 = \dot{S} = const$, $S = \left|\vec{\mathbf{S}}\right|$. Then expression (3.23) becomes

$$I_N = Bv \sin\beta \cos\lambda \int_0^t \exp(-p(t-s))ds \,. \tag{4.3}$$

Until the stress vector reaches the tangential planes, $\psi_N = 0$, formula (A) takes the form

$$H_N = I_N + S_P \,. \tag{4.4}$$

By integrating in (4.3) and inserting the result of the integration in (4.4), we obtain

$$H_N = \frac{Bv}{p}[1-\exp(-pt)]\sin\beta\cos\lambda + S_P \,, \quad t \in [0, t_1] \text{ in Fig. 4a.} \tag{4.5}$$

As seen from (4.5), the plane distances grow due to the increase in I_N. This means that formula (4.5) describes the movements of planes in the direction away from the origin of coordinate. Since these movements are not caused by the "pushing" action of stress deviator vector, they do not cause irreversible strain. The inner envelope of the planes with distances from (4.5) is shown in Fig. 5a (only tangent planes with $\lambda = 0$ are shown). As seen, the action of the integral of non-homogeneity does not result in the formation of a corner point.

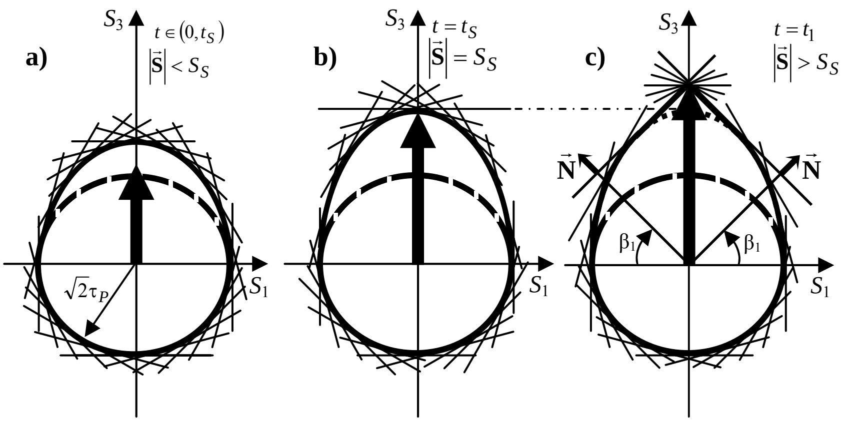

Figure 5

The transformation of yield (a and b) and loading (c) surface

Let S_S denote the length of the stress deviator vector which at time t_S ($t_S \in [0, t_1]$) reaches the first plane ($\beta = \pi/2$ and $\lambda = 0$), i.e. the plastic flow starts developing (Fig. 5b). For this plane, formula (2.7) gives $H_N = S_S$. The replacement of H_N by S_S in (4.5) leads to the equation for S_S:

$$S_S = \frac{Bv}{p}(1-\exp(-pt_S)) + S_P \,, \quad S_S = vt_S \tag{4.6}$$

The plot of S_S as the function of v constructed on the base of (4.6) is shown in Fig. 6. As follows from Eq. (4.6), curve $S_S = S_S(S_P)$ has a horizontal asymptote, which is at a distance of $S_P/(1-B)$ from the abscissa that corresponds to the case of an infinitely large loading rate.

For $S > S_S$, the stress deviator vector translates some set of plane (Fig. 5c), and angle β_1 gives the boundary planes on the endpoint of $\vec{\mathbf{S}}$.

Further, let us find the yield limit for an arbitrary proportional loading with a constant loading rate. Now, expression (3.23) is

$$I_N = B\cos\lambda \times$$
$$\times \int_0^t \left(\frac{dS_1}{ds}\cos\alpha\cos\beta + \frac{dS_2}{ds}\sin\alpha\cos\beta + \frac{dS_3}{ds}\sin\beta \right)\exp[-p(t-s)]ds \, . \tag{4.7}$$

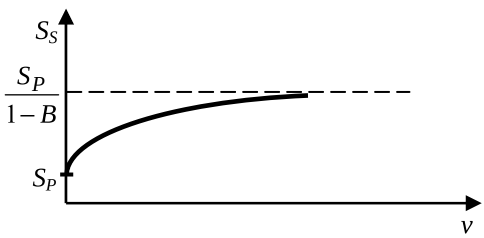

Figure 6
Yield limit vs loading rate plot

In the direction of the action of the stress deviator vector whose orientation is given by angles α_0 and β_0, relations $\cos\alpha_0 = S_1 \cdot \left(S_1^2 + S_2^2\right)^{-1/2}$ and $\sin\beta_0 = S_3 \cdot S^{-1}$ hold true and formula (4.7) yields the form

$$I_{N_0} = B\cos\lambda \int_0^t \frac{S_i}{S}\frac{dS_i}{ds}\exp[-p(t-s)]ds \, . \tag{4.8}$$

Since $\dot{S}$ can be expressed as $\dot{S} = \frac{dS}{dt} = \frac{S_i}{S}\frac{dS_i}{dt}$, Eq. (4.8) gives

$$I_{N_0} = B\cos\lambda \int_0^t \frac{dS}{ds}\exp[-p(t-s)]ds \, . \tag{4.9}$$

The integral I_{N_0} is identical to that from (4.3) at $\beta = \pi/2$ meaning that formula (4.6) is applicable to the determination of yield limit via the creep limit for an arbitrary state of stress.

Summarizing, formula (4.6) is of great importance due to the fact that it allows working with only one material constant, creep limit. In contrast to classical theories of plastic/creep deformation that use separately yield limit or creep limit depending on the problem to be solved, the generalized synthetic model is constructed in such a way that the creep limit plays the role of the material constant, while the yield limit is a function of loading rate.

4.2 The Modeling of Irreversible Deformation

Consider the case of proportional loading when the loading trajectory is a straight line in $\mathbf{S}^3$. Further, let the plot of $S(t)$ have the form as in Fig. 4. Since the synthetic theory provides the fulfillment of the law of the deviator proportionality [14-16, 18], the formulae obtained for the case of, e.g., pure shear are fully applicable (up to constants) to arbitrary rectilinear loading path in $\mathbf{S}^3$.

