# Vehicle Dynamic-based Approach for the Optimization of Traffic Parameters of the Intelligent Driver Model (IDM) and for the Support of Autonomous Vehicles' Driving Ability

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Abstract: The research identifies the dynamical parameters in the area of mechanics within the base of traffic model parameters used by the Intelligent Driver Model (IDM), which are the highest acceleration parameters set by the vehicle, the desired speed parameters of the vehicle and the distance-keeping parameters of the vehicle. All this facilitates the automatic control of autonomous electric vehicles in certain vehicle groups.

Keywords: IDM; dynamics-based approach; support for autonomous vehicles' driving

# 1 Introduction

The aim of the research is the longitudinal dynamic optimization of the driving processes. This means the support of an optimally attenuated, non-hectic traffic process that results in energy savings and noise reduction of vehicles in the vehicle group and in addition to this, emission reduction of conventional vehicles [14, 15]. During the course of the ride, minimal oscillations in speed occurs. Therefore, it reduces speed changes and the number of excessive braking. In the analysis the acceleration capabilities are vehicle characteristics. In the optimization task the variables are tracking distances and desired speed values. An important requirement is the definition of minimum distances for the former and the upper limit for the latter. In this paper, we investigate a group of vehicles with almost identical characteristics in terms of acceleration properties, but this analysis can naturally be extended to variable-component vehicle groups as well.

The research material discusses a highly complex problem, i.e. the re-formulation of the IDM models into mechanical-dynamic systems, and thus conducts studies in a physical parameter space and system in which significant knowledge in the field of physics and dynamics has accumulated.

It can be said that the theoretical foundation and application [12] of IDM models is very good, however, the complexity [7, 11] of real-world traffic processes poses serious problems to professionals in the applications. On the one hand, greater error tolerance than the accuracy of traffic parameters is to be expected than in many other dynamical systems. On, the other hand, it can be stated that complexity is extremely high [16, 28, 29, 30] either when assessing the entirety of effects of road traffic on a single vehicle, or only the information [1] to be processed by the driver or the autopilot. Similarly, the system is very complex if we consider only the surface traffic processes as a whole over a large-scale network [21, 22, 23, 24, 25]. The reality is, however, much more complex, since the above elements form a single large-scale dynamic system during their operation and are in constant interaction with each other. This complex system in its physical reality consists, on the one hand of the multitude of vehicle-dynamical systems [8] (which is a multitude of man-autopilot-machine systems) and, on the other hand, of the multitude of static and dynamic traffic network elements. Finally, all the above is surrounded by a very complicated dynamic external environment that, in addition to seasonality, also has different geographic, meteorological, economic and cultural characteristics [5, 6, 13].

In connection with the above, we will briefly review the traffic-related applications of IDM, as well as the relationship between the IDM model and the large-scale networks. We investigate the longitudinal dynamic properties of the vehicles during the catch-up with the leader vehicle.

For this complex dynamic process we interpret Lehr's relative attenuation factor  $\delta$  and  $\omega$  eigenfrequency from the field of mechanics as dynamic characteristics with well-known effects. The purpose of the analysis is to optimize traffic parameters and to support the driving of autonomous vehicles using the above introduced dynamic characteristics.

### 2 The IDM Model and Its Traffic-related Applications

Adaptive Cruise Control (ACC) is a vehicle system that allows the vehicle to adjust its speed to the environment. The Intelligent Driver Model (IDM) is an adaptive cruise control (ACC) model that is widely used in transportation research to model longitudinal movement. Treiber, Hennecke and Helbing developed the Intelligent Driver Model (IDM) (1), which is being used by the car company BMW, in 2000 at the transport laboratory of Dresden Technical University [33, 34].

$$\dot{\mathbf{v}}_{k} = \mathbf{a}_{k} \left[ 1 - \left( \frac{\mathbf{v}_{k}}{\mathbf{v}_{k}^{0}} \right)^{4} - \left( \frac{\mathbf{s}^{*} \left( \mathbf{v}_{k}, \Delta \mathbf{v}_{k} \right)}{\mathbf{s}_{k}} \right)^{2} \right]$$
(1)

