The DEA – FUZZY ANP Department Ranking Model Applied in Iran Amirkabir University

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Abstract: Proposed in this study is a hybrid model for supporting the department selection process within Iran Amirkabir University. This research is a two-stage model designed to fully rank the organizational departments where each department has multiple inputs and outputs. First, the department evaluation problem is formulated by Data Envelopment Analysis (DEA) and separately formulates each pair of units. In the second stage, the pair-wise evaluation matrix generated in the first stage is utilized to fully rank-scale the units via the Fuzzy Analytical Network Process (FANP). The FANP method adopted here uses triangular fuzzy numbers. ANP equipped with fuzzy logic helps in overcoming the impreciseness in the preferences. DEA-FANP ranking does not replace the DEA classification model; rather, it furthers the analysis by providing full ranking in the DEA context for all departments, efficient and inefficient.

Keywords: Data Envelopment Analysis (DEA); Fuzzy Analytical Network Process (FANP); Performance; Efficiency; Fully Rank

1 Introduction

Multi-attribute decision-making (MADM) ranks elements based on single or multiple criteria, where each criterion contributes positively to the overall evaluations. The decision maker often carries out the evaluations subjectively. However, DEA deals with classifying the units into two categories, efficient and inefficient, based on two sets of multiple outputs contributing positively to the overall evaluation [1].

Many researchers (Belton & Vickers, 1993) highlight the relationship between DEA and MCDM: “According to them, DEA utilizes a process of allocating weights to criteria, just like other approaches to multi criteria and analysis”.

Ranking is very common in MCDM literature, especially when we need to describe lists of elements or alternatives with single or multiple criteria that we wish to evaluate, and then compare or select. Various approaches have been proposed in the literature for full- ranking of the element, ranging from the utility theory approach to AHP developed by Saaty [2], [3].

Throughout the process of reviewing the literature, it appeared that limited research has been carried out regarding DEA-FANP methods, and only the DEA-AHP method in which connections among factors are not considered has been addressed. The idea of combining AHP and DEA is not new, and there have been several attempts to use them in actual situations. Some of these examples include: Bowen [4], Shang and Sueyoshi [5], Zhang and Cui [6], Zilla Sinuany-Stern et al. [3], Taho Yang, Chunwei Kuo [7], Takamura and Tone [8], Saen et al. [9], Ramanathan [10], and Wang et al. [11].

This paper is divided into four sections. In Section 1, the studied problem is introduced. Section 2 briefly describes the DEA-FANP method and the stages of the proposed model and steps are determined in detail. How the proposed model is used in an example in the real world is explained in Section 3. Finally, in Section 4, conclusions and future research areas are discussed.

2 The DEA-FUZZY ANP Method

2.1 Fuzzy Sets and Fuzzy Number

Zadeh (1965) introduced the Fuzzy Set Theory to deal with the uncertainty due to imprecision and vagueness. A major contribution of this theory is its ability to represent vague data; it also allows mathematical operators and programming to be applied to the fuzzy domain. A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one [12].

A tilde '$\sim$' will be placed above a symbol if the symbol represents a fuzzy set. A triangular fuzzy number (TFN), $\tilde{M}$ is shown in Fig. 1. A TFN is denoted simply as (l/m, m/u) or (l,m,u). The parameters l, m and u ($l \leq m \leq u$) denote respectively the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event. The membership function of triangular fuzzy numbers is as follows:

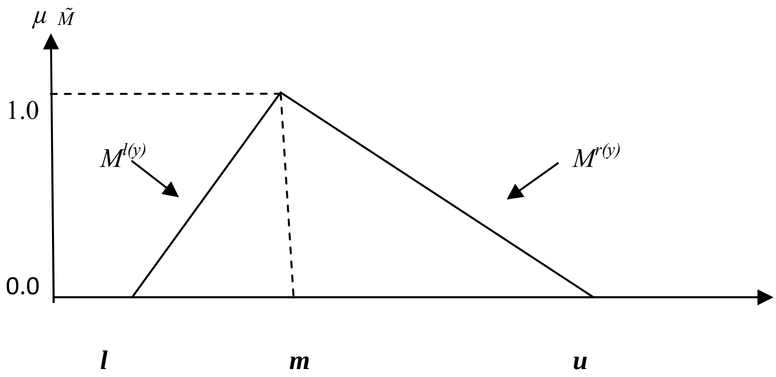

Figure 1
A triangular fuzzy number $\tilde{M}$

Each TFN has linear representations on its left and right side, such that its membership function can be defined as

$$\mu(x/\tilde{M}) = \begin{cases} 0, & x<l, \\ (x-l)/(m-l), & l \le x \le m, \\ (u-x)/(u-m), & m \le x \le u, \\ 0, & x>u. \end{cases} \quad (1)$$

