Mathematical Description of the Universal IDM - some Comments and Application

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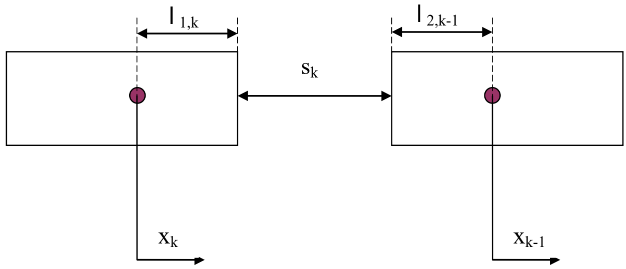
Abstract: The aim of the study is to define and mathematically describe the universal IDM. An important result of this research is that the model uses a single system of differential equations. It is able to simultaneously describe the dynamic operations of the IDM systems for all different vehicle sequences. The aim of the study is to support the driving of autonomous vehicles by taking into account the dynamic variations in the state characteristics of traffic processes. The approach used is motivated by important issues in current modelling techniques that address significant economic problems in the application of large-scale ITS network models. This also points to a new opportunity in the key area of vehicle traffic management, in the related targeted fundamental research, particularly in the analysis of traffic processes in large-scale dynamic networks.

Keywords: Universal IDM; differential equations system; dynamics-based approach; autonomous vehicles

1 Introduction

The Intelligent Driver Model (IDM) belongs to the family of adaptive cruise control (ACC) system models. It was developed in 2000 by Treiber, Hennecke and Helbing at the Transport Laboratory of the Technical University of Dresden and used by the car manufacturer BMW. For the multi-model open source road traffic simulator, Treiber and Helbing [10] use the IDM to simulate the longitudinal motion of the vehicle and this simulator also presents a lane change strategy with a software solution. The inhomogeneity of model-based single-lane traffic was investigated by Treiber et al. [13]. Treiber et al. [14], examines the stability of vehicle traffic and the parameter sensitivity of the IDM. Kesting et al. [16] propose the extension of the driver parameters of the IDM model. The authors investigate the impact of IDM-equipped vehicles on traffic flow and travel time. Jerath [4] also uses the IDM and studies the effect of adaptive cruise control on traffic processes. The results of the above works show that increasing the proportion of ACC-equipped vehicles leads to an increase in traffic efficiency by reducing travel time. Treiber and Kesting [5, 15] investigated the instability of congested traffic using IDM. The classic IDM is a chain model-like microscopic model consisting of n vehicles that describes the longitudinal dynamics of the vehicles, Figure 1 and Figure 2. Each driver looks only forward and tries to maintain an appropriate following distance. There is no overtaking in the model, i.e. the vehicles maintain their order. The first vehicle in the group and slow moving vehicles also play a key role in the model. The longitudinal dynamics of the vehicle traffic system are determined by the parameters and relationship functions of the system, and this allows the vehicles to adapt their speed to the environment (1).



$$s_k = (x_{k-1} - l_{2,k-1}) - (x_k + l_{1,k}) = (x_{k-1} - x_k) - (l_{2,k-1} + l_{1,k}) = (x_{k-1} - x_k) - l_k$$

Figure 1

The distance s_k between successive vehicles

Regarding the longitudinal dynamics, we followed the classic dynamic principle for all vehicles when creating the model. The coordinates x_k (k=1,2, …, n) describe the longitudinal movement of the vehicle's centers of gravity. In the model defined by us, the length of each k-th vehicle can be obtained from the sum of $l_{1,k}$ and $l_{2,k}$, ($L_k = l_{1,k} + l_{2,k}$). If we use the simplifying condition - that the center of gravity is at the geometric center of the length of each vehicle, then in this case $0.5\ L_k = l_{1,k}$ (k=1,2, …, n) and $s_k = (x_{k-1} - x_k) - 0.5(L_{k-1} + L_k)$.

$$\dot{v_k} = a_k \left[1 - \left(\frac{v_k}{v_k^0} \right)^{\delta} - \left(\frac{s^*(v_k, \Delta v_k)}{s_k} \right)^2 \right] \tag{1}$$