Where:

ak is the maximal acceleration of the kth vehicle,

- $\mathbf{v}_{k} = \dot{\mathbf{x}}_{k}$  is the speed of the *k*th vehicle,
- $\mathbf{v}_{k}^{0}$  is the desired speed of the *k*th vehicle,

 $S_k$  is the distance between the *k*th and the preceding vehicle,

 $\Delta x_k = x_{k-1} - x_k$  the difference between the positions of the centre of gravity of the (*k*-1)th vehicle and that of the *k*th vehicle,



 $Figure \ 1$  The  $s_k$  distance between consecutive vehicles

$$\mathbf{s}^{*}\left(\mathbf{v}_{k},\Delta\mathbf{v}_{k}\right) = \mathbf{s}_{k}^{0} + \mathbf{T}_{k}\cdot\mathbf{v}_{k} + \frac{\mathbf{v}_{k}\cdot\Delta\mathbf{v}_{k}}{2\sqrt{a_{k}b_{k}}};$$
(2)

Where:

 $s_k^0$  is the congestion speed of the *k*th vehicle,

 $T_k$  is the safety time slot of the *k*th vehicle, which is pre-set by the drivers,

 $\Delta v_k = v_{k-1} - v_k$  is the difference between the speeds of the *k*th and (*k*-1)th vehicles,

 $b_k$  is the maximal deceleration of the *k*th vehicle.

Taking into account that:

 $\mathbf{X}_{k}$  is the position of the *k*th vehicle,

 $\ddot{\mathbf{x}}_k = \dot{\mathbf{v}}_k$  is the acceleration of the *k*th vehicle,

equation (1) can be rewritten based on the above into the second order differential equation (3) representing the positions, here neglecting the second and third members of formula (2):

$$\ddot{x}_{k} = a_{k} \left[ 1 - \left( \frac{\dot{x}_{k}}{v_{k}^{0}} \right)^{4} - \left( \frac{s_{k}^{0}}{\left( x_{k-1} - x_{k} \right) - l_{k}} \right)^{2} \right]$$
(3)

By rearranging this, the following *k*th differential equation may be written:

$$\frac{\ddot{x}_{k}}{a_{k}} + \left(\frac{\dot{x}_{k}}{v_{k}^{0}}\right)^{4} + \left(\frac{s_{k}^{0}}{\left(x_{k-1} - x_{k}\right) - l_{k}}\right)^{2} = 1$$

(k=1,2,...,n), where 0 is the leading point of the track followed by the first vehicle.

For each vehicle, the position-based movements are defined by the following differential equation system for the 1., 2., ..., n. members of the vehicle group. According to the proposals in the works of [2, 3]:

$$\frac{\ddot{x}_{1}}{a_{1}} + \left(\frac{\dot{x}_{1}}{v_{1}^{0}}\right)^{4} + \left(\frac{s_{1}^{0}}{(x_{0} - x_{1}) - l_{1}}\right)^{2} = 1$$

$$\frac{\ddot{x}_{2}}{a_{2}} + \left(\frac{\dot{x}_{2}}{v_{2}^{0}}\right)^{4} + \left(\frac{s_{2}^{0}}{(x_{1} - x_{2}) - l_{2}}\right)^{2} = 1$$

$$\frac{\ddot{x}_{n}}{a_{n}} + \left(\frac{\dot{x}_{n}}{v_{n}^{0}}\right)^{4} + \left(\frac{s_{n}^{0}}{(x_{n-1} - x_{n}) - l_{n}}\right)^{2} = 1$$

The IDM model is used for modeling continuous traffic flows in simulations of highway and city traffic. As a vehicle tracking model, IDM describes the dynamics of the position and speed of each vehicle. In the case of Multi-model Open Source Traffic Simulator, [32] use IDM to simulate the longitudinal movement of the vehicle and this simulator also introduces a lane change strategy. Model-based single-lane traffic inhomogeneity is studied by [35].

