Orders in Semirings of Transition Bistochastic Matrices Induced by Mobility Measure of Social Sciences

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Abstract: The order of transition matrices induced by mobility measure is presented. A semiring is formed over the set of all bistochastic matrices in which the order is induced by mobility measure which satisfies relaxed Shorrocks monotonicity condition and all other Shorrocks axioms.

Keywords: mobility measure, transition matrix, semiring

1 Introduction

In 1978, Shorrocks [11] defined mobility index in the social sciences, as a continuous function over the set of transition matrices, and he was first to provide axiomatic approach to mobility indices, see [1], [4]. However, Shorrocks himself showed that the axioms he proposed are not consistent for all mobility indices, i.e., there is no mobility index which satisfies all axioms. Different indices detect various mobility aspects. Mobility is defined as movement of dynamic system from one state to the other in time. In social sciences, there are important different types of mobility, as social classes mobility, intergenerational mobility, intragenerational mobility, etc [4]. The selection of the mobility index is very important. Namely, it should satisfy different motivations for measuring mobility. It is common practice, when selecting mobility indices, to initially define the desired features which these indices should satisfy, and that these features have to be consistent. The applied mobility indices have a great influence on transition matrices ranking.

In this paper, the motivation is to measure mobility as movement by using mobility indices which induce partial order on the set of transition matrices. Then every two transition matrices can be compared by using such mobility indices. Transition matrices, which have more movement in them, must have higher mobility index. In this paper the set of transition matrices is reduced to the set of transition bistochastic matrices, as well as the properties of mobility indices are given. Mobility indices with selected properties induces partial order on the set of transition bistochastic matrices and semirings are formed.

Since mobility index is a bounded function, the mobility index minimal value is zero, and the maximal is one. There are also controversies over the selection of transition matrices with minimal and maximal mobility. Transition matrices, as non-negative matrices, are closely related to the class of stochastic processes which are Markov chains. Markov chains are used as theoretical models for description of a system which can be found in different states. In Section 2, an overview of definitions related to Markov Chains and transition matrices are given, and specially important homogenous regular Markov Chains. In Section 3, Shorrocks axiomatic approach to mobility measures is described with stress laid on the inconsistency of these axioms and possibilities of overcoming of this inconsistency. A brief overview of the influences of mobility measure on the order of transition matrices and the problem area of selecting transition matrices which have the values of mobility indices 0 or 1 are presented. In Section 4 a semiring is formed over the set of all the bistochastic transition matrices in which partial ordering is induced by mobility measure, which satisfies some of the Shorrocks axioms. In other words, mobility measure which induces the order in the formed semiring satisfies all the Shorrocks axioms except the monotonicity axiom. The way out is that the mobility measure satisfies the relaxed monotonicity condition thereby achieving the consistency of the axioms.

2 Transition Matrix of Markov Chain

Markov chains (MCs) are used to describe a system which can be found in different states, see [11]. The system passes from one state to the other in time, and this transition is described by the set of transition probabilities $p_{ij}(k)$. If the behaviour of the system is known at the initial time (time 0), the set of transition probabilities determines the behaviour of the system. According to [10] we have the following definitions.

Definition 2.1: For a given a countable state space $S=\{s_0, s_1, s_2,..\}$ a sequence of random variables $(X_k)_{k\in N'}$, where $N' = N \cup \{0\}$, taking values in S, is called *Markov Chain* if it has the following property: if $x_0, x_1, ..., x_{k+1}$ are elements of S, then

$$P(X_{k+1} = x_{k+1}/X_k = x_k,...,X_0 = x_0) = P(X_{k+1} = x_{k+1}/X_k = x_k)$$

if $P(X_k = x_k,...,X_0 = x_0) > 0$. We call the probability $P(X_{k+1} = s_j \,/\, x_k = s_i)$ the *transition probability* from state s_i to state s_j and write it as $p_{ij}(k+1)$, s_i, $s_j \in S$, $k \in$N.

We denote the row vector of the initial distribution by $\prod_0'$. We have by [3], [10].

Definition 2.2: (i) For fixed k in N the matrix $\mathbf{P_k} = [p_{ij}(k)]$, s_i, $s_j \in S$, is called *the transition matrix* with non-negative elements.