A fuzzy number can always be given by its corresponding left and right representation of each degree of membership:

$$\tilde{M} = (M^{l(y)}, M^{r(y)} = (l+(m-l)y, u+(m-u)y), \qquad y \in [0,1] \quad (2)$$

where $l(y)$ and $r(y)$ denote the left side representation and the right side representation of a fuzzy number, respectively. Many ranking methods for fuzzy numbers have been developed in literature. These methods may provide different ranking results, and most of them are tedious in graphic manipulation, requiring complex mathematical calculation [13].

2.2 Fuzzy ANP

ANP, also introduced by Saaty, is a generalization of the analytic hierarchy process (AHP). Whereas AHP represents a framework with a uni-directional hierarchical AHP relationship, ANP allows for complex interrelationships among decision levels and attributes. The ANP feedback approach replaces hierarchies with networks in which the relationships between levels are not easily represented as higher or lower, dominant or subordinate, direct or indirect. For instance, not only does the importance of the criteria determine the importance of the alternatives, as in a hierarchy, but also the importance of the alternatives may impact on the importance of the criteria [15].

ANP does not require this strictly hierarchical structure; it allows factors to `control' and be `controlled' by the varying levels or `clusters' of attributes. Some controlling factors are also present at the same level. This interdependency among factors and their levels is defined as a systems-with-feedback approach.

The ANP approach is capable of handing interdependent relationships among elements by obtaining composite weights through the development of a supermatrix. The supermatrix concept contains parallels to the Markov chain process [14-15], where relative importance weights are adjusted by forming a supermatrix from the eigenvectors of these relative importance weights. The weights are then adjusted by determining the products of the supermatrix.

The AHP method provides a structured framework for setting priorities on each level of the hierarchy using pair-wise comparisons that are quantified using a 1-9 scale, as demonstrated in Table 1. In contrast, the ANP method allows for more complex relationships among decision layers and their properties.

Table 1
The 1-9 scale for AHP [15]

Importance intensity	***Definition***	**Explanation**
1	Equal importance	Two activities contribute equally to the objective
3	Moderate importance of one over another	Experience and judgment slightly favor one over another
5	Strong importance of one over another	Experience and judgment strongly favor one over another
7	Very strong importance of one over another	Activity is strongly favored and its dominance is demonstrated in practice
9	Extreme importance of one over another	Importance of one over another affirmed on the highest possible order
2,4,6,8	Intermediate values	Used to represent compromise between the priorities listed above

The inability of ANP to deal with the impression and subjectiveness in the pair-wise comparison process has been improved in fuzzy ANP. Instead of a crisp value, fuzzy ANP applies a range of values to incorporate the decision maker's uncertainly [16]. In this method, the fuzzy conversion scale is as in Table 2. This scale will be used in the Mikhailov [17] fuzzy prioritization approach.

Table 2
The 1-9 Fuzzy conversion scale [17]

Importance intensity	***Triangular fuzzy scale***
1	(1,1,1)
2	(1.6,2.0,2.4)
3	(2.4,3.0,3.6)
4	(3.2,4.0,4.8)
5	(4.0,5.0,6.0)
6	(4.8,6.0,7,2)
7	(5.6,7.0,8.4)
8	(6.4,8.0,9.6)
9	(7.2,9.0,10.8)

2.3 Data Envelopment Analysis (DEA)

DEA has been successfully employed for assessing the relative performance of a set of firms, usually called decision-making units (DMU's), which use a variety of identical inputs. The concept of Frontier Analysis, suggested by Farrel (1957), forms the basis of DEA, but the recent series of discussions started with an article by Charnes et al. [18].