The work of [36] studies vehicle stability and IDM parameter sensitivity. The work of [10] proposed extending the driver parameters of the IDM model. They study the impact of vehicles equipped with IDM on traffic flow and travel times as bottlenecks. The work of [9] also uses the IDM model and examines the impact of Adaptive Cruise Control on traffic flows. The results of the above work show that increasing the proportion of ACC vehicles will result in increased traffic efficiency by reducing travel times. The work of [37] used IDM to study instability in congested traffic.

The IDM model has many advantages over other ACC models from a calibration and intuitive parameters point of view, and also modeling requires simple simulation. However, there are also disadvantages in respect of assuring the proper features of the vehicle and the driver. The IDM is a collision-free model. Therefore, in critical accident situations, the desired minimum distance is no longer sufficient to guarantee driver safety and, in the event of an emergency braking, it tends to overshoot the actual deceleration of the vehicle.

The works of [2, 3] developed a proposal for a more accurate operation of the IDM model and studied possible modifications to IDM, taking into account driver's safety and the real capabilities of the vehicle. As a result of this amendment, the driver has to take into account the behavior of the following vehicles, and thus a modified IDM model has been developed and tested with a microscopic simulator considering string stabilization. This modified IDM model already highlights the proper vehicle capabilities.

Based on this, the IDM model is already providing greater performance in driver security by following real reactions in near-collision critical situations. The paper shows the modification and the state-of-the-art operation of the intelligent driver model in connection with the proper capabilities of the vehicle.

Modeling and research work encompasses a complex area and includes approaches of both microscopic and macroscopic modeling, [4]. The complex macroscopic traffic environment is generated by the large-scale network model, in which the microscopic traffic simulation model provides the individual vehicle movement in traffic on the sections of the defined trajectories. However, this microscopic model must properly reproduce dynamic traffic processes and must also be validated. Accordingly, at this stage of our work we rely on Intelligent Driver Model research and development of [33, 34, 4]. The features of the classical IDM model are the following: a single system of differential equations that analyses the case of n vehicles traveling on a single lane. The microscopic model describes a chain model-like longitudinal dynamics. Each driver looks only forward and aims to keep an appropriate distance. There is no overtaking, the vehicles keep their order and the first vehicle has a dominant role, as do the slow-moving vehicles in the group.



The n element vehicle group

#### Figure 2

The n-element vehicle group and the environment determining their movement

According to the above the classical IDM model is written with separate differential equations, member by member. The works of [2, 3], are summarized in the following system of differential equations (4), where the current position of the *i*th vehicle is described by function  $x_i(t)$ . The parameters and functions used in the model are as follows:

 $a_i$  is the maximal acceleration of the *i*th vehicle,

 $v_i$  is the desired speed of the *i*th vehicle,

 $s_i$  is the required distance between the *i*th and the preceding vehicle (i=1,2,...,n),

$$\left\langle \underline{\underline{A}} \right\rangle^{-1} \ddot{x}(t) + \left\langle \underline{\underline{V}} \right\rangle^{-1} \underline{\underline{f}_{1}}(\dot{x}(t)) + \left\langle \underline{\underline{S}} \right\rangle \underline{\underline{f}_{2}}(x(t)) = \underline{1}$$

$$\left\langle \underline{\underline{A}} \right\rangle^{-1} = \left\langle \frac{1}{a_{1}}, \frac{1}{a_{2}}, \dots, \frac{1}{a_{n}} \right\rangle; \left\langle \underline{\underline{V}} \right\rangle^{-1} = \left\langle \frac{1}{v_{1}^{4}}, \frac{1}{v_{2}^{4}}, \dots, \frac{1}{v_{n}^{4}} \right\rangle; \left\langle \underline{\underline{S}} \right\rangle = \left\langle s_{1}^{2}, s_{2}^{2}, \dots, s_{n}^{2} \right\rangle$$

$$s_{i} = s_{0i} = const., \text{ or: } s_{i} = s_{i}(\dot{x}_{i-1}, \dot{x}_{i}) \text{ (i=1,2,...,n)}.$$

$$\underline{f}_{1}(\dot{x}(t)) = \begin{bmatrix} \dot{x}_{1}^{4} \\ \dot{x}_{2}^{4} \\ \dots \\ \dot{x}_{n}^{4} \end{bmatrix}, \quad \underline{f}_{2}(x(t)) = \begin{bmatrix} \frac{1}{(x_{0} - x_{1})^{2}} \\ \frac{1}{(x_{n-1} - x_{n})^{2}} \\ \dots \\ \frac{1}{(x_{n-1} - x_{n})^{2}} \end{bmatrix}, \quad \underline{1} = \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix}$$