(ii) $\Omega = \{ \mathbf{P} \,/ p_{ij} \geq 0 \;\forall p_{ij},\; \sum_{j=1}^{n} p_{ij} = 1 \}$ is called the class of *stochastic matrices*.

(iii) $\Gamma = \{ \mathbf{P} \,/ p_{ij} \geq 0 \;\forall p_{ij},\; \sum_{i=1}^{n} p_{ij} = 1,\; \sum_{j=1}^{n} p_{ij} = 1 \}$ is called the class of *bistochastic matrices.*

(iv) If $\mathbf{P_1 = P_2 = ... = P_k}$... the Markov chain is said to have *stationary transition probabilities* or is said to be *homogeneous*. Otherwise it is non-homogeneous.

Let $\mathbf{P_k}$ be a transition matrix. Denote by $\prod_k'$ the row vector of the probability distribution of X_k. We shall use notation

$T_{p,r} = \mathbf{P_{p+1}P_{p+2}...P_{p+r}},$

and write $\prod_k' = \prod_0' T_{0,k}$. For $k > p$ it is

$\prod_k' = \prod_p' T_{p,k-p}$.

For homogeneous Markov chain it holds: $T_{p,k} = \mathbf{P}^k$.

Definition 2.3: (i) A square non-negative matrix $\mathbf{P}$ is said to be *primitive* if there exists a positive integer k such that $\mathbf{P}^k > 0$.

(ii) Any initial probability distribution $\prod_0$ is said to be *stationary*, if $\prod_0' = \prod_k'$, and a Markov chain with such an initial distibution is itself said to be stationary.

Let us denote the *stationary distributon* by π.

Definition 2.4: (i) An n×n non-negative matrix $\mathbf{P}$ is *irreducible* if for every pair i, j of its index set, there exist a positive integer $m \equiv m(i,j)$ such that $p_{ij}^{(m)} > 0$.

(ii) MCs is *irreducible* when its transition matrix is irreducible.

Irreducible matrix cannot have a zero row or column, see [10].

Theorem 2.5: An irreducible MCs has a unique stationary distribution π' , given as a solution of the equations $\pi'\mathbf{P} = \pi'$ and $\pi'\mathbf{1} = 1$, where **1** is vector column with unity in each position, and $\mathbf{1}\pi'$ is a transition matrix whose rows are all equal to π'.

We have by [10].

Theorem 2.6: (Ergodic Theorem for primitive MCs) For a primitive MCs we have: $\lim_{k\to\infty} \mathbf{P^k} = \mathbf{1}\pi'$ elementwise, where π is the unique stationary distribution of the MCs, and the rate of approach to the limit is geometric.

Following the literature on mobility indices, we assume that the transition matrix **P** is homogeneous, irreducible and primitive. Then there exist a unique stationary distribution π (vector column of probability distribution). Morever, $\lim_{k\to\infty} \mathbf{P^k} = \mathbf{1}\pi'$.

3 Mobility Measure on Transition Matrices

In 1978, Shorrocks [11] defined mobility measure as a continuous real function *M* over the set $\mathsf{P}_\mathbf{P}$ of all transition matrices.

Definition 3.1: Mobility index in the Schoorrocs sense is a function M: $\mathsf{P}_\mathbf{P} \to$ R, which satisfies the following axioms:

(N) *Normalization:* $0 \le M(\mathbf{P}) \le 1$, for all $\mathbf{P} \in \mathsf{P}_\mathbf{P}$.

(M) *Monotonicity:* Mobility index reflects the change of increase in the matrix off-diagonal elements at the expense of diagonal elements. Thus, we write $\mathbf{P} \succ \mathbf{P'}$ when $p_{ij} \ge p'_{ij}$ for all the $i \ne j$ and $p_{ij} > p'_{ij}$ for a $i \ne j$. We have that $\mathbf{P} \succ \mathbf{P'}$ implies $M(\mathbf{P}) > M(\mathbf{P'})$.

(I) *Immobility:* $M(\mathbf{I}) = 0$, where **I** is identity matrix.

(PM) *Perfect Mobility*: Matrices with identical rows have maximal mobility 1.