For each vehicle, the motions according to their position are determined by the following system of differential equations (2) for vehicle groups 1, 2, ..., n. Derbel, O.; Peter, T.; Zebiri, H.; Mourllion, B.; Basset, M. [1, 2]. The following Derbel, O.; Peter, T.; Zebiri, H.; Mourllion, B.; For the Basset, M. [1, 2] model, we have already taken into account the more precise correlation using s*.

$$s^*\left(\dot{x}_k, (\dot{x}_{k-1} - \dot{x}_k)\right) = s_0 + \dot{x}_k T + \frac{\dot{x}_k(\dot{x}_{k-1} - \dot{x}_k)}{2\sqrt{ab}}; \ (k=1,2,\dots,n).$$

This can be found in the material of Treiber, M and Kesting, A [17], which discusses the various research areas of transport processes in great detail.

$$\frac{\ddot{x_1}}{a_1} + \left(\frac{\dot{x}_1}{v_1^0} \right)^{\delta} + \left(\frac{s_0 + \dot{x}_1 T + \frac{\dot{x}_1(\dot{x}_0 - \dot{x}_1)}{2\sqrt{ab}}}{(x_0 - x_1) - l_1} \right)^2 = 1$$

$$\frac{\ddot{x_2}}{a_2} + \left(\frac{\dot{x}_2}{v_2^0} \right)^{\delta} + \left(\frac{s_0 + \dot{x}_2 T + \frac{\dot{x}_2(\dot{x}_1 - \dot{x}_2)}{2\sqrt{ab}}}{(x_1 - x_2) - l_2} \right)^2 = 1$$

$$\frac{\ddot{x_n}}{a_n} + \left(\frac{\dot{x}_n}{v_n^0} \right)^{\delta} + \left(\frac{s_0 + \dot{x}_n T + \frac{\dot{x}_n(\dot{x}_{n-1} - \dot{x}_n)}{2\sqrt{ab}}}{(x_{n-1} - x_n) - l_n} \right)^2 = 1 \tag{2}$$

Where meaning of the parameters:

a_k is the maximum acceleration of the k-th vehicle,

x_k is the position of the k-th vehicle,

$\dot{x}_k = v_k$ is the speed of the k-th vehicle,

$\ddot{x}_k$ is the acceleration of the *k*-th vehicle,

v_0^k is the desired speed of the *k*-th vehicle,

$x_{k-1} - x_k$ is the distance between the centre of gravity of the *(k-1)*-th and *k*-th vehicles,

s_k is the distance between the (k-1)-th and k-th vehicles (the vehicle length, in the case of our article, varies by vehicle, Figure 1.

$\Delta v_k = v_{k-1} - v_k$ is the difference between the speed of *(k-1)*-th and *k*-th

T Safe time headway

a the Maximum acceleration

b Comfortable deceleration

δ Acceleration exponent

s_0 Minimum distance

Modeling and research work covers a complex field and includes both microscopic and macroscopic modeling approaches, Treiber et al. [11, 12], as well as, e.g., the Generalized Velocity–Density Model based on microscopic traffic simulation, Derbel, O., Peter, T., Mourllion B., and Basset M. [3], Regarding the complex macroscopic traffic environment, the generation of the large-scale network model is the important task, for example, Péter T. and Bokor J. [7, 8, 9].

In these studies [1, 2, 3, 6], we already applied one (3) generalized, structural method to the IDM. At the same time, this is only an IDM in which the vehicle positions and vehicle numbers are the same (in the first position is vehicle 1, in the second position is vehicle 2 and in the nth position is the nth vehicle).

In our article, the procedure presented below is, how to write down and store all possible sequential IDMs in a single matrix-difference system?

$$\left\langle \underline{\underline{A}} \right\rangle^{-1} \ddot{x}(t) + \left\langle \underline{\underline{V}} \right\rangle^{-1} \underline{f_1}(\dot{x}(t)) + \left\langle \underline{\underline{S}} \right\rangle \underline{f_2}(x(t)) = \underline{1} \tag{3}$$

Where:

$$\left\langle \underline{\underline{A}} \right\rangle^{-1} = \left\langle \frac{1}{a_1}, \frac{1}{a_2}, \ldots, \frac{1}{a_n} \right\rangle; \quad \left\langle \underline{\underline{V}} \right\rangle^{-1} = \left\langle \frac{1}{v_1^4}, \frac{1}{v_2^4}, \ldots, \frac{1}{v_n^4} \right\rangle; \quad \left\langle \underline{\underline{S}} \right\rangle = \left\langle s_1^2, s_2^2, \ldots, s_n^2 \right\rangle$$