For the case of pure shear, expressions (A), (2.5) and (2.7) give the defect intensity as

$$\psi_N = [S_3 - I]\sin\beta\cos\lambda - S_P = \left(\frac{\Omega}{a} - 1\right)S_P, \quad S_3 > S_S, \tag{4.10}$$

$$\Omega = m_3 \cos\lambda = \sin\beta\cos\lambda, \tag{4.11}$$

$$a = \frac{S_P}{S_3 - I}. \tag{4.12}$$

In formulae (4.10) and (4.12)

$$I = B\int_0^t \frac{dS_3}{ds}\exp[-p(t-s)]ds, \tag{4.13}$$

$$I\big|_{t=t_1} \equiv I_1 = \frac{Bv}{p}\left[1 - \exp(-pt_1)\right], \quad v = \dot{S}_3 = \dot{S} = const, \tag{4.13a}$$

$$I\big|_{t>t_1} \equiv I_2 = \frac{Bv}{p}\left[\exp(pt_1) - 1\right]\exp(-pt). \tag{4.13b}$$

According to (3.27) and (3.28), the defect intensity in expression (4.10) is positive for

$$0 \le \alpha \le 2\pi, \quad \beta_1 \le \beta \le \pi/2, \quad 0 \le \lambda \le \lambda_1, \quad \cos\lambda_1 = \frac{\sin\beta_1}{\sin\beta}, \quad \sin\beta_1 = a \tag{4.14}$$

The loading surface at $t = t_1$ is shown in Fig. 5c or 7a.

The defect intensity increment is expressed from (4.10) as

$$d\psi_N = (dS_3 - dI)\Omega . \tag{4.15}$$

Beyond the angles-diapason given by (4.14), we have $\psi_N = d\psi_N = 0$ and $\varphi_N = d\varphi_N = 0$.

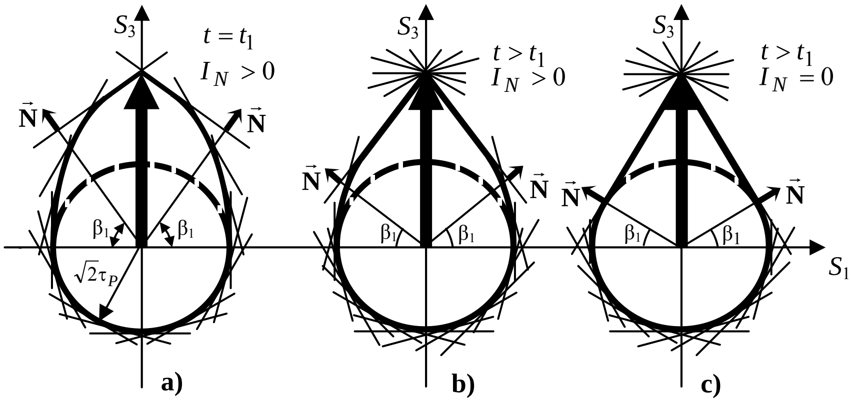

Figure 7
Kinetics of loading surface at creep

Consider the transformation of the loading surface for $t > t_1$ when $\dot{S}_3 = 0$. For the tangent planes that are beyond the diapason (4.14), formula (A) at $\psi_N = 0$ gives

$$H_N = S_P + I_2 \sin\beta\cos\lambda . \tag{4.16}$$

Because of the descending character of I_2, we infer that H_N decreases for $t > t_1$ meaning that planes that are not on the endpoint of the stress deviator vector at $t = t_1$ start to move towards the origin of the coordinates. These movements result in the greater number of planes becoming located at the endpoint of the stress deviator vector. This situation is illustrated by Fig. 7b from which it is seen that the boundary angle β_1 determined by (4.14) and (4.13b) decreases with time. As integral I_2 tends to zero, formula (4.16) gives $H_N = S_P$ meaning that the planes stop moving and the boundary angle β_1 takes its minimal value (Fig. 7c).

Further, formula (B) gives the strain intensity as

$$r d\varphi_N = d\psi_N + K\psi_N dt = (dS_3 - dI)\Omega + KS_P\left(\frac{\Omega}{a} - 1\right)dt . \tag{4.17}$$

Finally, formulae (C) gives the increment in irreversible-strain-vector-component, Δe_3^i, as

$$\Delta e_3^i = \frac{1}{2r}\int_0^{2\pi} d\alpha \int_{\beta_1}^{\pi/2} \sin 2\beta d\beta \int_0^{\lambda_1} \Delta\varphi_N \cos\lambda d\lambda\ , \tag{4.18}$$

where $\Delta\varphi_N$ is given by (4.17). In formula (4.18), the symbol Δ stands for the time-dependent increment of Δe_3^i and $\Delta\varphi_N$. By integrating over α, β and λ in (4.18), we obtain

$$\Delta e_3^i = a_0\left[\Delta\Phi(a) + K\Phi(a)\Delta t\right], \tag{4.19}$$

where

$$a_0 = \frac{\sqrt{2}\pi\tau_p}{3r} = const,\ \Phi(a) = \frac{\arccos a}{a} - 2\sqrt{1-a^2} + a^2 \ln\frac{1+\sqrt{1-a^2}}{a},\ \Phi(1) = 0\ . \tag{4.20}$$

The analysis of (4.20) shows that the function Φ is in inverse proportion with its argument *a*, Fig. 8.

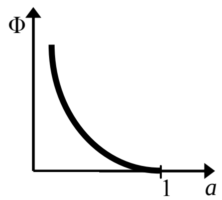

Figure 8
Φ(*a*) function

By integrating over time in (4.19), we obtain the formula for the irreversible strain component in pure shear as:

$$e_3^i = a_0\left[\Phi(a) + \int_{t_S}^{t} K\Phi(a)dt\right]. \tag{4.21}$$

To evaluate the integral (4.21), one needs to know the function $K(S_3(t),\Theta)$. This question will be considered in detail in 5.

Following the law of deviator proportionality [14], formula (4.21) can be rewritten for the case of an arbitrary stress state as

$$e_k^i = a_0\left[\Phi(a) + \int_{t_S}^{t} K\Phi(a)dt\right]\frac{S_k}{S} \qquad k = 1,2,3 \tag{4.22}$$

where, instead of (4.12),

$$a = \frac{S_P}{S - I}, \tag{4.23}$$

and I is calculated by (4.13a) and (4.13b) where $v = \dot{S} = const$.

Formula (4.22) is of a general character; it is applicable to the modeling of any type of deformation, both plastic and unsteady/steady-state creep. At $t = t_1$, we obtain the plastic strain vector components; at $t > t_1$ we get the total, plastic and creep, strain components.

4.3 The Analysis of the System of Constitutive Equations. Partial Cases

1) Consider the case of steady-state creep when $dS = 0$ and $I_N = 0$. It is clear that expression (4.15) gives $d\psi_N = 0$, i.e. the defect intensity (density) does not change during the steady state creep, reflecting the well-known fact that the steady-state creep deformation develops under the equilibrium between the processes of hardening and softening. Therefore, formula (C) gives the constant strain intensity rate:

$$r\dot{\varphi}_N = K\psi_N = const \quad , \quad K(S, \Theta) = const \,. \tag{4.24}$$

Another consequence from conditions $dS = 0$ and $I_N = 0$ is $a = S_P / S = const$ and $\Phi(a) = const$ (see (4.20) and (4.23)). According to (4.22), the steady-state creep strain(rate) components, e_k^P, can be written as

$$e_k^P = a_0 \Phi(a) \frac{S_k}{S} + a_0 K \Phi(a) \frac{S_k}{S} t \quad \text{or} \quad \dot{e}_k^P = a_0 K \Phi(a) \frac{S_k}{S} = const \tag{4.25}$$

where $a_0 \Phi(a) \cdot (S_k / S)$ is the value of strain at the end of primary creep. Formula (A) shows that the plane distances do not change with time, meaning that the steady state creep deformation is “produced” by the set of motionless planes which are located on the endpoint of the stress deviator stress (Fig. 7c). Since function K appears in the formula for steady-state creep rate, we can infer that it takes very small values, and the manifestation of the second term in (4.22) becomes material only under long-termed loadings. At the same time, it is important to emphasize that the role of the time integral in (4.22) grows with the increase in the duration of loading especially at elevated temperatures.