(SI) *Strong immobility*: $M(\mathbf{P}) = 0$ if and only if $\mathbf{P} = \mathbf{I}$.

(SPI) *Strong perfect mobility*: $M(\mathbf{P}) = 1$ if and only if **P** has identical rows.

Example 3.2: Shorrocks gives in [11] an example which show that (M) and (PM) are into conflict. Consider the following matrices:

$$P_1 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \qquad P_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Axioms (M) and (PM) imply $M(\mathbf{P_2}) > M(\mathbf{P_1})=1$, which violets (N).

Shorrocks assumes that a perfectly mobile structure is given by the maximum value of mobilty index, and that the exact index ranking is unimportant, so that the basic conflict is between the axioms (PM) and (M). As one of the ways for solving this conflict Shorrocks proposes adjusting the monotonicity condition by substituting the $M(\mathbf{P}) > M(\mathbf{P'})$ by a weaker condition.

Relaxed monotonicity: Mobility index reflects the change of increase in the matrix off-diagonal elements at the expense of diagonal elements. Thus, we write $\mathbf{P} \succ \mathbf{P'}$ when $p_{ij} \geq p_{ij}^{'}$ for all the $i \neq j$ and $p_{ij} > p_{ij}^{'}$ for a $i \neq j$. Then $\mathbf{P} \succ \mathbf{P'}$ implies $M(\mathbf{P}) \geq M(\mathbf{P'})$.

In this way, consistence is restored, since maximum mobility is assigned to all the transition matrices whose off-diagonal elements are not smaller than some perfectly mobile structure.

Different mobility measures can give different transition matrices ranking. Dardanoni (1993), gives an illustration of ranking of these matrices on the example of three transition matrices and five mobility measures [2]. Dardanoni examines ordering of transition matrices by applying the following mobility measures:

1 *Eigenvalue:* The second highest characteristic square according to the module $|\lambda_2|$.

2 *Trace: trace* = $\frac{trace(P)-1}{n-1}$. This mobility index ignores the extradiagonal transition probabilities.

3 *Determinant:*

Determinant = $|P|^{\frac{1}{n-1}}$.

This mobility index gives the minimum mobility value to the transition matrices which have any two rows or columns equal.

4 *Mean first passage*:

Mean first passage $= \pi^{'} M^{P} \pi$.

5 *Bartholomew*

$$\text{Bartholomew} = \frac{1}{n-1}\sum_i \sum_j \pi_i p_{ij} |i-j|$$

where π_i is the i-th coordinate of π.

Mean first passage and Bartholomew mobility indices are called *equilibrium indices*. This indices measure mobility where the probability distribution remains unchanged over time, i.e. remains equal to unique stationary distribution π.

Example 3.3: Consider the following three matrices:

$$P_1 = \begin{bmatrix} 0.6 & 0.35 & 0.05 \\ 0.35 & 0.4 & 0.25 \\ 0.05 & 0.25 & 0.7 \end{bmatrix}; P_2 = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}; P_3 = \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.3 & 0.4 & 0.3 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}.$$

Each of these three matrices can be most mobile, depending on the selected mobility measure. The ordering of the transition matrices $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$ induced by the selected mobility measures as most mobile are the following:

1 Eigenvalue $\mathbf{P_2}$,

2 Trace $\mathbf{P_2}$, $\mathbf{P_3}$,

3 Determinant $\mathbf{P_1}$,

4 Mean First Passage $\mathbf{P_3}$,

5 Bartholomew $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$.

4 Semirings

Let S be non-empty set endowed with a partial order $\leq$. The operation $\oplus$ (pseudo-addition) is function $\oplus : S \times S \to S$ which is commutative, non-decreasing, associative and has a zero element, denoted by $\mathbf{0}$. Let $S_+ = \{x: x \in S, x \geq \mathbf{0}\}$. The operation $\otimes$ (pseudo-multiplication) is a function $\otimes: S \times S \to S$ which is positively non-decreasing, i.e., $x \leq y$ implies $x \otimes z \leq y \otimes z$, $z \otimes x \leq z \otimes y$, $z \in S_+$, associative and for which there exist a unit element $\mathbf{1} \in S$, i.e., for each $x \in S$, $\mathbf{1} \otimes x = x$. We suppose $\mathbf{0} \otimes x = \mathbf{0}$ and that $\otimes$ is a distributuve pseudo-multiplication with respect to $\oplus$, i.e., $x \otimes (y \oplus z) = (x \otimes y) \oplus (x \otimes z)$. The structure $(S, \oplus, \otimes)$ is a semiring (see [5], [6], [7], [8], [9]).