(In the formulas, the value of the acceleration exponent δ was set to 4.)

$$s_i = s_i(\dot{x}_{i-1}, \dot{x}_i) \qquad (i=1,2, \ldots, n).$$

$$\underline{f_1}(\dot{x}(t)) = \begin{bmatrix} \dot{x}_1^4 \\ \dot{x}_2^4 \\ \ldots \\ \dot{x}_n^4 \end{bmatrix} \qquad \underline{f_2}(x(t)) = \begin{bmatrix} \frac{1}{(x_0 - x_1 - l_1)^2} \\ \frac{1}{(x_1 - x_2 - l_2)^2} \\ \ldots \\ \ldots \\ \frac{1}{(x_{n-1} - x_n - l_n)^2} \end{bmatrix} \qquad \underline{1} = \begin{bmatrix} 1 \\ 1 \\ \ldots \\ 1 \end{bmatrix}$$

2 The Universal IDM

The model presented in this article, a mathematically, it is a complex model containing n different vehicles in every possible order. In the model, the parameter structure can be fixed, but the parameters can also be stochastic if this is necessary in connection with further investigations.

This allows for a very efficient, automatic model generation in connection with a large number of differential equation systems belonging to different IDMs.

The Universal IDM will be, a generalization of classic IDM, which still requires certain further research.

In this case, this generalization means that in a single complex mathematical model, vehicle elements i = 1,2, ..., n can freely implement overtaking strategies, on a network sector.

In our case, since we used a mathematical construction, it is advisable to continue further mathematical investigations based on this.

3 Relationship between the IDM Model and the Network Domain

For a given vehicle, the speed and the tracking distance are determined by the driver. Your decision depends on both your own perceptions and the signals sent by your vehicle from the physical environment. All of this has a crucial impact on network traffic. Accordingly, the quality of the road and the physical effects of meteorological and visual conditions determine the selectable speed range for a given vehicle density. The IDM can be used to describe the effect of dynamic relationships between successive vehicles in a given section. At the same time, the dynamics of the motion of the IDM group is not only self-regulating, but is also determined by the control speeds of the large-scale network and network sectors [18, 19, 20, 21, 22, 23, 24, 28].

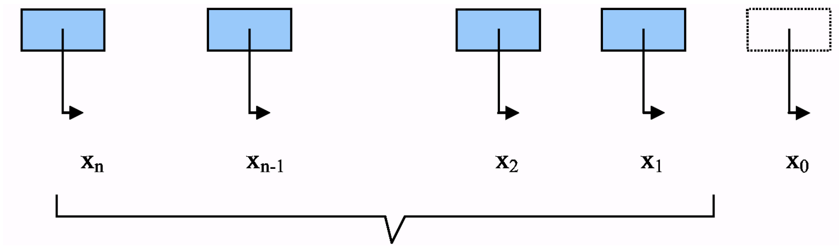


Figure 2
n elements, n! possible orders, the control function $x_0(t)$

The vehicles slow down when congestion occurs, stop when the traffic light turns red, but accelerate to the maximum speed limit after a reaction time delay if the road section ahead is clear. This is indicated in Fig. 2 by the control function $x_0(t)$, which is determined by the large-scale macroscopic network processes of each trajectory.

Based on the above, the IDM groups of all sectors are also continuously being reorganized. On this basis, it is important to emphasize that in each sector, according to reality:

1) The number of vehicles changes dynamically.

2) The order of the vehicles changes dynamically.

3) The composition of vehicles changes dynamically.

4) Overtaking manoeuvres are carried out in accordance with the parameters of the vehicles in the system taking into account the actual conditions.

5) These processes can be controlled according to the needs of the different vehicles and the traffic situation.

The usage of the model:

1) Each sector independently controls the addition and deletion of continuous input and output flows to and from the IDM operating on it.

2) Each sector independently controls and manages the input and output flows of the IDM system operating on it, and also takes into account the regulations and prohibitions resulting from the management of network traffic.

4 Mathematical Approach and Modelling Procedure

In the model, a fixed initial state is described by the vector **p**, where, for example, for n = 3, the default state is the following: p[1] = 1, p[2] = 2, p[3] = 3.