2) On the basis of the above, we can neglect the second term in (4.22) or the term $K\psi_N \, dt$ in formula (C) when plastic or unsteady state creep strains are investigated. This is absolutely justifiable due to the fact that the second term in (4.22) is comparable with the term $a_0 \Phi(a)$ only under very long-termed loading (at least several tens of hours). Therefore, formula (C) at $K = 0$ takes the form

$$rd\varphi_N = d\psi_N \tag{4.26}$$

and expression (4.22) gives

$$e_k^i = a_0\Phi(a)\frac{S_k}{S}, \tag{4.27}$$

where e_k^i is the total, plastic + primary creep, strain components. The function $\Phi(a)$ is not to be thought of as being of a time-independent nature because its argument a contains the integral of non-homogeneity, which regulates the time-dependent development of deformation.

Formula (4.26) reflects another well-known fact that the increase in plastic deformation causes that in the defects of crystal lattice leading to the work-hardening of material and, consequently, the plastic deformation progress requires an increase in acting stress.

The plastic deformation (e_k^S) at the end of active loading ($t = t_1$) is calculated by (4.27) in which

$$a \equiv a_1 = \frac{S_P}{S - I_1}. \tag{4.28}$$

The deformation produced for $t > t_1$, when $dS/dt = 0$, is calculated by (4.27) in which

$$a \equiv a_2 = \frac{S_P}{S - I_2}. \tag{4.29}$$

The analysis of expressions (4.27) and (4.20) shows that the increase in plastic deformation during active loading is modeled by the growth of S in (4.28) (since the condition $S > S_S$ is hold, the growth in S prevails over that in I_1). Further, the progress of deformation for $t \geq t_1$ is regulated by the decrease of I_2 in (4.29). The condition $I_2 \to 0$ symbolizes the end of primary creep.

Pure primary creep strain components (without the initial plastic strain components e_k^S), e_k^C, are calculated as the difference between the total and plastic strain components:

$$e_k^C = a_0\left[\Phi(a_2) - \Phi(a_1)\right]\frac{S_k}{S}. \tag{4.30}$$

It is easy to see that

$$e_k^S + e_k^C = a_0\Phi\left(\frac{S_P}{S}\right)\frac{S_k}{S} = const \text{ as } I_2 \to 0. \tag{4.31}$$

As follows from (4.31), the synthetic theory states that a material possesses some reserve of irreversible deformation which can be manifested in the form of plastic or creep deformation dependent on the loading rate in active loading. Formulae (4.28), (4.29) and (4.27) lead to the following relations between the magnitudes of plastic and primary creep: the growth in loading rate, on the one hand, leads to a decrease in plastic deformation, but, on the other hand, causes the increase in the magnitude of primary creep strain. Another conclusion is that slow loading does not result in following primary creep at all due to $I = 0$ at slow loading.

It is worthwhile to emphasize that only the function Φ is applicable to the calculation of any type of deformation.

3) Consider the case when a complete or partial unloading follows the loading which has produced some irreversible deformation. It is clear that $d\varphi_N = 0$ in unloading and formula (C) becomes

$$d\psi_N = -K\psi_N\,dt \ . \tag{4.32}$$

The solution of the differential equation above is

$$\psi_N = \psi_{N0}\exp(-Kt), \tag{4.33}$$

where ψ_{N0} is the defect intensity accumulated during the initial irreversible straining. Expression (4.32) describes the process of defects relaxation.

4.4 Loading Criterion

While the limits of integration in formula (D) for proportional loading can be determined relatively simply, this is not the case for arbitrary (curvilinear) loading paths. To express analytically the integration limits for curvilinear loading paths is a very difficult task and, consequently, computer assisted methods must be applied. Nevertheless, a general criterion for the development of irreversible straining must be formulated. Let a current stress vector $\vec{\mathbf{S}}$ have produced some irreversible strain, i.e. some set of tangent planes are on its endpoint. For these planes, formulae (A) and (2.3) give

$$S_P + \psi_N + I_N = \vec{\mathbf{S}}\cdot\vec{\mathbf{N}} \ . \tag{4.34}$$

If the vector $\vec{\mathbf{S}}$ acquires increment $d\vec{\mathbf{S}}$, for planes that are on the endpoint of vector $\vec{\mathbf{S}} + d\vec{\mathbf{S}}$ we have

$$S_P + \psi_N + d\psi_N + I_N + dI_N = \vec{\mathbf{S}}\cdot\vec{\mathbf{N}} + d\vec{\mathbf{S}}\cdot\vec{\mathbf{N}} \ . \tag{4.35}$$

Therefore, formulae (4.34) and (4.35) give that

$$d\psi_N = d\vec{\mathbf{S}}\cdot\vec{\mathbf{N}} - dI_N \ . \tag{4.36}$$

We propose the following criterion: the planes that produce irreversible strains due to a given vector $\vec{\mathbf{S}}$ continue to do this due to the vector $\vec{\mathbf{S}} + d\vec{\mathbf{S}}$ if for these planes $d\psi_N \geq 0$:

$$d\vec{\mathbf{S}} \cdot \vec{\mathbf{N}} - dI_N \geq 0 . \tag{4.37}$$

Eq. (4.37) must be applied to the determination of integration limits in formula (D) for arbitrary loading paths.

As seen from the series of Rusinko's works [14-16], the synthetic theory demonstrates good agreement with experimental data.

5 Steady-State Creep. Function K

The steady-state creep rate is governed by expression (4.25), which for the case of uniaxial tension is

$$\dot{e}_1^P = Ka_0\Phi(a), \quad a = \sigma_P / \sigma_x . \tag{5.1}$$

The function *K* is proposed as the product of two functions:

$$K = K_1(\Theta)K_2(\sigma). \tag{5.2}$$

Let us split the range of homological temperature into three diapasons: $0 < \Theta \leq \Theta_1$ (low temperature), $\Theta_1 \leq \Theta \leq \Theta_2$ (elevated temperature), and $\Theta_2 \leq \Theta \leq \Theta_3$ (high temperature). The values of Θ_i are $\Theta_1 \approx 0{,}25$, $\Theta_2 \approx 0{,}5$ and $\Theta_3 \approx 0{,}7$ for pure metals and $\Theta_1 \approx 0{,}3$, $\Theta_2 \approx 0{,}55$ and $\Theta_3 \approx 0{,}75$ for alloys. The range $\Theta_3 \leq \Theta \leq 1$ is not considered below.