Example 4.1: Two special important real cases are ($[0, \infty)$, *min*, $+$) and *g-calculus*, i.e., when there exist a bijection $g:[a, b]\to[0,\infty]$ such that $x \oplus y=g^{-1}(g(x)+g(y))$ and $x \otimes y = g^{-1}(g(x)g(y))$, where $[a, b] \subset [-\infty, \infty]$.

We denote by P_P the set of transition matrices $\mathbf{I}$, $\mathbf{P^1}$, $\mathbf{P^2}$, $\mathbf{P^3}$,..,$\mathbf{P^k}$,.. , where $\mathbf{P}$ is a primitive homogenous transition matrix. According to Theorem 2.6, the sequence $(\mathbf{P^k})_{k \in N}$ converges with exponential growth to stationary regime which has all rows equal.

Theorem 4.2: ($\mathsf{P}_{\mathbf{P}}$*, min,* *) is a *semiring*, where *min:* $\mathsf{P}_{\mathbf{P}}^2 \to \mathsf{P}_{\mathbf{P}}$ is an idempotent operation, which induces the order on $\mathsf{P}_{\mathbf{P}}$, and it is defined for every two matrices $\mathbf{P^i}$, $\mathbf{P^j}$ from $\mathsf{P}_{\mathbf{P}}$ in the following way

$$min(\mathbf{P}^i, \mathbf{P}^j) = \mathbf{P}^i \text{ } if \text{ } M(\mathbf{P}^i) \leq M(\mathbf{P}^j), \tag{1}$$

where M is the *mobility measure* which satisfies all Shorrocks axioms, and * is matrix multiplication.

Proof. Let us observe, without loss of generality, transition matrices $\mathbf{P^1}$, $\mathbf{P^2}$ and $\mathbf{P^3}$, and mobilty index M which satisfies all Shorrocks axioms. By matrix multiplication transition probabilities increase at the expense of diagonal elements, and thus

$$M(\mathbf{P^1}) < M(\mathbf{P^2}) < M(\mathbf{P^3}).$$

Operation min given by (1) is closed in the set $\mathsf{P}_{\mathbf{P}}$. It is associative:

$$\begin{aligned} min(\mathbf{P^1}, min(\mathbf{P^2},\mathbf{P^3})) &= min(\mathbf{P^1}, \mathbf{P^2}) \\ &= \mathbf{P^1} \\ &= min(\mathbf{P^1}, \mathbf{P^3}) \\ &= min(min(\mathbf{P^1}, \mathbf{P^2}), \mathbf{P^3}). \end{aligned}$$

It is commutative: $min(\mathbf{P^1}, \mathbf{P^2}) = min(\mathbf{P^2}, \mathbf{P^1})$, and the neutral element $\mathbf{0}$ is the matrix which has all rows equal, and it is stationary distribution matrix $\mathbf{1}\pi'$. Mobility index of this matrix is 1. For every $\mathbf{P} \in \mathsf{P}_{\mathbf{P}}$ we have $min(\mathbf{P}, \mathbf{1}\pi') = min(\mathbf{1}\pi', \mathbf{P}) = \mathbf{P}$.

Operation * of the multiplication operation of transition matrices is closed in set the $\mathsf{P}_{\mathbf{P}}$, and associative, since the matrix multiplication, in general is associative. The neutral element $\mathbf{1}$ is the unit matrix $\mathbf{I}$, and its mobilty index is zero, i.e., for every $\mathbf{P} \in \mathsf{P}_{\mathbf{P}}$ we have $\mathbf{I} * \mathbf{P} = \mathbf{P} * \mathbf{I} = \mathbf{P}$.