As we move forward, of course, positions may change and the value of the vector coordinate p_i will be, for example, k.

The meaning of $p_i = k$ is that the *k*-th vehicle is in the *i*-th position in the queue at the given time (i = 1, 2, …, n; k = 1,2, …, n;).

In the model, the existence of a direct relationship between two vehicles is determined by the values of the elements of matrix **U** defined by us. The existence of a relationship is represented by the value 1, while the fact that there is no relationship between them is defined by the value 0.

Consequently, these relationships are contained in the matrix u[i, k], (i =1,2, …, n; k =1,2, …, n), in which in each *i*-th row, only u[i, p[i]] = 1, thus u[i, k] = 1 and the other elements of the *i*-th row are zero (i = 1,2, …, n).

In the system of differential equations of the universal IDM, the elements of the matrix u[i, k] are used by the matrices **A** and **B**. Matrix **A** takes into account the maximum acceleration a_i of the *i*-th vehicle, while matrix **B** takes into account the desired speed v_i of the *i*-th vehicle.

In the case of the system of differential equations the 3-dimensional matrix k[j, p, i] is used in the vector ***f***. The vector ***f*** determines the relations for the quotients of the squared distances between the two vehicles or determines the non-existence of these relations.

Let us consider first the first equation of the system of differential equations. Then the value of the matrix k[j, p, i] will be 1 only in the case (4) when:

k[0, p[1],1]=1 (4)

Namely, it is the value of p[1] that determines which *k*-th vehicle is at the front of the queue and follows the control signal $x_0(t)$.

Accordingly, the value of the other elements in the sum is zero.

Next, let us examine the values of the elements of the matrix k[j, p, i] in the second equation of the system of differential equations.

Here, a value of 1 only applies to vehicles with serial numbers at positions p_1 and p_2, according to the following conditions:

if ((p[1] < p[2])) then k[p[1], p[2], 2] =1 (5)

if ((p[1] > p[2])) then k[p[2], p[1], 2] =1

the other elements in the sum are zero, and so on.

In the last line of the system of differential equations, 1 value also occurs in only two cases. Exactly, only for vehicles with serial numbers at positions p_{n-1} and p_n, under the following conditions:

if ((p[n-1]<p[n])) then k[p[n-1],p[n],n] =1; (6)

if ((p[n-1]>p[n])) then k[p[n],p[n-1],n] =1

the other elements in the sum are zero.

One result of this method is that the IDM for all permutations of the n-element vehicle group can be summarized in a single model. Based on this, the IDM for any vehicle sequence can be written by appropriately setting the elements of the matrices u[i, k] and k[j, p, i]. This model definition thus allows for complex modeling of definitively formed sequences based on a single model.

Another important result of the method is that it integrated all possible n-element IDMs into a single complex model.

In this way, it also prepares the possibility of a mathematical analysis of structural changes occurring during system transitions.

5 The System of Differential Equations of the Universal IDM for n Vehicles

The requirements are described by the following system of matrix differential equations:

$$A \cdot \ddot{x} + B \cdot \dot{x}^4 + f(x) = e(t) \tag{7}$$

(In the formulas, the value of the acceleration exponent δ was set to 4. Naturally, 4 can be replaced with δ and the acceleration exponent can be used in general.)

Where, on the left side of the above system of matrix differential equations, the products, are column vectors. The elements of the matrices **A** and **B** comprise the matrix elements $u_{i,j}$ discussed above, as follows:

$$A = [u_{i,j} 1/a_j];\ B = [u_{i,j}\ 1/v_j^4]$$

In mathematics, this is the **Hadamard product**, also known as the **Schur product**, which is the product of the elements in the same place in two matrices, resulting in a matrix of the same dimension as the two matrices. It is named after the French mathematician Jacques Hadamard and the German mathematician Issai Schur. The vector ***f*** contains the matrix elements $k_{j,\,p,\,i}$.