Within the range $0 < \Theta < \Theta_1$, the temperature is not enough for the thermal activation of dislocation motion, so steady creep does not occur, $K_1(\Theta) = 0$. For $\Theta_2 \leq \Theta \leq \Theta_3$ the dislocation climb is a dominating mechanism of creep. Since the rate of dislocation climb is controlled by the intensity of the diffusional processes, it is natural to assume that the function $K_1(\Theta)$ is proportional to the quantity of migrating (activated) atoms that regulate the vacancy-motion intensity. The relative quantity of activated atoms is

$$\int_{U_0}^{\infty} pdU = \exp\left(-\frac{U_0}{RT}\right), \tag{5.3}$$

where U_0 is the atom-migration-activation energy. In arriving at the result (5.3) we have utilized the Maxwell-Boltzmann energy distribution $p = \frac{1}{RT}\exp\left(-\frac{U}{RT}\right)$. Thus, (denoting through $\widetilde{T}$ the melting point of metal):

$$K_1(\Theta) = \exp\left(-\frac{U_0}{RT}\right) = \exp\left(-\frac{U_0}{R\widetilde{T}\Theta}\right). \tag{5.4}$$

Within the range of elevated temperatures, $\Theta_1 \le \Theta \le \Theta_2$, various processes govern the steady-state creep, none dominating over another. Therefore, the establishing of function $K_1(\Theta)$ from physical reasoning can lead to unreliable results. As a result, we propose the linear form of function $K_1(\Theta)$ so that $K_1(\Theta_1) = 0$ and $K_1(\Theta_2)$ takes the form of expression (5.4) (Fig. 9):

$$K_1(\Theta) = \frac{\Theta - \Theta_1}{\Theta_2 - \Theta_1}\exp\left(-\frac{U_0}{R\Theta_2\widetilde{T}}\right) \text{ for } \Theta_1 < \Theta < \Theta_2 . \tag{5.5}$$

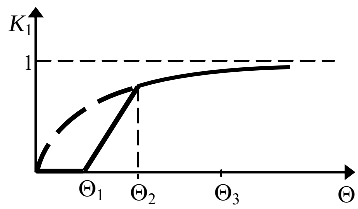

Figure 9
K_1 (Θ) function

Function K_2 can be found by making use of the empirical formula for steady-state creep rate in uniaxial tension:

$$\dot{\varepsilon}_x^P = Cf(\Theta)\sigma_x^k, \quad \dot{\varepsilon}_x^P = 0 \text{ at } \sigma_x < \sigma_P . \tag{5.6}$$

It is clear that expressions (5.1) and (5.6) give $\dot{\varepsilon}_x^P = 0$ as $\sigma_x < \sigma_P$.

Further, as follows from (4.20), function Φ behaves as $\pi\sigma_x/(2\sigma_P)$ for $\sigma_P/\sigma_x \to 0$ and the strain-rate in (5.1) (together with (2.12)) can be written as

$$\dot{\varepsilon}_x^P = \frac{\pi^2}{9r}K_1(\Theta)K_2(\sigma_x)\sigma_x . \tag{5.7}$$

Taking function f in (5.6) to be in the form of K_1 and equating the right-hand sides of formulae (5.6) and (5.7) to each other, we obtain

$$K_2(\sigma_x) = \frac{9Cr}{\pi^2}\sigma_x^{k-1} . \tag{5.8}$$

6 Creep Delay

Consider the case when a loading regime is as in Fig. 10a.

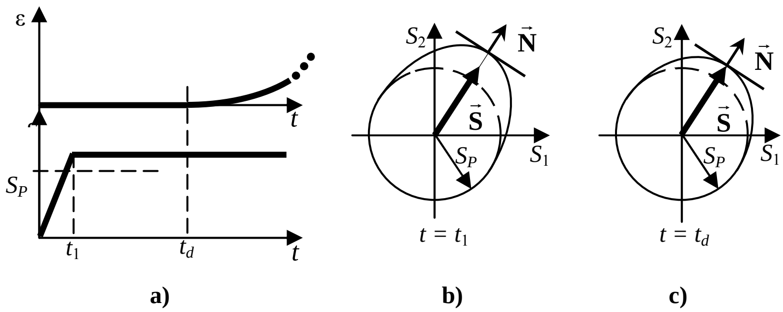

Figure 10
Creep delay

Let the modulus of stress deviator vector at $t = t_1$ ($S > S_P$) not reach any tangential plane (Fig. 10b), i.e. there is no plastic deformation ($\psi_N = 0$ and $\varphi_N = 0$ for all the planes). According to formula (3.26), once $dS/dt = 0$, the integral of non-homogeneity starts to decrease and formula (A) becomes

$$H_N = S_P + \frac{vB}{p}\left[\exp(pt_1) - 1\right]\exp(-pt), \quad t > t_1 . \tag{6.1}$$

The formula above means that the tangent planes move back. The instant that a first plane touches the endpoint of the stress vector, $t = t_d$, symbolizes the start of creep deformation (Fig. 10c). The period of time when the creep deformation is absent, $[0, t_d]$, is referred to as creep delay [10]. It is clear that the first plane to be on the endpoint of the stress deviator vector is perpendicular to it, $H = \vec{\mathbf{S}} \cdot \vec{\mathbf{N}} = S$. Replacing H_N in (6.1) by S gives the following equation for t_d:

$$S_P + \frac{vB}{p}\left[\exp(pt_1) - 1\right]\exp(-pt_d) = S \tag{6.2}$$

As seen from (6.2), the duration of creep delay grows with the loading rate and is absent at slow loading when $I_N = 0$.

6.1 Steady-State Creep as a Function of Initial Deformation

According to formulae (5.1), (5.4), (5.5) and (5.8), the steady-state creep rate is a single-valued function of acting stress and temperature independently of whether the stress magnitude exceeds the yield limit of the material or not. This is in full agreement with numerous experiments. In this context, the investigations of Namestnikov, V. and Chvostunkov, A. [11] carried out as long ago as in the 1960s are of great importance. They deal with the comparison of the strain-hardening creep theory to experimental results for the cases when a creep deformation develops from an elastic or plastic state. First, the case when the creep diagram starts with some initial plastic deformation is considered. The model constants are determined so that the calculated steady-creep rates best fit experimental ones. As it has turned out, if we use these constants for the case when the creep deformation develops from an elastic state, the calculated results show considerable discrepancy with experiments.

