On the Complexity of the Channel Routing Problem in the Dogleg-free Multilayer Manhattan Model

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Abstract:Let l_H *and* l_V *denote the number of layers reserved for horizontal (vertical) wire segments in the multilayer dogleg-free Manhattan model.* $[d/l_H]$ *is a lower bound for the minimum width for routing a channel of density* d *where* $[x]$ *denotes the upper integer part of* x*. A greedy interval packing algorithm realizes every channel routing problem with width* $[d/(l_V-1)]$ *in linear time if* $l_V \geq 2$*.*

In case $l_V=l_H+1$ *the resulting width is* $[d/l_H]$*, hence it is best possible.*

The second author has shown that in case $l_V=l_H=k>1$*, it is NP-complete to decide whether a channel routing problem can be solved with width* $[d/k]$ *in the dogleg-free* $2k$*-layer Manhattan model.*

Here we turn to the remaining case. In the special case $l_V=1$ *and* $l_H=2$ *we show its relation to the NP-complete problem for the usual 2-layers Manhattan model and we point out a relation of the routing problem to a 2-processor job sheduling problem.*

Keywords:VLSI, detailed routing,channel routing, Manhattan model, multilayer routing.

1 Basic Definitions

A <u>channel</u> is a rectangular grid of rows and columns.

n	denotes the length of the channel (number of the columns),
w	denotes the width of the channel, the width of a certain routing (number of the rows).

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The rows of the routing will be called tracks.

The principal direction of the channel will be called horizontal, this means, the pins of the devices of the electric equipment are placed on the Northern and Southern boundaries of the channel. These pins are called terminals (or nodes) and the important task of the routing: to interconnect some terminals by wires. A subset of terminals to be interconnected will be called a net. In the examples terminals of the same net are denoted by the same number.

If $X_{i,l}$ and $X_{i,r}$ denote the X-coordinates of the leftmost and the rightmost terminal of net N_i then the interval $[X_{i,l}, X_{i,r}]$ will be called the interval of the net N_i If a net has only two opposite terminals then the interval reduces to a single point. Such a net is called trivial like N_5 in Figure 1.

A routing is dogleg-free if the realization of each nontrivial net contains a single horizontal wire segment only. (For example net N_1 of Figure 1 is dogleg-free, N_3 has a dogleg.) The congestion of a vertical line is the number of nets, whose intervals intersect the line. The maximum congestion is called the density of the channel routing problem and is denoted by ***d***.

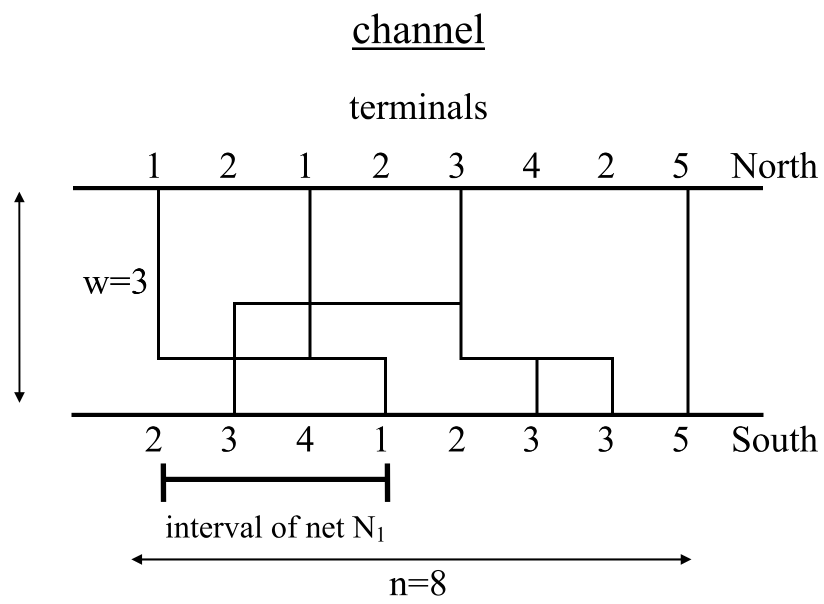


Figure 1

Let ***l*** denote the number of layers in the routing, and we consider the „Manhattan model”. The „usual” Manhattan model has two layers for the routing, one layer for the horizontal and one layer for the vertical wire segments. The multilayer Manhattan model has ***l*** layers, let l_H denote the number of layers reserved for horizontal and $\boldsymbol{l_V}$ denote the number of layers reserved for vertical wire segments. Consecutive layers in the Manhattan model may contain wire segments of different direction only.