Distributivity of the matrix multiplication according to min follows in the following way

$$\mathbf{P^1} * min(\mathbf{P^2}, \mathbf{P^3}) = \mathbf{P^1} * \mathbf{P^2} = min(\mathbf{P^1} * \mathbf{P^2}, \mathbf{P^2} * \mathbf{P^3}).$$

For every $\mathbf{P} \in \mathsf{P}_{\mathbf{P}}$ we have $\mathbf{P} * \mathbf{0} = \mathbf{0} * \mathbf{P} = \mathbf{0}$. Matrix $\mathbf{1}\pi$ 'is the left zero for matrix multiplication: $\mathbf{1}\pi' * \mathbf{P} = \mathbf{1}\pi'$. If we multiply the equation from the right side by matrix $\mathbf{P}$, we get: $\mathbf{1}\pi' * \mathbf{P}^2 = \mathbf{1}\pi' * \mathbf{P} = \mathbf{1}\pi'$. Continuing this procedure, it follows that

$$\mathbf{1}\pi' * \mathbf{P}^k = \mathbf{1}\pi' * \mathbf{P}^{k-1} = \ldots = \mathbf{1}\pi'.$$

Matrix $\mathbf{1}\pi'$ is the right zero for matrix multiplication: $\mathbf{P} * \mathbf{1}\pi' = \mathbf{1}\pi'$. Let us show, witout loss of generality, that this equation is fulfilled for n = 3,

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \end{bmatrix} = \begin{bmatrix} b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \end{bmatrix},$$

and this is valid because $\sum_{j=1}^{n} p_{ij} = 1$. If we multiply the last equation from the left side by **P** matrix, we get that for every $\mathbf{P^i}$ stationary probabilty matrix $\mathbf{1}\pi'$ is the right zero for multiplication of homogenous transition matrices. □

Remark 4.3: In Theorem 4.2 the semiring is formed on the set $\mathsf{P}_\mathbf{P}$ of transition matrices **I**, $\mathbf{P^1}$, $\mathbf{P^2}$, $\mathbf{P^3}$,..,$\mathbf{P^k}$,.. , where **P** is a primitive homogenous transition matrix. By matrix multiplication, transition probabilities increase at the expense of diagonal elements, and mobility index reflects this change. According to theorem 2.6, for some *k,* the matrix $\mathbf{P^k}$ is stationary distribution matrix with all rows equal. For $m > k$, $\mathbf{P^m}$ is equal to stationary distribution matrix. Counterexample from Example 3.2 is out of the present situation, and the axiom of monotonicity is satisfied.

Theorem 4.4: (P, min, *) is a semiring where P is the set of all primitive homogenous transition bistochastic matrices and unit matrix **I**, *min*: $\mathsf{P}^2 \to \mathsf{P}$ is an idempotent operation, which induces the order on P, and it is defined for every two matrices $\mathbf{P_i}$, $\mathbf{P_j}$ from P in the following way

$$min(\mathbf{P_i}, \mathbf{P_j}) = \mathbf{P}_i \text{ if } M(\mathbf{P}_i) \leq M(\mathbf{P}_j), \quad (2)$$

where M is the mobility measure, which fulfils the condition of relaxed monotonicity and all Shorocks' axioms (except monotonicity), and * is the matrix multiplication.

Proof. Let us observe, by not taking away from generality, homogenous transition bistochastic matrices $\mathbf{P_1}$, $\mathbf{P_2}$ i $\mathbf{P_3}$ and mobility index M which fulfils the conditions of theorem. Without loss of generality we suppose that

$M(\mathbf{P}_1) \leq M(\mathbf{P}_2) \leq M(\mathbf{P}_3)$.