Let us investigate, in terms of the generalized synthetic theory, whether the presence or absence of the initial plastic deformation affects the steady-state creep rate. Consider the case when at a given temperature one and the same stress deviator vector produces or not plastic deformation at $t = t_1$ (Figs. 11a and 11b) in the loading regime shown in Fig. 4a. Such a situation can be obtained at different loading rates. Following the techniques of the construction of loading surface discussed in 4.1. and 4.2, it is easy to see that the loading surfaces for the steady-state creep state are identical in both cases (Figs. 11c and 11d). This simple illustration shows that the generalized synthetic theory provides experimentally confirmed results that a secondary creep rate is a single-valued function of stress and temperature.

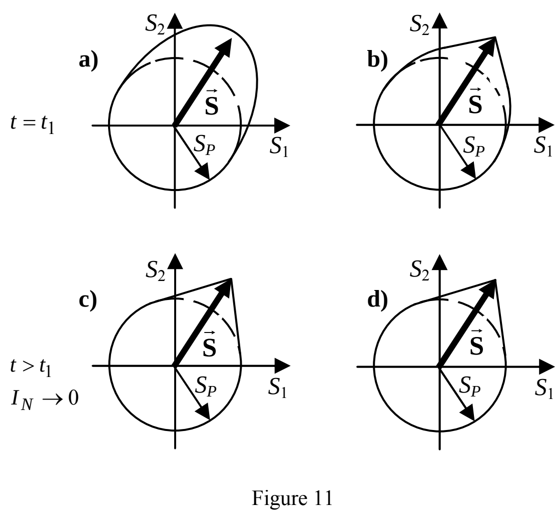

Figure 11

Identical steady-state creep surfaces with different initial straining states

Therefore, in contrast to classical creep theories, the generalized synthetic theory leads to correct results independently of whether the creep deformation develops from a plastic or elastic state (after some creep-delay-period).

7 The Bauschinger Negative Effect

The Bauschinger effect – an initial plastic deformation of one sign reduces the resistance of the material with respect to a subsequent plastic deformation of the opposite sign – is explained by the fact that the repulsive forces, which arise within dislocations conglomeration generated in initial loading, hasten the onset of the plastic deformation of opposite sign and, consequently, the smaller stresses are needed to induce plastic deformation. Starting from a certain initial-plastic-strain, the so-called Bauschinger negative effect is observed [7] when the compressive plastic deformation starts to develop at a positive magnitude of acting stress, Fig. 12a.

In order to allow for the Bauschinger effect, one must replace the equation proposed in [18] by the following:

$$\psi_{-N} = -\psi_N , \tag{7.1}$$

where an index -N symbolizes the plane whose outward-pointing normal vector $-\vec{\mathbf{N}}$ is in the opposite direction to the vector $\vec{\mathbf{N}}$.

a)

b)

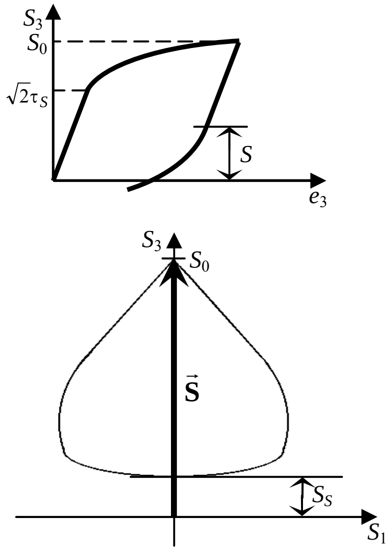

Figure 12
Bauschinger negative effect

The parameter of non-homogeneity for tangent planes with normal vectors $-\vec{\mathbf{N}}$, I_{-N}, is

$$I_{-N} = B\int_0^t \frac{d\vec{\mathbf{S}}}{ds} \cdot \left(-\vec{\mathbf{N}}\right)\exp[-p(t-s)]ds = -B\int_0^t \frac{d\vec{\mathbf{S}}}{ds} \cdot \vec{\mathbf{N}}\exp[-p(t-s)]ds = -I_N \quad (7.2)$$

It is clear that a rate-hardening occurring due to a plastic loading does not exert influence upon that in the following loading of opposite sign. To rephrase this in terms of synthetic theory, we say that if I_N is positive, then I_{-N} is set to be zero and vice verse.

The distance to the planes with vectors $-\vec{\mathbf{N}}$, on the basis of expressions (A), (7.1) and (7.2), can be written as

$$H_{-N} = S_P + \psi_{-N} + I_{-N} = S_P - \psi_N - I_N . \quad \text{(E)}$$

Formula (E) symbolizes that an initial plastic deformation of one sign reduces the resistance of material with respect to a subsequent plastic deformation of the opposite sign. Indeed, distance H_{-N} decreases with the growth in defect intensity ψ_N and integral I_N due to the plastic straining in directions $\vec{\mathbf{N}}$. The decrease in H_{-N} means that tangential planes with normals $-\vec{\mathbf{N}}$ near the origin of coordinates. If the initial loading is of such a magnitude that $\psi_N + I_N > S_P$, the distance to the planes calculated by formula (E) becomes negative, meaning that the planes with normals $-\vec{\mathbf{N}}$ have gone over the origin of coordinate, i.e. the Bauschinger negative effect is manifested. Fig. 12b shows the loading surface whose lower part is constructed on the basis of formula (E) for the case of the Bauschinger negative effect in pure shear.

Formula (E), which governs the relation between the hardening/softening processes occurring in opposite directions, must be included into the system of constitutive equations (A)-(D).

8 Reverse Creep

Consider the time-dependent deformation of a specimen of aluminum alloy PA4 (chemical composition 0.7-1.2% Mg, 0.6-1.0% Mn, 0.7-1.2% Si, 0.5% Fe, the rest Al) under the stepwise uniaxial loading shown in Fig. 13 [12]. The $\varepsilon - t$ curve consists of the following portions:

(1-2) unsteady creep under constant tensile stress σ_1 at room temperature for $t \in [0, t_c]$ (portion 0-1 is the initial plastic deformation due to σ_1);

(2-3) the drop of stress by the amount of $\Delta\sigma$ that results in the plastic ***compressive!*** strain $\Delta\varepsilon^S$; this is the manifestation of the Bauschinger negative effect;

(3-4) ***compressive reverse creep!*** Despite the stretching stress acts, the specimen undergoes compressive creep deformation for $t \in [t_c, t_c + t_r]$;

(4-5) the creep deformation does not progresses for the range $t \in [t_c + t_r, t_c + t_r + t_d]$;

beyond point 5, for $t > t_c + t_r + t_d$, the creep in the direction of acting stress is resumed.

As stated in [12], the reverse creep is observed only if the Bauschinger negative effect takes place.