Detailed description of the above is provided by [2, 3]. This model, using a function h(t) also takes into account the fact that drivers monitor the movement of the following vehicles as well, see Fig. 3.

$$\langle \underline{A} \rangle^{-1} \ddot{x}(t) + \underline{V} \underline{f}_{1}(\dot{x}(t)) + \underline{S} \underline{f}_{2}(x(t)) = \underline{1}(t) + \underline{h}(t)$$

$$\langle \underline{A} \rangle^{-1} = \left\langle \frac{1}{a_{1}}, \frac{1}{a_{2}}, \dots, \frac{1}{a_{n}} \right\rangle;$$

$$\underbrace{V}_{\underline{A}} = \begin{bmatrix} \frac{1}{v_{1}^{4}} & \frac{h_{1}}{v_{2}^{4}} & & \\ & \frac{1}{v_{2}^{4}} & \frac{h_{2}}{v_{3}^{4}} & & \\ & & \frac{1}{v_{1}^{4}} & \frac{h_{1}}{v_{1+1}^{4}} & & \\ & & & \frac{1}{v_{n}^{4}} & \frac{h_{i}}{v_{n+1}^{4}} \\ & & & & \frac{1}{v_{n}^{4}} \end{bmatrix}; \quad \underline{S} = \begin{bmatrix} s_{1}^{2} & h_{1}s_{2}^{2} & & \\ & s_{2}^{2} & h_{2}s_{3}^{2} & \\ & & s_{i}^{2} & h_{i}s_{i+1}^{2} \\ & & & & s_{n}^{2} \end{bmatrix};$$

$$h_{i}(t) = \begin{bmatrix} h_{1}(t) \\ h_{2}(t) \\ & \cdots \\ & h_{n}(t) \end{bmatrix}; \quad h_{i}(t) = hf_{i}(t) \cdot \frac{a_{i+1}}{a_{i}}; \quad (i=1,2,\dots,n-1); \quad h_{n}(t) = 0.$$

Applying  $h_i(t)$ , we assume that the driver takes into account the follower vehicle behavior. Where "*h*" is a human factor (dimensionless parameter), which is a calibrated parameter according the driver characteristic.

$$\begin{split} s_{i} &= s_{0i} = const., \text{or: } s_{i} = s_{i}(\dot{x}_{i-1}, \dot{x}_{i}) \text{ (i=1,2,...,n)}.\\ \\ \underline{f_{1}}(\dot{x}(t)) &= \begin{bmatrix} \dot{x}_{1}^{4} \\ \dot{x}_{2}^{4} \\ \vdots \\ \dot{x}_{n}^{4} \end{bmatrix}; \quad \underline{f_{2}}(x(t)) = \begin{bmatrix} \frac{1}{\varepsilon_{1}^{2} + (x_{0} - x_{1})^{2}} \\ \frac{1}{\varepsilon_{2}^{2} + (x_{1} - x_{2})^{2}} \\ \vdots \\ \frac{1}{\varepsilon_{n}^{2} + (x_{n-1} - x_{n})^{2}} \end{bmatrix}; \quad \underline{1}(t) = \begin{bmatrix} 1(t) \\ 1(t) \\ \vdots \\ 1(t) \end{bmatrix}; \end{split}$$



Figure 3 Setting of the stabilized speed state after the starting of a vehicle group

# 3 Relationship between the IDM Model and the Large-Scale Network

The speed of a given vehicle and the distance kept are determined by the driver. Their decision, however, depends on their own perceptions, on signals that are transmitted by the physical environment and received by the vehicle and on the local and general effects of network traffic [17, 18, 19, 20, 21, 22]. Physical impacts resulting from road quality, meteorological and visibility conditions at a given vehicle density determine a selectable speed range. The modified IDM model discussed in the previous section can be used to describe the dynamic traffic connections originating from forward-moving vehicle-vehicle effects in a given section.