A gridpoint, where the wire turns from horizontal to vertical direction (or vice versa), and the wire must leave a layer for another adjacent one is called via. In case $\boldsymbol{l=2}$ one layer is reserved for horizontal, and one for vertical wire segments. If $\boldsymbol{l}$ is even, the number of horizontal and vertical layers are equal. If $\boldsymbol{l}$ is odd, the number of horizontal and vertical layers are different. For example in case $\boldsymbol{l=3}$ we can distinguish between **HVH** and **VHV** models. In general for any odd number $\boldsymbol{l}$ one has two types of $\boldsymbol{l}$ -layer- Manhattan models.

In the multilayer Manhattan model we have a lower bound for the minimum width for routing a channel of density $\boldsymbol{d}$: $\boldsymbol{[d/l_H]}$, where $\boldsymbol{[x]}$ denotes the upper integer part of $\boldsymbol{x}$. We also have an upper bound for $\boldsymbol{l_V \geq 2}$: a greedy interval packing algorithm gives a solution with width $\boldsymbol{[d/(l_V - 1)]}$ in linear time. It is the best possible if $\boldsymbol{l_V = l_H + 1}$.

2 Gallai's Algorithm

First let us suppose that all the terminals of the nets are on the Northern boundary of the channel. This case is the so called single row routing problem [10]. In this case the intervals of the nets can be packed into $\boldsymbol{d}$ horizontal lines, so called tracks, by a greedy interval packing algorithm:

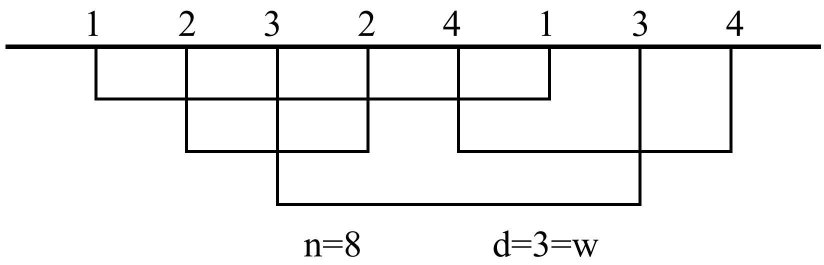


Figure 2

Let us consider the list **L** of the interval of the nets.

Step 1 If the list **L** is empty, stop. Otherwise consider the interval with minimum left end coordinate, place it to a new track, denote its right end coordinate by **X**; and delete the interval from **L**.

Step 2 Consider those intervals whose left end coordinate is greater than **X**. If there are none, go to step **1**. Otherwise choose the one with minimum left end coordinate, place it to the actual track, denote its right end coordinate by **X**, delete this interval from **L** and go to step **2**.

Based on Tibor Gallai's research in the fifties [7], this algorithm gives the following classical result:

<u>Theorem</u>

- Every single row routing problem can be solved in linear time in the 2-layer Manhattan model.
- The resulting width equals the density, hence it is the best possible.
- The routing is dogleg-free, and the width could not be reduced if doglegs were permitted.

If we return to <u>the general channel routing</u> problem, the lower bound $\boldsymbol{w \geq d}$ clearly remains valid in the 2-layer model, and in general, $\boldsymbol{w \geq \lceil d/l_H \rceil}$ is valid for every solution, where $\boldsymbol{d}$ is the density and $\boldsymbol{l_H}$ is the number of horizontal layers.

But the statements of Theorem do not remain true. Let us show some examples.