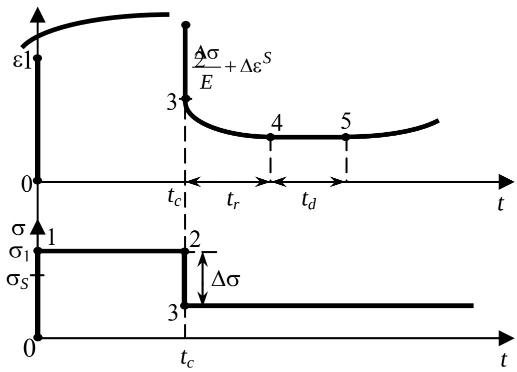

Figure 13
Creep diagram under stepwise loading

Let the magnitude of time-periods t_r and t_d be the main task of this Section.

For the case of uniaxial tension, according to expression (2.1), the stress deviator tensor components are $S_1 = \sqrt{2}\sigma_1/\sqrt{3}$ and $S_2 = S_3 = 0$, i.e. vector $\vec{\mathbf{S}}(S_1,0,0)$ is co-directed with S_1-axis. To determine t_r and t_d, it is quite sufficient to study the displacements of two planes with normals $\vec{\mathbf{N}}$ and $-\vec{\mathbf{N}}$, which are tangential to sphere (4.2) in the virgin state ($\lambda = 0$) and perpendicular to the vector $\vec{\mathbf{S}}$. These planes will be denoted by **I** and **I'**, respectively. The perpendicularity of planes **I** and **I'** to S_1-axis implies that the orientations of their normals $\vec{\mathbf{N}}$ and $-\vec{\mathbf{N}}$ are set by angles $\alpha = 0$ and $\beta = 0$, and $\alpha = \pi$ and $\beta = 0$, respectively.

Formulae (2.5) and (2.7) give that

$$H_N = \vec{\mathbf{S}} \cdot \vec{\mathbf{m}} \cos\lambda = S_1 m_1 \cos\lambda = S_1\Omega, \quad \Omega = m_1 \cos\lambda = S_1 |\cos\alpha| \cos\beta \cos\lambda \qquad (8.1)$$

It is clear that $\Omega = 1$ for planes **I** and **I'** and, further throughout, all formulae will be written at $\Omega = 1$.

By making use of the basic formulae of the generalized synthetic theory, let us study the positions of planes **I** and **I'** on each portion of loading (the indexes in further formulae follow the marks in Fig. 13). Fig. 14 illustrates the positions of planes **I** and **I'**. In Fig. 14 a, b, d and e, together with planes **I** and **I'**, the set of planes shifted by the stress deviator vector are also shown. Since we restrict ourselves only to the determination of t_r and t_d, the number of these planes is immaterial and thus they are shown purely schematically.

Portion 0-1. The plane **I** is displaced by the stress deviator vector $\vec{\mathbf{S}}(S_1,0,0)$ (Fig. 14a). Assuming the loading rate on potion ***0-1*** to be infinitely large, formulae (3.25) and (A), together with (8.1), give that

$$I_{N1} = BS_1 > 0\,, \quad H_{N1} = S_1\,, \quad \psi_{N1} = S_1(1-B) - S_P\,. \tag{8.2}$$

The distance to the plane **I'** can be calculated by formula (E) as

$$H_{-N1} = S_P - \psi_{N1} = 2S_P - S_1(1-B)\,. \tag{8.3}$$

In arriving at the result (8.3) we have taken into account that $I_{-N1} = 0$. Dependent on the values of S_P, S_1 and B, the distance H_{-N1} can take both positive and negative values.

Portion 1-2. The distance to plane **I** remains unchangeable for $0 \le t \le t_c$ due to the fact that it continues to be on the endpoint of the stress deviator vector. Expressions (3.26) and (A) are

$$I_{N(1-2)} = BS_1 \exp(-pt)\,, \tag{8.4}$$

$$\psi_{N(1-2)} = S_1(1 - B\exp(-pt)) - S_P\,, \quad \psi_{N2} = \psi_{N(1-2)}\Big|_{t=t_c}\,. \tag{8.5}$$

The distance to the plane **I'** is governed by formula (E) at $I_{-N(1-2)} = 0$ and formula (8.4):

$$H_{-N(1-2)} = S_P - \psi_{N(1-2)} = 2S_P - S_1(1 - B\exp(-pt))\,,$$

$$H_{-N2} = H_{-N(1-2)}\Big|_{t=t_c}\,. \tag{8.6}$$

To ensure the arising of the Bauschinger negative effect at the unloading in portion ***2-3,*** we require that $S_1 \rangle\rangle S_P$ so that the magnitude of distance H_{-N2} is negative, i.e. plane **I'** has gone over the origin of coordinates. The positions of planes **I** and **I'** at $t = t_c$ are shown in Fig. 14b.

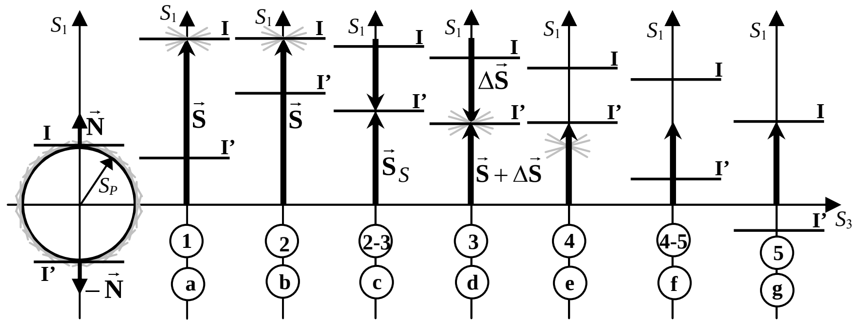

Figure 14
Positions of planes I and I'

Portion 2-3 (stress drop). The stress increment vector $\Delta\vec{\mathbf{S}}$ shifts a set of planes with normals $-\vec{\mathbf{N}}$ (plane **I'** is also among them) thereby producing the compressive plastic-strain-increment $\Delta\varepsilon^S$. Since $\Delta\varepsilon^S$ is produced under the action of positive stress, $\vec{\mathbf{S}}+\Delta\vec{\mathbf{S}}$, it is clear that the Bauschinger negative effect occurs. Let us find a yield limit, S_S, in the partial unloading. To do this, we need to determine the distance to the plane **I'** when the vector $\Delta\vec{\mathbf{S}}$ reaches this plane. At $\Delta S = S_1 - S_S$, the compressive plastic strain starts to develop (Fig. 14c).