Where:

$$A=\begin{bmatrix} \frac{u_{11}}{a_1} & \frac{u_{12}}{a_2} & \dots & \frac{u_{1n}}{a_n} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \frac{u_{ij}}{a_j} & \dots \\ \frac{u_{n1}}{a_1} & \frac{u_{n2}}{a_2} & \dots & \frac{u_{nn}}{a_n} \end{bmatrix}; \quad B=\begin{bmatrix} \frac{u_{11}}{v_1^{\;4}} & \frac{u_{12}}{v_2^{\;4}} & \dots & \frac{u_{1n}}{v_n^{\;4}} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \frac{u_{ij}}{v_j^{\;4}} & \dots \\ \frac{u_{n1}}{v_1^{\;4}} & \frac{u_{n2}}{v_2^{\;4}} & \dots & \frac{u_{nn}}{v_n^{\;4}} \end{bmatrix};$$

$$f=\begin{bmatrix} \sum_{p=1}^{n}\frac{k_{0,p,1}\cdot s_{0,p}^{\;\;2}}{(x_0-x_p-l_p)^2} \\ \dots \\ \sum_{j=1}^{n-1}\sum_{p=j+1}^{n}\frac{k_{j,p,i}\cdot s_{j,p}^{\;\;2}}{(x_j-x_p-l_p)^2} \\ \sum_{j=1}^{n-1}\sum_{p=j+1}^{n}\frac{k_{j,p,n}\cdot s_{j,p}^{\;\;2}}{(x_j-x_p-l_p)^2} \end{bmatrix}$$

$x\in\Re^n$ is the state characteristic vector of the vehicle positions, where n is the number of vehicles,

$\dot{x}\in\Re^n$, $\ddot{x}\in\Re^n$ are the vectors of vehicle speeds and accelerations

$A, B\in\Re^{nxn}$ $f,e\in\Re^n$

In a more concise notation, the matrices and vectors in the system of equations can be summarized as follows:

$$A=\begin{bmatrix} \frac{u_{ij}}{a_j} \end{bmatrix}; \quad B=\begin{bmatrix} \frac{u_{ij}}{v_j^{\;4}} \end{bmatrix} \quad f=\begin{bmatrix} f_i \end{bmatrix}$$

($i=1, 2, \dots, n; j=1, 2, \dots, n$)

for i=1:

$$\sum_{p=1}^{n}\frac{k_{0,p,1}\cdot s_{0,p}^{\;\;2}}{(x_0-x_p-1_p)^2}$$

Where:

$$s_{0,p}^2=\left(s_0+\dot{x}_pT+\frac{\dot{x}_p(\dot{x}_0-\dot{x}_p)}{2\sqrt{ab}}\right)^2; \ (p=1,2,\dots,n);$$

for i>1:

$$f_i = \sum_{j=1}^{n-1} \sum_{p=j+1}^{n} \frac{k_{j,p,i} \cdot s_{j,p}{}^2}{(x_j - x_p - 1_p)^2}$$

Where:

$$s_{j,p}^2 = \left(s_0 + \dot{x}_p T + \frac{\dot{x}_p(\dot{x}_j - \dot{x}_p)}{2\sqrt{ab}}\right)^2; \; (j=1,2,\ldots,n\text{-}1; p=j+1,\ldots,n);$$

The total number of all IDM is n! Let us consider, as an example, the collective system of differential equations containing all sequences for n = 3:

$$\begin{bmatrix} \frac{u_{1,1}}{a_1} & \frac{u_{1,2}}{a_2} & \frac{u_{1,3}}{a_3} \\ \frac{u_{2,1}}{a_1} & \frac{u_{2,2}}{a_2} & \frac{u_{2,3}}{a_3} \\ \frac{u_{3,1}}{a_1} & \frac{u_{3,2}}{a_2} & \frac{u_{3,3}}{a_3} \end{bmatrix} \begin{bmatrix} \frac{d^2}{dt^2} x1(t) \\ \frac{d^2}{dt^2} x2(t) \\ \frac{d^2}{dt^2} x3(t) \end{bmatrix} + \begin{bmatrix} \frac{u_{1,1}}{v_1^4} & \frac{u_{1,2}}{v_2^4} & \frac{u_{1,3}}{v_3^4} \\ \frac{u_{2,1}}{v_1^4} & \frac{u_{2,2}}{v_2^4} & \frac{u_{2,3}}{v_3^4} \\ \frac{u_{3,1}}{v_1^4} & \frac{u_{3,2}}{v_2^4} & \frac{u_{3,3}}{v_3^4} \end{bmatrix} \begin{bmatrix} \left(\frac{d}{dt} x1(t)\right)^4 \\ \left(\frac{d}{dt} x2(t)\right)^4 \\ \left(\frac{d}{dt} x3(t)\right)^4 \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{k_{0,1,1} s_{0,1}^2}{(x_0 - x_1 - l_1)^2} + \frac{k_{0,2,1} s_{0,2}^2}{(x_0 - x_2 - l_2)^2} + \frac{k_{0,3,1} s_{0,3}^2}{(x_0 - x_3 - l_3)^2} \\ \frac{k_{1,2,2} s_{1,2}^2}{(x_1 - x_2 - l_2)^2} + \frac{k_{1,3,2} s_{1,3}^2}{(x_1 - x_3 - l_3)^2} + \frac{k_{2,3,2} s_{2,3}^2}{(x_2 - x_3 - l_3)^2} \\ \frac{k_{1,2,3} s_{1,2}^2}{(x_1 - x_2 - l_2)^2} + \frac{k_{1,3,3} s_{1,3}^2}{(x_1 - x_3 - l_3)^2} + \frac{k_{2,3,3} s_{2,3}^2}{(x_2 - x_3 - l_3)^2} \end{bmatrix} = \begin{bmatrix} e(t) \\ e(t) \\ e(t) \end{bmatrix} \quad (8)$$

In total, six sequences can be generated for each initial case. All the cases that can be automatically generated by computer algebra are described below.

Case No.1: where the order of the vehicles is 1, 2, 3, contained in the vector p:

$$p^1=1,\; p^2=2,\; p^3=3$$

In this case, the calculated values of $u_{i,j}$ are as follows for u[i,p[i]]:=1; (i = 1,2,3):

$$u^{1,1}=1,\; u^{2,2}=1,\; u^{3,3}=1$$

The following algorithm is used to calculate the values of **k**[j,p,i], for **k**[0,p[1],1]:=1:

if ((p[1] <p[2])) then k[p[1],p[2],2]:=1; end; if ((p[1]>p[2])) then k[p[2],p[1],2]:=1; end if;
if ((p[2] <p[3])) then k[p[2],p[3],3]:=1; end; if ((p[2]>p[3])) then k[p[3],p[2],3]:=1; end if; (9)

The values of $k_{i,j,l}$ calculated using the algorithm are as follows:

$$k^{0,1,1}=1,\ k^{1,2,2}=1,\ k^{2,3,3}=1$$

Based on the above, the following final system of differential equations was determined in case No. 1 (10):

$$\begin{bmatrix} \dfrac{\frac{d^2}{dt^2} x1(t)}{a_1} \\ \dfrac{\frac{d^2}{dt^2} x2(t)}{a_2} \\ \dfrac{\frac{d^2}{dt^2} x3(t)}{a_3} \end{bmatrix} + \begin{bmatrix} \dfrac{\left(\frac{d}{dt} x1(t)\right)^4}{v_1^4} \\ \dfrac{\left(\frac{d}{dt} x2(t)\right)^4}{v_2^4} \\ \dfrac{\left(\frac{d}{dt} x3(t)\right)^4}{v_3^4} \end{bmatrix} + \begin{bmatrix} \dfrac{s_{0,1}^2}{(x_0 - x_1 - l_1)^2} \\ \dfrac{s_{1,2}^2}{(x_1 - x_2 - l_2)^2} \\ \dfrac{s_{2,3}^2}{(x_2 - x_3 - l_3)^2} \end{bmatrix} = \boldsymbol{e}(t) \quad (10)$$

Following the above algorithm, the computer algebraic method automatically provides the further results.

Case No. 2: the order of the vehicles is 1, 3, 2. The calculated values of $u_{i,j}$ and $k_{i,j,l}$ are:

$$p^1=1,\ p^2=3,\ p^3=2$$

$$u^{1,1}=1,\ u^{2,3}=1,\ u^{3,2}=1$$

$$k^{0,1,1}=1,\ k^{1,3,2}=1,\ k^{2,3,3}=1$$

In case No. 2, the final system of differential equations is as follows:

$$\begin{bmatrix} \dfrac{\frac{d^2}{dt^2} x1(t)}{a_1} \\ \dfrac{\frac{d^2}{dt^2} x3(t)}{a_3} \\ \dfrac{\frac{d^2}{dt^2} x2(t)}{a_2} \end{bmatrix} + \begin{bmatrix} \dfrac{\left(\frac{d}{dt} x1(t)\right)^4}{v_1^4} \\ \dfrac{\left(\frac{d}{dt} x3(t)\right)^4}{v_3^4} \\ \dfrac{\left(\frac{d}{dt} x2(t)\right)^4}{v_2^4} \end{bmatrix} + \begin{bmatrix} \dfrac{s_{0,1}^2}{(x_0 - x_1 - l_1)^2} \\ \dfrac{s_{1,3}^2}{(x_1 - x_3 - l_3)^2} \\ \dfrac{s_{2,3}^2}{(x_2 - x_3 - l_3)^2} \end{bmatrix} = \boldsymbol{e}(t) \quad (11)$$

Case No. 3: the order of the vehicles is 2, 1, 3. The calculated values of $u_{i,j}$ and $k_{i,j,l}$ are:

$$p^{1}=2\; p^{2}=1,\; p^{3}=3$$

$$u^{1,2}=1,\; u^{2,1}=1,\; u^{3,3}=1$$

$$k^{0,2,1}=1,\; k^{1,2,2}=1,\; k^{1,3,3}=1$$

In case No. 3, the final system of differential equations is as follows:

$$\begin{bmatrix} \dfrac{\frac{d^2}{dt^2} x2(t)}{a_2} \\ \dfrac{\frac{d^2}{dt^2} x1(t)}{a_1} \\ \dfrac{\frac{d^2}{dt^2} x3(t)}{a_3} \end{bmatrix} + \begin{bmatrix} \dfrac{\left(\frac{d}{dt} x2(t)\right)^4}{v_2^4} \\ \dfrac{\left(\frac{d}{dt} x1(t)\right)^4}{v_1^4} \\ \dfrac{\left(\frac{d}{dt} x3(t)\right)^4}{v_3^4} \end{bmatrix} + \begin{bmatrix} \dfrac{s_{0,2}^2}{(x_0 - x_2 - l_2)^2} \\ \dfrac{s_{1,2}^2}{(x_1 - x_2 - l_2)^2} \\ \dfrac{s_{1,3}^2}{(x_1 - x_3 - l_3)^2} \end{bmatrix} = \boldsymbol{e}(t) \qquad (12)$$

Case No. 4: the order of the vehicles is 2, 3, 1. The calculated values of $u_{i,j}$ and $k_{i,j,l}$ are:

$$p^{1}=2,\; p^{2}=3,\; p^{3}=1$$

$$u^{1,2}=1,\; u^{2,3}=1,\; u^{3,1}=1$$

$$k^{0,2,1}=1,\; k^{2,3,2}=1,\; k^{1,3,3}=1$$

In case No. 4, the final system of differential equations is as follows:

$$\begin{bmatrix} \dfrac{\frac{d^2}{dt^2} x2(t)}{a_2} \\ \dfrac{\frac{d^2}{dt^2} x3(t)}{a_3} \\ \dfrac{\frac{d^2}{dt^2} x1(t)}{a_1} \end{bmatrix} + \begin{bmatrix} \dfrac{\left(\frac{d}{dt} x2(t)\right)^4}{v_2^4} \\ \dfrac{\left(\frac{d}{dt} x3(t)\right)^4}{v_3^4} \\ \dfrac{\left(\frac{d}{dt} x1(t)\right)^4}{v_1^4} \end{bmatrix} + \begin{bmatrix} \dfrac{s_{0,2}^2}{(x_0 - x_2 - l_2)^2} \\ \dfrac{s_{2,3}^2}{(x_2 - x_3 - l_3)^2} \\ \dfrac{s_{1,3}^2}{(x_1 - x_3 - l_3)^2} \end{bmatrix} = \boldsymbol{e}(t) \qquad (13)$$