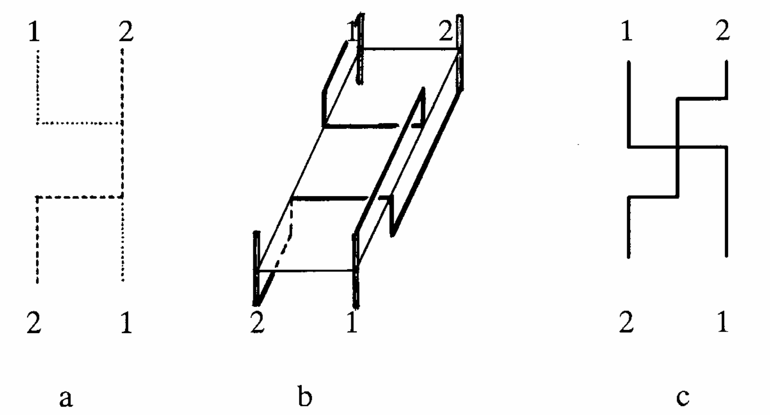
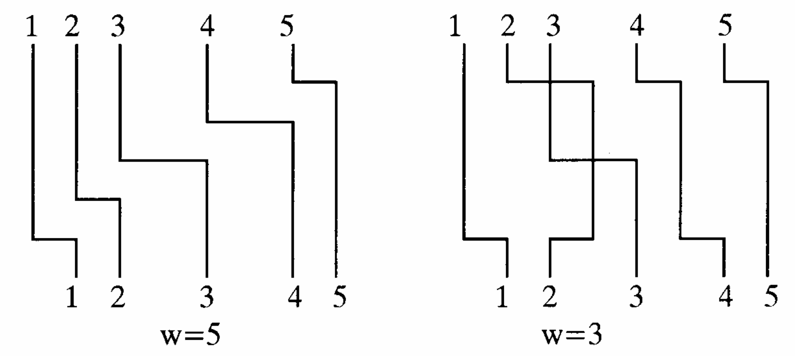


Figure 3

1. 3a is unsolvable in the 2-layer dogleg-free Manhattan model, but it has a solution in VHV model (see 3b), and it is solvable in the 2-layer Manhattan model, if doglegs are permitted (see 3c).



a b

Figure 4

2. We have a solution in Figure 4a with width 5, and the same channel-routing problem has a solution with width 3, if doglegs are permitted (see 4b).

It is interesting to consider what can we say in general about channel-routing in the dogleg-free multilayer Manhattan model.

1. Every channel routing problem can be solved in linear time in the (VHVH....V) type Manhattan model, and the resulting width is $[d/l_H]$, hence it is the best possible. The proof in the VHV case comes from the above theorem [3] and the generalization for more layers is obvious [1], [2], [6].
2. In case $l_V=l_H=k$ it is NP complete to decide whether a channel routing problem can be solved with width $[d/k]$ in the dogleg-free 2k-layer Manhattan model. (The case k=1 has been well known for many years [8], [12], the more recent result [11] refers to the case $k>1$.)

Now we are going to study the remaining case.

3 The Case l_V=1 and l_H=2

In this case we have a trivial lower bound, but only some trivial upper bounds can be formulated. In fact, the dogleg-free realizability of a channel routing problem in this model is equivalent to that in the 2-layer Manhattan model and actual width requirements in then two models are closely related, as shown by the following observations [11]:

Lemma 1

If a channel routing problem can be solved with width **w** in the dogleg-free way in the HVH model – using one vertical and two horizontal layers – , then it has a dogleg-free solution with width at most **2w** in the „usual" Manhattan model (using one vertical and one horizontal layer).