Operation min is given by (2) is closed in the set P and it is associative:

$$\begin{aligned} min(\mathbf{P}_1, min(\mathbf{P}_2, \mathbf{P}_3)) &= min(\mathbf{P}_1, \mathbf{P}_2) \\ &= \mathbf{P}_1 \\ &= min(\mathbf{P}_1, \mathbf{P}_3) \\ &= min(min(\mathbf{P}_1, \mathbf{P}_2), \mathbf{P}_3). \end{aligned}$$

It is commutative: $min(\mathbf{P_1}, \mathbf{P_2}) = min(\mathbf{P_2}, \mathbf{P_1})$, and the neutral element **0** is the matrix which has all rows equal. On the set of all homogenous bistochastic matrices, every stationary distribution matrix of the $\mathbf{1}\pi'$ form for a sequence $\mathbf{P_i^k}$ has the mobility index 1. From the set of matrices with the mobility index 1, only the form matrix $\left[\frac{1}{n}\right]_{nxn}$ is at the same time zero element for multiplying, so let us take this matrix as a neutral element for the min operation. For every **P** ∈ P we have min(**P**, $\left[\frac{1}{n}\right]_{nxn}$) = min($\left[\frac{1}{n}\right]_{nxn}$,**P**) = **P**.

Operation * is multiplication operation of transition matrices, and it is closed in set P, and associative, since matrix multiplication, in general, is associative. The neutral element **1** is the unit matrix **I**, and its mobility index is zero, i.e., for every **P** ∈ P we have **I** * **P** = **P** * **I** = **P**.

Distributivity of the matrix multiplication according to min follows in the following way

$\mathbf{P_1} * min(\mathbf{P_2},\mathbf{P_3}) = \mathbf{P_1} * \mathbf{P_2} = min(\mathbf{P_1} * \mathbf{P_2}, \mathbf{P_2} * \mathbf{P_3})$.

For every **P**∈ P we have **P** * **0** = **0** * **P** = **0.** Without loss of generality, let n = 3. The following then applies:

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \end{bmatrix} = \begin{bmatrix} \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \end{bmatrix},$$

this is valid because $\sum_{j=1}^{3} p_{ij} = 1$.

$$\begin{bmatrix} \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & \frac{1}{n} \end{bmatrix}$$

this is valid because $\sum_{i=1}^{3} p_{ij} = 1$. □

Conclusions

In sociological researches often occurs the problem of determining the minimal and maximal mobile transition matrices, as well as the problem of ordering transition matrices whose mobility measures are between 0 and 1. Many authors keep considering this problem by introducing partial order on the set of transition matrices, induced by mobility measure. So far, this problem has not been considered by forming a semiring on the set of transition bistochastic matrices. In this paper, a semiring on the set of transition bistochastic matrices has been formed, which is induced by mobility measures, which represents a consistent set of axioms. The research will be continued in the direction of determining mobility measures which satisfy the mentioned set of axioms, as well as comparing orders which induce these measures on the set of transition bistochastic matrices.

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References

[1] Arrow, K. J., Sen, A. K., Suzamura, K.: Handbook of Social Choice and Welfare, Elsevier, Amsterdam, (2002)

[2] Checchi, D., Dardanoni, V.: Mobility Comparisons: Does Using Different Measures Matter?, EconPapers, (2002)

[3] Gear, D., Schokkeart, M., Martinez, M.: Three Meanings of Intergenerational Mobility, Economica 68 (2001) 519-537

[4] Gear, D., Schokkeart, M., Martinez, M.: Measuring Intergenerational Mobility and Equality of Opportunity, Public Economics, Center for Economic Studies, Discussion Paper Series DPS 98.10 (1998)

[5] Gondran, M., Minoux, M.: Graphes, dioides et semi-anneaux, Editions TEC & DOC, Londres-Paris-New York, 2001

[6] Klement, E. P., Mesier, R., Pap, E.: Triangular Norms. Kluwer Academic Publishers, Dordrecht, 2000

[7] Kuich, W.: Semirings, Automata, Languages, Berlin, Springer-Verlag, 1986

[8] Pap, E., Null-Additive Set Functions, Kluwer Academic Publishers, Dordrecht-Boston-London, 1995

[9] Pap, E., Pseudo-Additive Measures and Their Aplications, Handbook of Measure Theory (Ed. E. Pap), Elsevier, Amsterdam, 2002, 1403-1465

[10] Seneta, E.: Non-negative Matrices and Markov Chains, Second Edition, Springer - Verlag, New York, Heidelberg, Berlin, (1981)

[11] Shorrocks, A. F.: The Measurement of Mobility, Econometrica 46, (1978) 1013-1024