At the same time, the dynamics of the movement of the IDM model group is not arbitrary. It is determined by control speeds formed in the large-scale network or network sections. The vehicles slow down if a congestion occurs, stop when the traffic light switches to red, but after the reaction delay time, they will accelerate to the maximum permitted speed if the road section ahead is free. This is indicated in Fig. 2 by the control speed function  $x_0(t)$  defined by the large-scale macroscopic network processes for each trajectory.

### 4 The Impact of the Vehicles'maximal Acceleration Parameter a on the Motion Process during Catchup to the Leader Vehicle

The following diagrams show simulated route-time and speed-time diagrams for different *a* acceleration capabilities. In our case, stationary vehicles in the same lane start from different starting points at  $t_0 = 0$  and follow the movement of the leader vehicle (leading point).



Figure 4 The catch-up motion process at a=10 [m/s<sup>2</sup>]

Figure 5 The catch-up speed process at a=10 [m/s<sup>2</sup>]





The catch-up motion process at  $a=5 \text{ [m/s^2]}$ 

Figure 7 The catch-up speed process at a=5  $[\rm m/s^2]$ 





Figure 8 The catch-up motion process at a=3 [m/s<sup>2</sup>]

Figure 9 The catch-up speed process at a=3 [m/s<sup>2</sup>]



The simulations in this case included three successive vehicles from the vehicle group outside the leader vehicle. For vehicles with less acceleration, slower movements can be seen. Acceleration capabilities have an effect on the overshoot, and on the time of speed stabilization. All of these features naturally determine the movements of other participants in the traffic, energy consumption, emissions and traffic noise, so it is worth taking a deeper analysis that will support the smoother traffic processes in the vehicle groups.

### 5 The Relationship between the Dynamical Characteristics in the Area of Mechanics and the Traffic Parameters

The IDM model investigates the longitudinal movement of vehicle groups in traffic, thus the essence of the analysis is a longitudinal dynamic traffic analysis.

The parameters of the IDM basic system are traffic system parameters with uncertainties. Therefore, let us look at the following interpretation and appropriately transcription of the mathematical model that is useful for further analysis of dynamical properties and for exploring new relationships.

In addition to the following model considerations, the applied model structure can be used to carry out an analysis of a suitable multi-mass dynamical model by using purely dynamics and vibrations concepts and deducting conclusions from these in respect to the original traffic system parameters.

$$\left\langle \underline{\underline{A}} \right\rangle^{-1} \ddot{x}(t) + \left\langle \underline{\underline{V}} \right\rangle^{-1} \underline{f}_{1}(\dot{x}(t)) + \left\langle \underline{\underline{S}} \right\rangle \underline{f}_{2}(x(t)) = \underline{1}$$
(6)

The system of differential equations (6) contains dimensionless members on the left-hand side because the dimensions at the multipliers are reciprocal to each other. It follows from the above that the vector on the right-hand side is necessarily dimensionless. The numerical values of the solutions of the system of differential equations do not change if the following physical dimensions are applied to each element of the matrix and of the vectors: A<sup>-1</sup> [kg]; V<sup>-1</sup>  $f_1$  [N], S  $f_2$ [N] and 1(t) [N]; x(t) [m]; (consequently: x(t) for the first derivative [m/s]; x(t) for the second derivative  $[m/s^2]$ ; The importance of this approach is that while the mathematical model is equivalent in the two cases, the physical interpretation is completely different. In the first case, we examine a traffic dynamics model and in the second case a mechanical dynamics one. However, such conclusions can be drawn based on the parameters of the second physical model, that cannot be drawn based on the first model. On that basis in the model (6) the mass matrix M, the non-linear attenuation vector  $\Psi(\dot{\mathbf{x}}(t))$  and the non-linear spring force vector  $\phi(x(t))$  can be interpreted, which defines the classical nonlinear vehicle dynamics model (7).  $(M \in \Re^{n \times n}; \psi(\dot{x}(t)) \in \Re^{n}; \phi(x(t)) \in \Re^{n};)$ 

$$\underline{\underline{\mathbf{M}}} = \left\langle \underline{\underline{\mathbf{A}}} \right\rangle;$$
  
$$\psi(\dot{\mathbf{x}}(t)) = \left\langle \underline{\underline{\mathbf{V}}} \right\rangle^{-1} \underline{\mathbf{f}}_{1}(\dot{\mathbf{x}}(t));$$
  
$$\phi(\mathbf{x}(t)) = \left\langle \underline{\underline{\mathbf{S}}} \right\rangle \underline{\mathbf{f}}_{2}(\mathbf{x}(t));$$

