Cogging Torque Reduction by Magnet Pole Pairing Technique

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Abstract: A high performance electrical drive needs a smooth torque waveform and a high torque to inertia ratio. The power density and performance needs can be, in most cases, fulfilled by using a permanent magnet synchronous machine (PMSM). This paper explores a new cogging torque reduction technique. This method can be used without reducing the power density of the machine and it can also be applied in a mass production process.

Keywords: cogging torque; finite element analysis; optimization; pulsating torque reduction

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1 Introduction

Pulsating torque is usually harmful for electric drives. They can create disturbing vibration and noise which need to be eliminated. The pulsating torque of PMSM machines is the sum of torque ripple and cogging torque.

Torque ripple is produced if the induced voltage graph of the machine or the power inverter has harmonic content. Cogging torque is a magnetostatic effect: pulsating torque arises due to magnetic energy variation in the air gap as the rotors magnet pole passes over a slot opening. The pulsating torque components have a usual user accepted level: it is defined in the ratio of the rated torque, which is usually 0.5% for cogging torque and 3% for torque ripple.

There are many well-known cogging torque minimizing techniques that are able to maintain a defined pulsating torque range. These methods may use of dummy slots, magnet poles or slot skewing and can effectively decrease a pulsating torque component, but they can also decrease the torque to inertia ratio of the machine.

There are many other cogging torque and torque ripple reducing methods presented, but usually these methods are effective for only one pulsating torque component. A method which is effective for cogging torque reduction is usually not effective for torque ripple reduction. Moreover, sometimes a cogging torque reducing method produces high torque ripple. The designer has to find a solution that is effective, for both pulsating torque components, this paper presents such a solution.

2 Physical Background

A PMSM machine has several magnet poles and several slots. Pulsating torque wave is generated if a magnet pole edge passes a slot opening. The shape of the wave is the function of the slot opening and magnet pole design. Different magnet pole shape may produce a significantly different cogging wave for the same slot opening (Figure 1). The designers try to create such a magnet pole shape that produces the most optimal cogging wave. The wider slot opening produces the higher amplitude for cogging wave. The slope of the cogging wave is usually the function of the magnet pole shape. If the magnet width is constant, the cogging wave is like the red one on Figure 1. If the magnet width is maximal at the middle of the magnet and becomes narrower towards the pole edges the slope of the cogging torque wave gets lower, like the blue (dashed) wave in Figure 1.

The individual magnet pole-slot opening related cogging torque graphs can be simulated by finite element analysis for each slot. The identical waves are summarized mechanically by the stator and rotor lamination for all slots and poles. The key question is the physical distribution of the magnet poles and stator slots which is defined by the magnet pole-stator slot number combination. In another aspect of view, the resultant cogging torque wave of the machine is the summing of different graphs which are in the same phase or have phase offset between each other, this phase offset is defined by the mechanical position of the slots and magnet poles.

In some cases the slot number-pole number combination gives the opportunity to use such a pole pitch ratio, that the cogging torque graph of the two magnet pole edge of an individual magnet pole is in the same phase, but with opposite sign. That is, the two cogging waves may cancel each other.

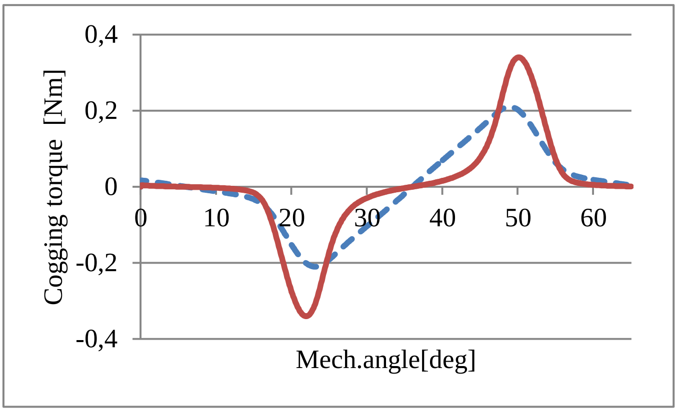

Figure 1

Cogging torque graph of an individual magnet pole for different magnet pole shape design.