Before we turn to the determination of S_S, special attention must be paid to the integral of non-homogeneity I_{-N}. Since the reverse creep is of a compressive nature, its development can be modeled only by means of planes with normals $-\vec{\mathbf{N}}$. Therefore, we require that I_{-N3} be positive. This means that the reverse creep will develop if the compressive (negative) plastic deformation has occurred, i.e. the material needs to obtain some strain energy which can be released in the form of time-dependent deformation. To meet condition $I_{-N3} > 0$, we require that the integral of non-homogeneity I_{N3} be negative. I_{N3}, due to the stress drop of ΔS, is $I_{N3} = B[S_1 \exp(-pt_c) - \Delta S]$. The inequality $I_{N3} < 0$ holds true if to require that

$$\Delta S > S_1 \exp(-pt_c). \tag{8.7}$$

As a result,

$$I_{-N3} = -I_{N3} = B[\Delta S - S_1 \exp(-pt_c)] > 0. \tag{8.8}$$

The yield limit S_S is equal to the magnitude of the stress vector when it reaches the plane **I'**, which is at the following distance from the origin of coordinate (Fig. 14c):

$$H_{-N} = \vec{\mathbf{S}} \cdot (-\vec{\mathbf{N}}) = -S_S \tag{8.9}$$

Since the planes with normals $\vec{\mathbf{N}}$ are not on the stress deviator vector $\vec{\mathbf{S}}+\Delta\vec{\mathbf{S}}$, there is no increment in the defect intensity, $\psi_{N3}=\psi_{N2}$. Therefore, expressions (8.5) and (A) give at $\Delta S = S_1 - S_S$ that

$$H_{-N3} = S_P + \psi_{-N3} + I_{-N3} = 2S_P - S_1(1-B) - BS_S . \tag{8.10}$$

By letting H_{-N3} in (8.10) be equal to $-S_S$, the equation for the yield limit in the partial unloading is:

$$S_S = S_1 - \frac{2S_P}{1-B} . \tag{8.11}$$

Summarizing, the occurrence of $\Delta\varepsilon^S$ is possible if the magnitude of S_S is positive and the stress $S_1 - \Delta S$ is less than S_S (Fig. 14d). These conditions, in the view of (8.11), can be met if

$$S_1 > \frac{2S_P}{1-B} \quad \text{and} \quad \Delta S > \frac{2S_P}{1-B} . \tag{8.12}$$

As $\Delta S \le S_1$, the fulfillment of the second inequality in (8.12) provides the fulfillment of the first one.

Portion 3-4 (reverse creep). The integral of non-homogeneity for $t > t_c$ can be obtained if we multiply I_{-N3} from (8.8) by $\exp[-p(t-t_c)]$:

$$I_{-N(3-4)} = B[\Delta S - S_1 \exp(-pt_c)]\exp[-p(t-t_c)]. \tag{8.13}$$

Since plane **I'** is on the endpoint of the vector $\vec{\mathbf{S}}+\Delta\vec{\mathbf{S}}$, one can write that

$$H_{-N(3-4)} = S_P + \psi_{-N(3-4)} + I_{-N(3-4)} = -(S_1 - \Delta S), \tag{8.14}$$

$$\psi_{-N(3-4)} = -(S_1 - \Delta S) - I_{-N(3-4)} - S_P , \tag{8.15}$$

$$\dot{\psi}_{-N(3-4)} = -\dot{I}_{-N(3-4)} = pI_{-N(3-4)} . \tag{8.16}$$

Reverse creep strain rate intensity, $\dot{\varphi}_{-N(3-4)}$, can be found from formula (C):

$$r\dot{\varphi}_{-N(3-4)} = \dot{\psi}_{-N(3-4)} + K\psi_{-N(3-4)} = pI_{-N(3-4)} + K\psi_{-N(3-4)} . \tag{8.17}$$

Now, formulae (8.13) and (8.15)-(8.17) give that

$$r\dot{\varphi}_{-N(3-4)} = B(p-K)[\Delta S \exp(pt_c) - S_1]\exp(-pt) - K[(S_1 - \Delta S) + S_P]. \tag{8.18}$$

As seen from (8.14), tangent planes with negative normals start to move towards the origin of the coordinate leaving the endpoint of the stress deviator vector. The decrease in the number of planes on the endpoint of the stress deviator vector leads to a decrease in the $r\dot{\varphi}_{-N(3-4)}$ from (8.18), i.e. the decrease in the reverse creep rate is modeled. As long as the right-hand side in (8.18) is positive, tangent planes with negative normals produce irreversible (creep) strain at constant $S_1 - \Delta S$. By letting $\dot{\varphi}_{-N(3-4)} = 0$, we express the fact that the last plane (**I'** plane) has left the endpoint of stress deviator vector (Fig. 14e), and we can calculated the duration of reverse creep t_r as

$$t_r = \frac{1}{p} \ln \frac{B(p-K)(\Delta S - S_1 \exp(-pt_c))}{K(S_1 - \Delta S + S_P)}. \tag{8.19}$$

$t_r > 0$ if the numerator is greater than the denominator in (8.19):

$$[B(p-K)+K]\Delta S > [B(p-K)\exp(-pt_c)+K]S_1 + KS_P. \tag{8.20}$$

Therefore, the formulae derived are valid if the magnitude of partial unloading ΔS satisfies expressions (8.8), (8.12) and (8.20).

As seen from (8.19), the reverse creep time t_r grows with ΔS if we hold S_1 and t_c fixed; this is true for the whole range of ΔS , from $S_1 - S_S$ to S_1 (complete unloading). Another result is that the reverse creep time t_r grows with the initial creep duration t_c at fixed values of S_1 and ΔS . Furthermore, the function $t_r(t_c)$ is bounded above by horizontal asymptote

$$\max t_r = \frac{1}{p} \ln \frac{B(p-K)\Delta S}{K(S_1 - \Delta S + S_P)} \quad \text{as } t_c \to \infty. \tag{8.21}$$

These results agree with experimental data.

Portion 4-5 (creep delay). The defect intensity for plane **I** at $t = t_c + t_r$ is determined by formulae (7.1), (8.10) and (8.14) at $t = t_c + t_r$:

$$\psi_{N4} = -\psi_{-N4} = \frac{p}{p-K}(S_1 + S_P - \Delta S). \tag{8.22}$$

Since tangents planes with neither positive or negative normals are not on the endpoint of $\vec{\mathbf{S}} + \Delta\vec{\mathbf{S}}$ (Fig. 14f), irreversible straining does not occur for $t > t_c + t_r$, $d\varphi_N = 0$. Therefore, we arrive at defects relaxation equation (4.33) which, together with initial condition (8.22), takes the form

$$\psi_{N(4-5)} = \frac{p}{p-K}(S_1 + S_P - \Delta S)\exp(-K(t - t_c - t_r)). \tag{8.23}$$

As $I_{N(4-5)} = 0$, the distance to plane **I** is

$$H_{N(4-5)} = S_P + \frac{p}{p-K}\left(S_1 + S_P - \Delta S\right)\exp\left(-K\left(t - t_c - t_r\right)\right). \tag{8.24}$$

This means that plane **I** moves in the direction towards the origin of the coordinates. The instant of time when the plane is on the endpoint of vector $\vec{\mathbf{S}} + \Delta\vec{\mathbf{S}}$ (Fig. 14g) can be found from (8.24) by letting

$$H_{N(4-5)} = S_1 - \Delta S\ . \tag{8.25}$$

As a result, we obtain the duration of creep delay t_d as

$$t_d = \frac{1}{K}\ln\frac{p\left(S_1 + S_P - \Delta S\right)}{\left(p-K\right)\left(S_1 - S_P - \Delta S\right)}. \tag{8.26}$$

For $t > t_c + t_r + t_d$ the creep of positive sign develops in time due to the fact that the planes come back to the endpoint of vector $\vec{\mathbf{S}} + \Delta\vec{\mathbf{S}}$. It is worth noting that the formula for t_d holds true if

$$S_1 - \Delta S > S_P\ . \tag{8.27}$$

This inequality expresses an obvious condition for the occurring of the positive creep deformation that the acting stress $S_1 - \Delta S$ must exceed the creep limit S_P.