The elements of the diagonal matrix  $\underline{\underline{M}}$  take values of  $\mathbf{m}_i = \frac{1}{a_i}$ ; (i = 1, 2, ..., n).

$$\underline{\underline{M}}\ddot{x}(t) + \psi(\dot{x}(t)) + \underline{\phi}(x(t)) = \underline{1}$$
<sup>(7)</sup>

The special feature of the IDM system is that if the leading point takes constant  $v_0$  speed, the movement of the members of the vehicle group (i=1, 2, ..., n) is asymptotically set to a stable state:

$$\ddot{x}_{i}(t) \rightarrow 0$$

$$\dot{x}_{i}(t) \rightarrow v_{i}^{*} = \text{const} \, !$$

$$x_{i-1}(t) - x_{i}(t) \rightarrow s_{i}^{*} = \text{const} \, !$$
(8)

In formulas (8),  $v_i^*$  is the desired exact speed and  $s_i^*$  is the desired exact distance for the *i*th vehicle, which parameters the drivers want to maintain based on the longitudinal dynamics of the vehicles. Since they make mistakes and can differ from these, the above is taken into account with the coefficients  $\alpha > 0$  and  $\beta > 0$ , thus  $s_i = \alpha S_i^*$  and  $v_i = \beta v_i^*$  are considered to be the actual operating points. The setting of the coefficients is performed during validations, measurements, or using simulation results. Then we examine the movements around the operating point, because in this way system-specific parameters can be derived, which can be interpreted well from previous vehicle dynamics studies and important information can be obtained in the analysis of more complex IDM parameters.

Let us study the attenuation factor  $k_i$ , the spring rigidity coefficient  $S_i$  and the mass  $m_i$  of the *i*th element of the system linearized around the working point:

$$\mathbf{k}_{i} = \left[\frac{d}{d\dot{\mathbf{x}}_{i}} \left(\frac{\dot{\mathbf{x}}_{i}}{\mathbf{v}_{i}}\right)^{4}\right]_{\dot{\mathbf{x}}_{i}=\mathbf{v}_{i}} = \frac{4}{\mathbf{v}_{i}};$$
(9)

$$S_{i} = \left[\frac{d}{dx_{i}}\left(\frac{s_{i}}{x_{i-1} - x_{i}}\right)^{2}\right]_{x_{i-1} - x_{i} = s_{i}} = \frac{2}{s_{i}};$$
(10)

$$\mathbf{m}_{i} = \frac{1}{\mathbf{a}_{i}}; \tag{11}$$

On the basis of (9), (10) and (11) we can introduce the eigenfrequency  $\omega_i$  and Lehr's relative attenuation factor  $\delta_i$  around the operating point, which are the following based on the original IDM model parameters:

$$\omega_i^2 = \frac{\mathbf{S}_i}{\mathbf{m}_i} = 2\frac{\mathbf{a}_i}{\mathbf{s}_i}; \tag{12}$$

(13) and (14) applies to the relative attenuation factor  $\delta_i$  based on the definition and on (9) and (11), respectively:

$$\frac{\underline{k}_{i}}{\underline{m}_{i}} = 2 \cdot \delta_{i} \cdot \omega_{i}$$

$$\frac{\underline{k}_{i}}{\underline{m}_{i}} = \frac{\frac{4}{\underline{v}_{i}}}{\frac{1}{\underline{a}_{i}}} = 4 \frac{\underline{a}_{i}}{\underline{v}_{i}}$$
(13)
(14)