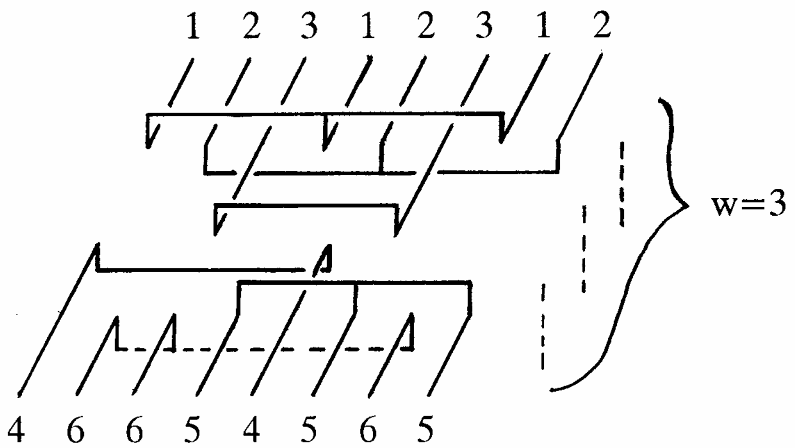


Figure 5a

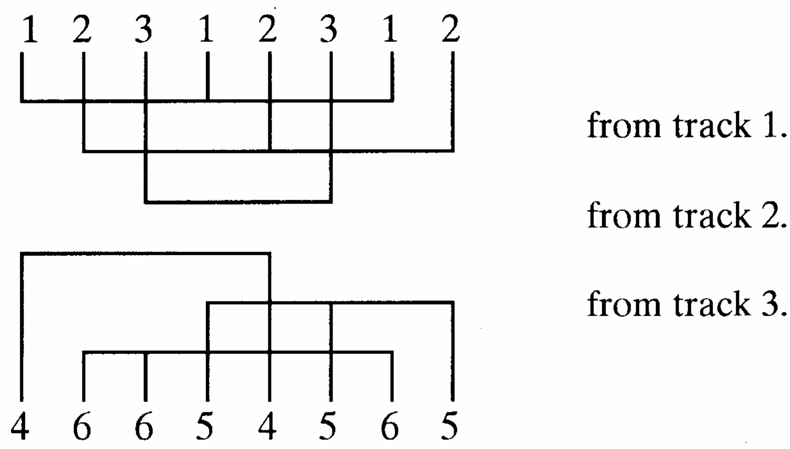


Figure 5b

<u>Proof</u>: Let us consider Figure 5a. This is a channel routing with length **8**, with width **3** in HVH Manhattan model. Let us proceed from North to South and realize the lines of the two horizontal layers in an alternating way (see Figure 5b) leading to two tracks of the two layer routing from each track of the original HVH routing. No conflict can arise, since we have a single vertical layer anyhow. This procedure transforms any HVH model into a „usual” 2-layer Manhattan solution [4].

<u>Lemma 2</u>

If a channel routing problem can be solved with width **w** in the dogleg-free 2-layer Manhattan model, it has a dogleg-free solution in the HVH model with width **w’**, where **w’** satisfies **w/2 ≤ w’ ≤w** .

The proof is obvious.

It is very important to see, however, that the transformation from HVH to „usual” Manhattan model cannot always be inverted. If a channel routing problem has a solution with width **w**, it <u>does not always have</u> a solution in HVH model with width **w/2**.

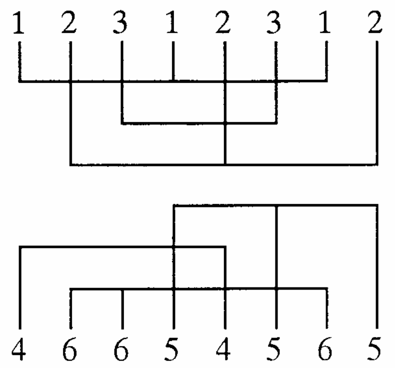


Figure 6a

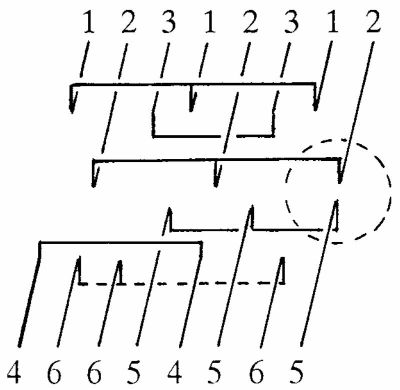


Figure 6b

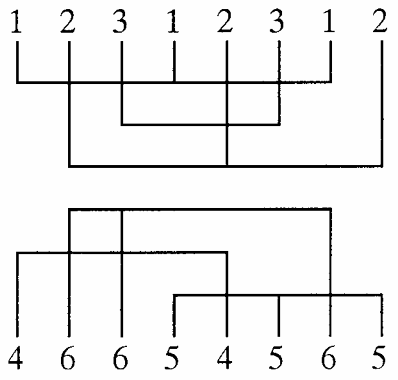


Figure 6c

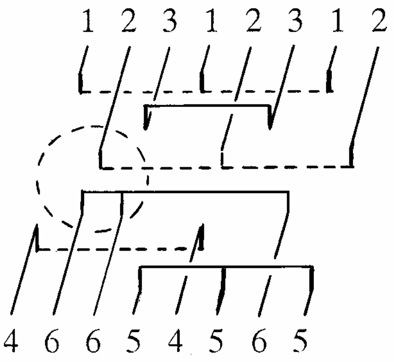


Figure 6d

For example, Figure 6a and 6c show two further solutions of the same channel routing problem as in Figure 5b, also with width **6**. However these solutions cannot be „folded back" like in Figure 5a (see 6b and 6d). More precisely, these solutions can be inverted to the HVH solution only with width **4** and not with width **3**. (In fact, one can enumerate that the actual problem can be routed in 40 different ways with width **6** in the usual Manhattan model and only 8 of them can be transformed back to the HVH model with width **3)**.

Let a dogleg-free channel-routing with width **w** in the usual Manhattan model and **w'** satisfies **w/2** ≤ **w'** ≤**w**. The problem of transforming the routing into a HVH routing with width **w'** (if it is possible at all) can be formulated as a 2-processor job scheduling problem [5].

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