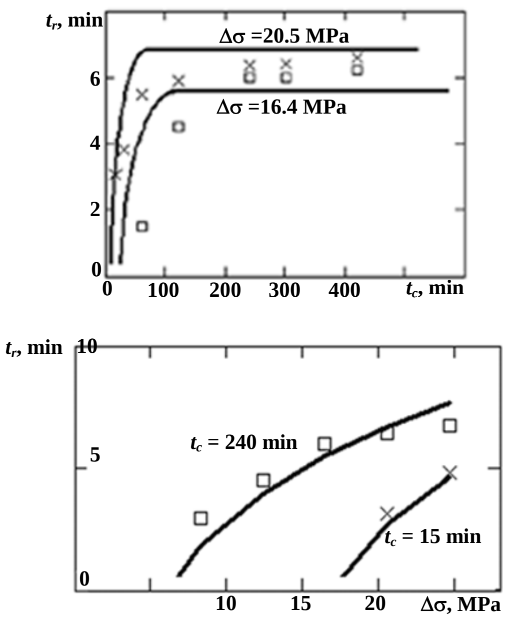

Figure 15
Experimental and calculated $t_r - t_c$ and $t_r - \Delta\sigma$ curves

In Fig. 15, $t_r \sim t_c$ and $t_r \sim \Delta\sigma$ curves are plotted on the basis of (8.19) (symbols □ and × indicate experimental points), for the following values of acting stress, creep limit, and the model constants, $\sigma_1 = 227\,\text{MPa}$, $\sigma_P = 10\,\text{MPa}$, $B = 0.05$, $K = 2.5 \cdot 10^{-4}$, $p = 0.2\,\text{min}^{-1}$. The comparison between the calculated result and experimental data shows satisfactory agreement.

Discussion

The phenomenon of reverse creep is of great importance relative to the theories based on the hypothesis of creep potential. According to the concept of potential, a creep rate is a single-valued function of the state of stress and stress values, meaning that a loading prehistory does not affect the creep rate. On the other hand, the reverse creep strongly depends on loading regime. Furthermore, the sign of the reverse creep is opposite to that of the acting stress. Therefore, the generalized theory provides broader possibilities than classical creep theories.

Conclusions

The generalized synthetic theory of irreversible deformation is capable of modeling a very wide circle of problems ranging from plastic and steady/unsteady-state creep deformation to non-classical problems of irreversible deformation such as creep delay, the Bauschinger negative effect and reverse creep. This capability results from (i) the uniformed approach to the modeling of irreversible deformation and (ii) the intimate connection between macro-deformation and the processes occurring on the macro-level of material.

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References

[1] Andrusik, J., Rusinko, K.: Plastic Strain of Work-Hardening Materials under Loading in Three-Dimensional Subspace of Five-Dimensional Stress-Deviator Space (in Russian). *Izv. RAN (Russian Academy of Sciences), Mekh. Tverd. Tela* **2**: pp. 92-101, 1993

[2] Batdorf, S. B., Budiansky, B.: A Mathematical Theory of Plasticity Based on the Concept of Slip. *NACA, Technical Note,* 1949

[3] Betten, J.: *Creep Mechanics*, Springer, Berlin, 2005

[4] Chaboche, J. L.: "Unified Cyclic Viscoplastic Constitutive Equations: Development, Capabilities and Thermodynamic Frame Work", Unified Constitutive Laws of Plastic Deformation, Kraus and Kraus zeds, Academic Press, 1996

[5] Chaboche, J. L., Lesne, P. M., Maire, J. F.: Thermodynamic Formulation of Constitutive Equations and Applications to the Viscoplasticity and Viscoelasticity of Metals and Polymers, *Int. J Solid Structures*, 1997

[6] Chakrabarty, J.: *Applied Plasticity*, Springer-Verlag, New York/Berlin/Heidelberg, 2000

[7] Chen, W. F., Han, D. J.: *Plasticity for Structural Engineers*, New York, 1988

[8] Ilyushin, A. A.: *Plasticity*, Moscow, 1963

[9] Kuksa, L., Lebedev, A., Koval'chuk, B.: Laws of Distribution of Microscopic Strains in Two-Phase Polycrystalline Alloys under Simple and Complex Loading, *Strength of Materials*, **18**: pp. 1-5, 1986

[10] McLean, D.: *Mechanical Properties of Metals and Alloys,* John Wiley, New York and London, 1977

[11] Namestnikov, V., Chvostunkov, A. 1960. Creep of Duralumin under Constant and Alternating Loading (in Russian), *Prikl. Mekh. i Tekhn. Fiz.* **4**

[12] Osipyuk, V.: Explanation and Analytical Description of Delayed Creep, *International Applied Mechanics,* **27***:* pp. 374-378, 1991

[13] Rabotnov, Y.: *Creep Problems in Structural Members*, North-Holland, Amsterdam/London, 1969

[14] Rusynko, A.: Creep with Temperature Hardening, *J. Materials Science* **33**: pp. 813-817, 1997

[15] Rusynko, A.: Mathematical Description of Ultrasonic Softening of Metals within the Framework of Synthetic Theory, *J. Materials Science* **37**: pp. 671-676, 2001

[16] Rusinko, A.: Analytic Dependence of the Rate of Stationary Creep of Metals on the Level of Plastic Prestrain, *J. Strength of Metals* **34**: pp. 381-389, 2002

[17] Rusinko, A.: Bases and Advances of the Synthetic Theory of Irreversible Deformation, *XXII International Congress of Theoretical and Applied Mechanics (ICTAM) 25-29 August 2008, Adelaide, Australia,* 2008

[18] Rusinko, A., Rusinko, K.: Synthetic Theory of Irreversible Deformation in the Context of Fundamental Bases of Plasticity, *Int. J. Mech. Mater*. **41**: pp. 106-120, 2009

[19] Sanders, I.: Plastic Stress-Strain Relations Based on Linear Loading funCtion. *Proc. 2nd US Nat. Congr. Appl. Mech.*, pp. 455-460, 1954