The other opportunity is to use special magnet positioning technique. The positioning of the magnets may be done so to have magnet pole edge pairs having the individual cogging waves in the same phase but with opposite sign. By this way some of the individual cogging torque graphs can be cancelled and the cogging torque level of the complete machine can be effectively reduced. This self-cancelling cogging torque reducing technique is useful because it doesn’t reduce the torque density of the machine. Special manufacturing technology is not needed either.

The individual cogging torque of one magnet pole and slot opening can be calculated by finite element method (FEM). The summing of these graphs can be done by analytical equations.

Let’s call the identical cogging torque wave for one magnet pole $\mathrm{f_{sp}}$. This wave can be accurately calculated by FEM. The slot number (Z) and pole number $(2\mathrm{p})$ is known for an analyzed machine. The mechanical angle between slots (β) and poles (α):

$$\alpha = \frac{360°}{2p}\ , \qquad \beta = \frac{360°}{Z},\ \gamma = \frac{360°}{LCM(2p, Z)} \tag{1}$$

The period of cogging torque of the complete machine is γ, $\mathrm{LCM}(2\mathrm{p},\mathrm{Z})$ is the least common multiple of the slot and pole number. The cogging torque wave of the machine is the summary of the unique cogging waves of each slot and magnet pole.

The cogging torque wave for one slot and the whole rotor for one complete mechanical rotor revolution can be generated: $\mathrm{f_{sp}}$ has to be summed for each magnet pole by adding α mechanical phase offset for each magnet pole. Also magnet positioning error (φ_i) can be taken into account. The cogging torque wave for one slot for one rotor revolution is the following:

$$\mathrm{f_{s\_360}} = \sum_{\mathrm{n}=1}^{2\mathrm{p}} \mathrm{f_{sp}}(\mathrm{x} + \mathrm{n} \cdot \alpha + \varphi_\mathrm{i}) \tag{2}$$

The cogging torque graph of the machine can be created by the sum of the cogging torque graphs of the slots:

$$\mathrm{f_{cogg}} = \sum_{\mathrm{m}=1}^{\mathrm{Z}} \mathrm{f_{s\_360\_m}}(\mathrm{x} + \mathrm{m} \cdot \gamma) \tag{3}$$

The phase offset marked by φ_i may be magnet positioning error or a defined magnet positioning phase offset. If $\mathrm{f_{sp}}$ is known, the optimal value for other parameters, like pole pitch ratio, etc., can be found.

These equations can be used to create the cogging torque waveform of a machine from the identical cogging torque wave of one magnet pole-slot opening interaction. This calculating technique is called hybrid method. Special magnet positioning or magnet pole width manipulation can be also taken into account and the resultant cogging torque wave of the modified machine can be calculated in a short time.

Cogging torque calculation by FEM for different pole positioning cases or for different pole width needs a different model. These models need to be created and calculated which usually takes quite a long time. The hybrid method can be used to find the optimal geometry in a short time and the final calculation and optimization can be done by FEM.

3 Cogging Torque Compensation by Pole Width Modification

Let's check some pole number-slot number combinations in Table 1. For best cogging torque compensation effect slot pitch, or multiplied slot pitch angle should be equal with the pole pitch. That is, f_{sp} waves should be summed in such way to compensate each other.

If the magnet pole pitch is too narrow, the harmonic content of the induced voltage graph will be high and in this case the ripple torque of the machine will be also high, usually higher than the application acceptable level. Table 1 contains the maximal magnet pole pitch values for some pole number cases.

The 27 slot 6 pole machine seems to be a good combination for self-cancelling cogging waves for each magnet pole. The maximal pole pitch for $2p = 6$ machine is 60º. The slot pitch for a $Z = 27$ slot machine is 13,33º. The nearest value around the maximal pole pitch value is 53,33º, which means four slot pitch.