DEA is a method for mathematically comparing different DMUs' productivity based on multiple inputs and outputs. The ratio of weighted inputs and outputs produces a single measure of productivity called relative efficiency. The DMUs which have a ratio of 1 are referred to as 'efficient', given the required inputs and produced outputs. The units that have a ratio less than 1 are 'less efficient' relative to the most efficient units. Because the weights for the input and the output variables of DMUs are computed to maximize the ratio and are then compared to a similar ratio of the best-performing DMUs, the measured productivity is also referred to as 'relative efficiency'.

2.4 The Proposed DEA- Fuzzy ANP Method

In this study, fuzzy ANP and DEA for efficiency measurement have advantages over other fuzzy ANP approaches. The priorities obtained from the Fuzzy ANP method based on DEA are defined as a two-staged approach. In the first stage, the pair-wise comparison of the results obtained from the model is based on DEA; in the second stage, a whole hierarchy is carried out by the Fuzzy ANP method on the results obtained from the first stage. A schematic diagram of the proposed model for measurement is shown in Figure 2.

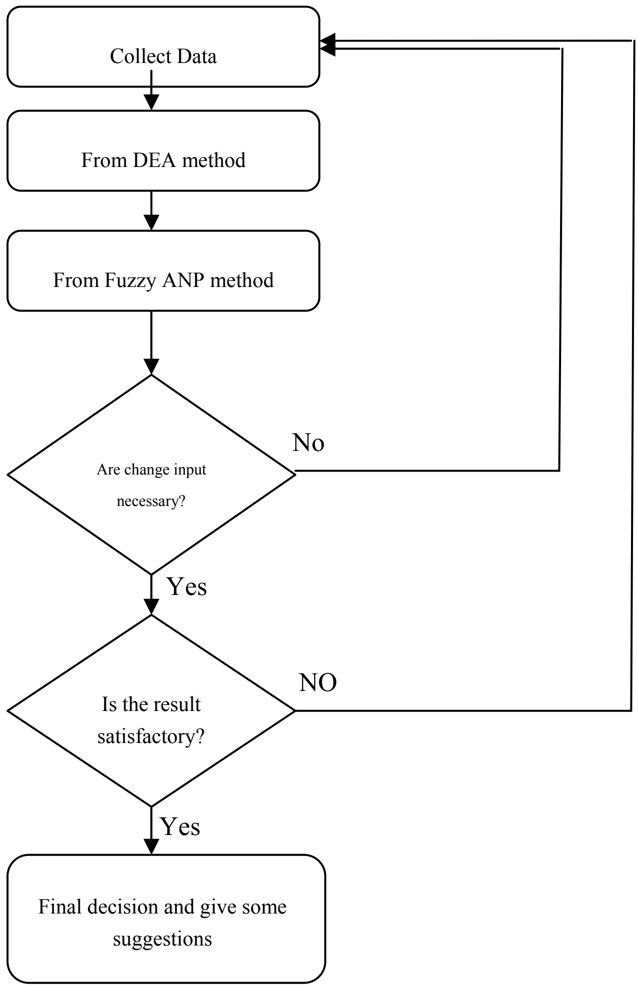

Figure 2
Schematic diagram of the proposed model for measurement

2.4.1 First Stage of the Method (DEA pair-wise comparisons)

In the Fuzzy ANP and DEA method, a pair-wise comparison in a decision-making unit is carried out. For instance, the DMUs are used for the production of x_{ij} (i=1, 2,…, m) entries and y_{rj} (r=1,2,...,s) outputs. X ($s \times n$) and Y ($m \times n$) are the amounts of the entries and outputs, respectively. In DEA, each unit is compared with all units, whereas in the DEA-Fuzzy ANP method, the DMUs are compared in a pair-wise method against each other.

Mathematical (Weighted Linear) Representation of the Problem:

$$e_{k,k'}=\max\sum_{r=1}^{s}u_r y \qquad (3)$$

s.t:

$$\sum_{i=1}^{m}v_i x_k=1 \qquad (4)$$

$$\sum_{r=1}^{s}u_r y_{rk}-\sum_{i=1}^{m}v_i x_{ik}\leq 0 \qquad (5)$$

$$\sum_{r=1}^{s}u_r y_{rk'}-\sum_{i=1}^{m}v_i x_{ik'}\leq 0 \qquad (6)$$

$u_r \geq 0 \quad r=1,2,\ldots,s \qquad v_i \geq 0 \quad i=1,2,\ldots,m$

By solving this mathematical model, $e_{k,k'}$ elements are solved and the pair-wise compared E matrix is obtained (k'=1,….,n, k=1,….,n and $k\neq k'$). In the second stage of the DEA- Fuzzy ANP method process, a two-level FANP model is given.