Based on (13) and (14):

$$\delta_{i} = \sqrt{2} \, \frac{\sqrt{a_{i} \cdot s_{i}}}{v_{i}} \tag{15}$$

The above formula was determined based on the IDM model parameters. The value of  $\delta$  can be approximated by simulation or measurement by applying the appropriate logarithmic decrement in a way that the attenuated oscillation process of the velocity function around the axis  $v=v_0$  is modelled with the function  $\dot{x}_i(t) = A \cdot \cos(\omega \cdot t) \cdot e^{-2 \cdot \delta \cdot \omega \cdot t}$ . In this case, we determine the amplitudes  $A_1, A_2, A_3, ..., A_k$ , according to local extremes  $t_1, t_2, t_3, ..., t_k$ . We also utilize that  $|\cos(\omega \cdot t_1)| = |\cos(\omega \cdot t_2)| = |\cos(\omega \cdot t_3)| = ... = |\cos(\omega \cdot t_k)|$ , consider the following arbitrary times  $t_i \neq t_j(t_1 \leq t_i \prec t_j \leq t_k)$  and apply the following formulas:

$$\frac{A_{i}}{A_{j}} = \frac{A \cdot \cos(\omega \cdot t_{i}) \cdot e^{-2 \cdot \delta \cdot \omega \cdot t_{i}}}{A \cdot \cos(\omega \cdot t_{j}) \cdot e^{-2 \cdot \delta \cdot \omega \cdot t_{j}}} = \frac{e^{-2 \cdot \delta \cdot \omega \cdot t_{i}}}{e^{-2 \cdot \delta \cdot \omega \cdot t_{j}}}$$
(16)

$$\ln \mathbf{A}_{i} - \ln \mathbf{A}_{j} = 2 \cdot \delta \cdot \boldsymbol{\omega} \cdot (\mathbf{t}_{j} - \mathbf{t}_{i})$$
<sup>(17)</sup>

$$\delta = \frac{\ln \mathbf{A}_{i} - \ln \mathbf{A}_{j}}{2 \cdot \boldsymbol{\omega} \cdot (\mathbf{t}_{j} - \mathbf{t}_{i})}$$
(18)

For practical calculations, the first 3-4 elements of the series  $t_1$ ,  $t_2$ ,  $t_3$ , ...,  $t_k$  can be used as a result of the rapid decrease in the amplitudes and the increase in the relative measurement error, therefore the amplitude of the highest value and the subsequent ones with acceptable measurement error can be taken into account.

Based on the above, according to the acceleration parameter *a*, the eigenfrequency  $\omega = \omega(a)$  and relative attenuation factor  $\delta = \delta(a)$  functions can be determined with a linearization method around the operating point.



Validation of the relative attenuation function for different acceleration capabilities. The functions  $\delta_1 = \delta_1$  (a) and  $\delta_2 = \delta_2$  (a) were determined based on the IDM model parameters and on the basis of the logarithmic decrement, respectively

Based on the above, for each vehicle, the relative attenuation factor  $\delta$  can be defined by a closed formula with the linearization of the nonlinear characteristics of the differential equation system around the operating point. At the same time, attenuation values can be measured in the dynamic vehicle system by examining the attenuating speed processes. Although the phenomenon is non-linear (the oscillation is anharmonic), based on our findings the above linearization can be validated surprisingly well using the logarithmic decrement method. The application of both methods together is very useful and important as it highlights the extent of the actual range to be taken into account in the linearization around the operating point, which means the determination of a related coefficient value. Lehr's relative attenuation factor  $\delta$  derived from measurement or simulation (the notation  $D_L$  is commonly used in the literature) is located in a well-defined range  $(1 < \delta < 1)$ . This physical parameter characterizing the attenuation of the system checks the  $\delta$  value calculated by linearization during our process. The relative error values calculated for the relative attenuation factors  $\delta$  during the validation are shown in the table below for the functions seen in Figure 17.