Table 1

Mechanical angle for slot pitch and magnet pole pitch for some pole number and slot number cases

Number of poles	**2**	**4**	**6**	**8**	**10**	**12**
Pole pitch [mech. angle]	180º	90º	60º	45º	36º	30º

Slot number	**9**	**12**	**15**	**18**	**21**	**24**	**27**
Slot pitch*1	40º	30º	24º	20º	17.14º	15º	13.33º
Slot pitch*2	80º	60º	48º	40º	34.29º	30º	26.67º
Slot pitch*3	120º	90º	72º	60º	51.43º	45º	40º
Slot pitch*4	160º	120º	96º	80º	68.57º	60º	53.33º

The f_{cogg} wave for the complete machine was calculated by FEM. The outer diameter of the analyzed machine was 150 mm, length of stator lamination was 100 mm. The air gap length is 1 mm. Rated torque of the machine was 20 Nm.

The cogging torque graphs for the machine for different magnet pole width value magnet can be seen on Figure 2.

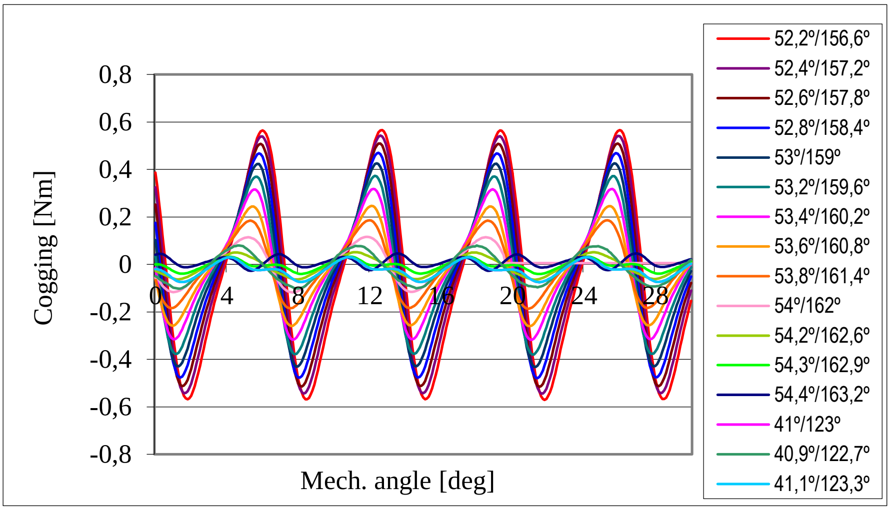

Figure 2

Magnet pole pitch dependence of cogging torque for a 6 pole 27 slot machine

Simulation was made by finite element analysis. The calculation of each graph took about 28 hours, so the complete simulation took more than 18 days. The drawing of the geometry for the rotor versions and also the model building for the simulation took about 2 hour per variant. This time is only calculating time without any rest for the computer. If the timing of the different models is not automatized, and the time gap between the different calculations is not minimal the time consumption may be much higher.

The lowest peak value cogging wave was calculated for the case of 54.3° magnet pole pitch. The 54.3° mechanical pole pitch means 162.9° in electrical angle which is a good value for low torque ripple level. The same wave can be produced if the magnet pole width is not 4 times but 3 times the value of the slot pitch. Please check the cogging torque wave for 54.3° and 41.1° magnet pole pitch. The analysis was made by 0.2° steps.

The self-compensating method has been validated:

$$f_{c_i} = (-1) \cdot f_{c_{i+N}} \tag{4}$$

where f_{c_i} is the identical cogging torque wave for the i^{th} slot, $N = 1,2,3 \ldots$ integer, so $f_{c_{i+N}}$ is an identical cogging torque for a slot in the neighborhood of the i^{th} one.

This comprehensive analysis was made to validate the modeling method and also to check the effect of the diagonal magnetized magnet poles. The optimal pole width was 54.3° and 41.1° in mechanical angle. The slot pitch is 13.33°, 3 slot pitch is 40º, 4 pole pitch is 53.33º. The pole width is more than 1º wider than the slot pitch.

If the pole pitch is different from the optimal value the amplitude of the sum cogging torque wave increases rapidly. This cogging torque reducing method act sensitively for magnet pole width related manufacturing tolerances. Normally cogging torque graph acts sensitively for magnet positioning error but by the case of the magnet pole self-cancelling method the sensitivity for magnet positioning error is low.