2.4.2 Second Stage of the Method (FANP ranking)

In the second level, based on the pair-wise comparison matrix E and after the hierarchy of FANP has been developed, the next stage creates matrices considering the interaction between pair-wise items for the factors and sub factors. We modify the selection process to a nine step method procedure, as follows:

Step 1. The calculation of $a_{k,k'}$: The components of the pair-wise comparative matrix are obtained via the following formula.

$$a_{k,k'}=e_{k,k'}/e_{k',k} \qquad (7)$$

Step 2. The calculation of Triangular Fuzzy Numbers: we setup the Triangular Fuzzy Numbers. Each expert makes a pair-wise comparison of the decision criteria and gives them relative scores.

$$\hat{G}_1=(l_i, m_i, u_i) \qquad (8)$$

Step 3. The calculation of $\hat{G}_1$: Establishing the Triangular Fuzzy Numbers, we setup the Triangular Fuzzy Numbers using the ANP method based on the Fuzzy numbers. Each expert makes a pair-wise comparison of the decision criteria and gives them relative scores.

$$\hat{G}_1=(l_i,m_i,u_i) \tag{9}$$

$$l_i=(l_{i1} \otimes l_{i2} \otimes \ldots \otimes l_{ik})^{1/k} \qquad i=1,2,\ldots,k \tag{10}$$

$$m_i=(m_{i1} \otimes m_{i2} \otimes \ldots\ldots \otimes m_{ik})^{1/k} \qquad i=1,2,\ldots,k \tag{11}$$

$$u_i=(u_{i1} \otimes u_{i2} \otimes \ldots. \otimes u_{ik})^{1/k} \qquad i=1,2,\ldots,k \tag{12}$$

Step 4. The calculation of $\hat{G}_T$: Establishing the geometric fuzzy mean of the total row using:

$$\hat{G}_T=(\sum_{i=1}^{k} l_i,\sum_{i=1}^{k} m_i,\sum_{i=1}^{k} u_i) \tag{13}$$

Step 5. The calculation of W: Fuzzy geometric mean of the fuzzy priority value calculated with normalization priorities for factors using:

$$\tilde{W} = \hat{G}_i /\hat{G}_T=(l_i ,m_i ,u_i)/(\sum_{i=1}^{k} l_i, \sum_{i=1}^{k} m_i,\sum_{i=1}^{k} u_i)=(l_i /\sum_{i=1}^{k} u_i, m_i /\sum_{i=1}^{k} m_i , u_i /\sum_{i=1}^{k} l_i) \tag{14}$$

Step 6. The calculation of $wi_{\alpha l}$: Factors belonging to nine different *α-cut* values α for the calculated, fuzzy priorities are applied for lower and upper limits for each α value:

$$wi_{\alpha l}=(wil_{\alpha l} , wiu_{\alpha}) \quad i=1,2,\ldots,k \quad l=1,2,\ldots,L \tag{15}$$

Step 7. The calculation of W_{il}, W_{iu}: Combine the entire upper values and the lower values separately, then divide by the total sum of the α value:

$$W_{il} = \sum_{l=1}^{L} \alpha\, (w_{il})_l \Big/ \sum_{l=1}^{L} \alpha_l \qquad i= 1,2,\ldots,k \qquad l= 1,2,\ldots,L \tag{16}$$

$$W_{iu} = \sum_{l=1}^{L} \alpha\, (w_{iu})_l \Big/ \sum_{l=1}^{L} \alpha_l \qquad i= 1,2,\ldots,k \qquad l= 1,2,\ldots,L \tag{17}$$

Step 8. The calculation of W_{id}: Use the following formula in order to defuzzify by the Combined upper limit value and lower limit value using the optimism index (λ)

$$w_{id} = \lambda W_{iu} + (1-\lambda)W_{il} \qquad \lambda \in [0,1] \qquad i = 1,2,...,k \tag{18}$$

Step 9. The calculation of W_{in}: Normalization of defuzzification value priorites using

$$W_{in} =w_{id} / \sum_{i=1}^{k} w_{id} \qquad i= 1,\ldots,k \tag{19}$$

Step 10. The calculation of $w_k \times W_{in}$: The final step deals with determining the degree of relations among different units by multiplying the matrices,