Case No. 5: the order of the vehicles is 3, 1, 2. The calculated values of $u_{i,j}$ and $k_{i,j,l}$ are:

$$p^{1}=3,\; p^{2}=1,\; p^{3}=2$$

$$u^{1,3}=1,\ u^{2,1}=1,\ u^{3,2}=1$$

$$k^{0,3,1}=1,\ k^{1,3,2}=1,\ k^{1,2,3}=1$$

In case No. 5, the final system of differential equations is as follows:

$$\begin{bmatrix} \dfrac{\frac{d^2}{dt^2} x3(t)}{a_3} \\ \dfrac{\frac{d^2}{dt^2} x1(t)}{a_1} \\ \dfrac{\frac{d^2}{dt^2} x2(t)}{a_2} \end{bmatrix} + \begin{bmatrix} \dfrac{\left(\frac{d}{dt} x3(t)\right)^4}{v_3^4} \\ \dfrac{\left(\frac{d}{dt} x1(t)\right)^4}{v_1^4} \\ \dfrac{\left(\frac{d}{dt} x2(t)\right)^4}{v_2^4} \end{bmatrix} + \begin{bmatrix} \dfrac{s_{0,3}^2}{(x_0 - x_3 - l_3)^2} \\ \dfrac{s_{1,3}^2}{(x_1 - x_3 - l_3)^2} \\ \dfrac{s_{1,2}^2}{(x_1 - x_2 - l_2)^2} \end{bmatrix} = \boldsymbol{e}(t) \tag{14}$$

Case No. 6: the order of the vehicles is 3, 2, 1. The calculated values of $u_{i,j}$ and $k_{i,j,l}$ are:

$$p^{1}=3,\ p^{2}=2,\ p^{3}=1$$

$$u^{1,3}=1,\ u^{2,2}=1,\ u^{3,1}=1$$

$$k^{0,3,1}=1,\ k^{2,3,2}=1,\ k^{1,2,3}=1$$

In case No. 6, the final system of differential equations is as follows:

$$\begin{bmatrix} \dfrac{\frac{d^2}{dt^2} x3(t)}{a_3} \\ \dfrac{\frac{d^2}{dt^2} x2(t)}{a_2} \\ \dfrac{\frac{d^2}{dt^2} x1(t)}{a_1} \end{bmatrix} + \begin{bmatrix} \dfrac{\left(\frac{d}{dt} x3(t)\right)^4}{v_3^4} \\ \dfrac{\left(\frac{d}{dt} x2(t)\right)^4}{v_2^4} \\ \dfrac{\left(\frac{d}{dt} x1(t)\right)^4}{v_1^4} \end{bmatrix} + \begin{bmatrix} \dfrac{s_{0,3}^2}{(x_0 - x_3 - l_3)^2} \\ \dfrac{s_{2,3}^2}{(x_2 - x_3 - l_3)^2} \\ \dfrac{s_{1,2}^2}{(x_1 - x_2 - l_2)^2} \end{bmatrix} = \boldsymbol{e}(t) \tag{15}$$

The above system of equations therefore describes an already formed and constant series over a certain period of time.

Conclusions

It can be concluded that the application of IDM chain models in itself has provided and is currently providing opportunities for many useful investigations related to traffic processes. In this regard, the automatic generation of a large number of these models provides a very useful additional contribution to these studies, model studies have been carried out, e.g., [30], [34]. The specified model, mathematically, it is a complex model containing n different vehicles in every possible order. In the model, the parameter structure can be fixed, but the parameters can also be stochastic, if this is necessary in connection with further investigations. The (7) is suitable for generating IDMs for different n-element vehicle lines since the entire connection system is located and stored in a single complex system of differential equations. This allows for a very efficient, automatic model generation in connection with a large number of differential equation systems belonging to different IDMs.

Based on the formulation of the principles, it is important to determine which objectives should be applied under the given traffic conditions. These can be: optimal proceeding through intersections, optimal energy consumption in traffic processes and the related optimal CO2 emissions, optimal environmental impact, and rapid transfer of vehicle convoys through the network domain [25, 26, 27, 29, 31, 32, 33]. In a more general approach, the parameters of the IDM system are transport system parameters. The vehicle dynamics parameters of the IDM can be investigated by an appropriately chosen rewriting of the mathematical model. The chosen model structure is also suitable for the analysis of the multi-mass vibrational dynamics model, using purely vibration theory concepts [6].

In the field of algorithms and programming related to overtaking and lane changes, there are indeed many excellent procedures. In our case, since we used a mathematical construction, it is advisable to continue further mathematical investigations based on this. The change in structure during system transitions is a very interesting problem and it really requires further important research, e.g. [35].

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