Table 1
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<b>a</b> [m/s <sup>2</sup> ]	1	2	3	5	6	8	10
Relative error [%]	8.59	6.65	4.50	0.13	1.77	5.53	7.94

# 6 The Variation of Relative Attenuation Factor $\delta$ at Acceleration a=5 m/s<sup>2</sup> and Fixed $\omega$ =0.2

It can be stated that in the case of a low relative attenuation factor ( $\delta$ =0.16-0.18) the speed overshoot is high and instead of 20 m/s it can reach up to 30 m/s, therefore, significant braking is needed so that the speed drops close to 0-10 m/s. Thus, the setting of vehicle speeds is very hectic and the speed oscillation can last for 1-1.5 minutes.

As the relative attenuation increases to  $\delta$ =0.2, the value of the speed overshoot and the setting time of the tracking speed decreases, e.g. at  $\delta$ =0.2 v=25, and the stability time is 1 min. At relative attenuation values  $\delta$ =0.2-0.24 the speed overshoot further decreases and the speed functions become smooth and oscillation-free, but the stability time starts to increase.



# 7 Variation of ω at Acceleration a=5 m/s<sup>2</sup> and Fixed Relative Attenuation Factor δ=0.2

It can be stated that in the case of low eigenfrequency range ( $\omega < 0.17$ ) the speed overshoot is high and instead of 20 m/s it can reach up to 29 m/s, therefore, significant braking is needed so that the speed drops close to 13-15 m/s. Thus, the setting of vehicle speeds is very hectic and the speed oscillation can last for 90 seconds. As the eigenfrequency increases to  $\omega=0.2$ , the value of the speed

overshoot and the setting time of the tracking speed decreases, e.g. at  $\omega$ =0.2 v=25, and the stability time is below 1 min. At eigenfrequency values  $\omega$ =0.2-0.24, 24 the speed overshoot further decreases (v=21) and the speed functions become smooth and oscillation-free, but the stability time starts to increase.



Tables 2 and 3 below show the calculated IDM parameters "s" and "v" related to  $\omega$  and  $\delta$ , while the last column shows the amount of specific energy consumption pertinent to the movement of 1 kg mass of the vehicle.

δ	s, dist. [m]	v [m/s]	Specific energy [Nm]
0,16	10	31,25	689,31
0,18	10	27,77	673,29
0,20	10	25,00	666,71
0,22	10	22,73	660,50
0,23	10	21,74	656,37

Table 2 At fixed a=5m/s^2 and  $\omega{=}0.2$  rad/s values

At optimal value  $\delta$ =0.23 a decrease of 4.779% in the energy consumption of the vehicle group occurred until the stable speed had been reached compared to the one calculated with the initial value  $\delta$ =0.16.

ω [rad/d]	s, dist. [m]	v [m/s]	Specific energy Nm]
0,17	13,84	29,41	673,45
0,18	12,35	27,78	670,08
0,20	10,00	25,00	666,71
0,22	8,26	22,73	662,98
0,24	6,94	20,83	632,74

Table 3 At fixed a=5m/s<sup>2</sup> and  $\delta$ =0.2 values

At optimal value  $\omega$ =0.24 a decrease of 6.045% in the energy consumption of the vehicle group occurred until the stable speed had been reached compared to the one calculated with the initial value  $\omega$ =0.17.

#### Conclusions

The parameters of the IDM base-system are traffic system parameters that can cause uncertainties due to the specificity or the hectic nature of traffic processes. This research investigated the vehicle-dynamical properties of the IDM model by the suitably chosen interpretation and transcription of the mathematical model. In order to further analyse the dynamical properties, it has led to the exploration of newer relations. The chosen model structure and the considerations used are suitable for analysing the multi-mass dynamical model using purely dynamics and vibrations concepts. Based on this, one can obtain important information on the more complex IDM system parameter structure and optimization.

The study takes into account the speed of the vehicle to be followed and determines the optimal maximum speed and the optimal distance for each of the specified acceleration capabilities. This affects both the rate of speed overshoot and the setting time of the stable speed state, as well as the optimization of the motion energy. All of these optimum properties have an impact on the other participants in the traffic, on the emissions and traffic noise. This facilitates smoother traffic processes in the vehicle groups and the automation of traffic-

dependent optimum decisions by automatically adjusting optimal parameters in the case of autonomous vehicles [26, 27, 31].

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