$W_k =$

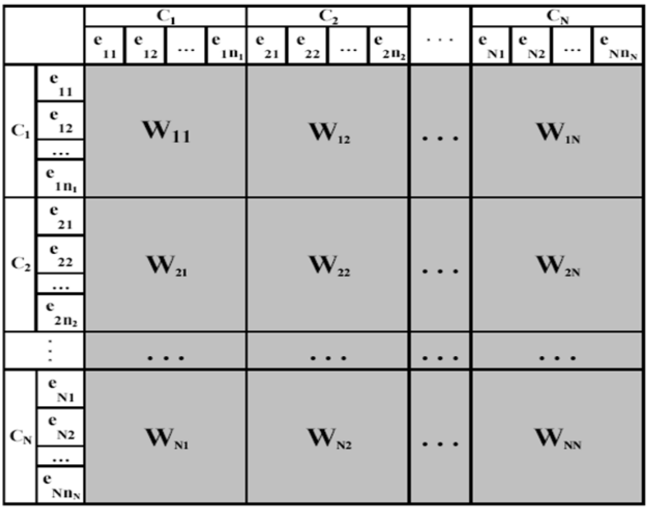

Figure 3
Relations among different units (super matrix) [15]

3 Applying the Sequential Methodology: An Illustrative Problem

The suggested hybrid model is demonstrated via an example of a selected department, supported by Iran Amirkabir University. Amirkabir University (Tehran Polytechnic) was established in 1958 as the first technical university of Iran. Through its rapid educational and research expansion, the university was able to gain a high ranking among all other universities and research centers. The achievements of this university in the area of research are evident from the many publications and the national and international prizes awarded for research activities. Thirteen departments have been considered in our evaluation. In our study, we employ a six-input evaluation criteria and four-output evaluation criteria:***Inputs:*** Number of Professor Doctors, Associated Professors, Assistant Professors, and Instructors; Budget of departments; and Number of credits.

Outputs: Number of alumni (undergraduates and graduate students), Evaluation of instructors, Number of academic congeries, and Number of academic papers (SCI-SSCI-AHCI).

Table 3
The DEA-Fuzzy ANP fully-ranking score

DMU	DEA-FANP score
D1	1.12449
D2	0.67602
D3	1.36825
D4	2.25443
D5	**2.82427**
D6	0.56335
D7	1.01403
D8	0.78684
D9	0.89915
D10	0.56435
D11	**0.55231**
D12	0.67926
D13	1.23937

The result score is always the-bigger-the-better. As visible in Table 3, department 5 has the largest score due to its highest efficiency and performance. Department 11 has the smallest score of the thirteen departments and is ranked in the last place. The relevant results can be seen in Table 3. Obviously, the best selection is candidate D5.

Conclusion

We have presented an effective model for rank scaling of the units with multiple inputs and multiple outputs using both DEA and FANP. In this paper, a two-stage hybrid methodology is provided where the binary comparison of the results obtained from the model is based on DEA. The second stage of the methodology assists in fully-ranking of the alternatives based on the results obtained from the first stage. The result of the methodology is a rank order of the alternatives, which can be used to select an individual project or a portfolio of projects.

The advantage of the DEA-FANP ranking model is that FANP pair-wise comparisons have been derived mathematically from multiple input/output data by running pair-wise DEA runs. Thus, there is no subjective evaluation.

The DEA and the FANP methods are commonly used in practice and, yet, both have limitations. The DEA-FANP method combines the best of both models by avoiding the pitfalls of each. ANP is designed for subjective evaluation of a set of alternatives based on multiple criteria organized in a hierarchical structure. In this model, we work with given tangible inputs and outputs of units, and no subjective assessment of the decision maker’s evaluation is involved. The Pareto optimum limitation of DEA is resolved by the full- ranking performed here by means of the FANP. It is important to note that DEA-FANP does not replace DEA, but rather, it provides further analysis of DEA to full ranking the units.

The performance measurement model developed here structured the performance measurement problem in a hierarchical form, critical areas and performance measures. The developed performance measurement model contributes to the previous performance measurement models by including and quantifying interdependencies that exist between system components. In addition, the involvement of fuzzy theory can adequately resolve the inherent uncertainty and imprecision associated with the mapping of a decision maker's perception to exact numbers.

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