The peak to peak value of cogging torque is 0.08 Nm, which is 0.4% of the rated torque of the machine so the usage of other cogging torque reducing method is not needed.

The finite element optimization of the machine was done on the way of step-by step analysis but time consumption was quite high, a faster calculation method would be preferred.

The shape of the cogging graph can be scaled for different magnet pole width situations and the scaled graphs can be summed. The result can be achieved in a short time by running for example a Matlab script which contains an identical cogging torque graph, the pole number-slot number combination and the different pole width cases. The cogging torque for the rotor variants can be easily plotted and the optimal pole width can be found in a short time. This optimal pole width can be used for the finite element analysis as a starting point and the optimizing process could be much faster.

Let’s check the error possibilities for the analytical method. If the magnetization of the magnet is radial, the identical cogging graph can be transformed narrower with same amplitude. If the magnetization is diagonal and the magnet pole width is modified, the shape of the identical cogging torque graph usually changes. If the pole width is thinner, the amplitude of the cogging wave gets higher. At the middle of the magnet pole the magnetization direction is perpendicular with the stator lamination but towards the magnet edges this angle changes and also the orthogonal projection changes. In this case the effective air gap length is higher towards the magnet pole edges and the amplitude of cogging torque decreases.

The variation of identical cogging torque graph was analyzed by finite element analysis for the case of several different magnet pole width magnet version (Figure 3.). If the identical magnet cogging wave would be only scaled to different width, amplitude error would arise. Equation (3) is still valid for the machine having the modified rotor configuration, which means only the exchanging of the f_{sp} wave. The scaling can be made for example by the *interp1* function in Matlab.

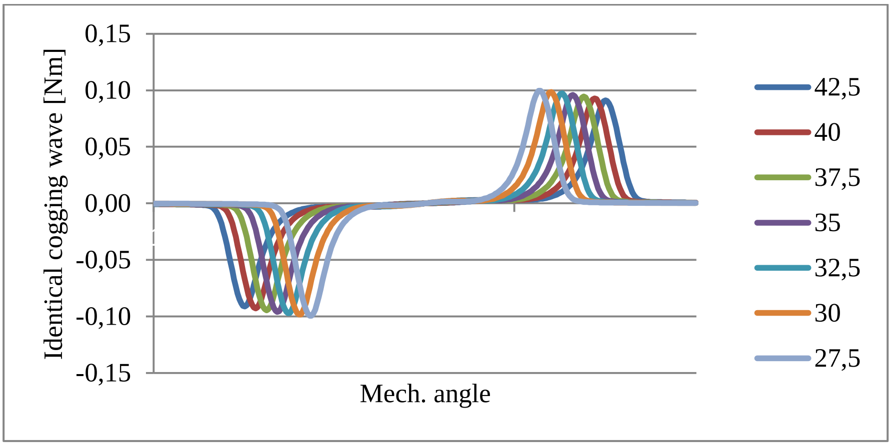

Figure 3
Identical cogging wave for different magnet pole width cases

If the geometry contains some geometry locations with high saturation level some unexpected cogging torque harmonics may appear which cannot be handled by the hybrid analytical method. For example, if the yoke is too narrow of the machine, the saturation level in the yoke region will be different for the identical cogging torque calculation. The identical cogging torque wave is calculated when only two magnet is present in the model. If all magnets are inserted in the model, the saturation level may be much higher.

Let's check the case of a 9 slot 6 pole machine. The magnetization of the magnets is also diagonal. The slot pitch of this machine is 40°. The magnet pole width analysis for cogging torque reduction was also made. The analysis was made by finite element method, results can be found on Figure 4.

The optimal pole width value for the actual slot opening, magnet pole shape and air gap length is 41.9° in mechanical angle. The maximal magnet pole pitch for a 6 pole machine is 60°. The 41.9° mechanical angle pole pitch means 125.7° in electrical angle which is very low and would create high torque ripple level. In this case the magnet pole self-compensating method is not applicable. The effect of diagonal magnetization acts also in this case: the magnet pole width is wider than the slot pitch.

In this case other pulsating torque reducing method should be used like the magnet pole-pairing method. This technique means such a magnet positioning that some magnets are placed by a defined phase offset of their original symmetrical position.

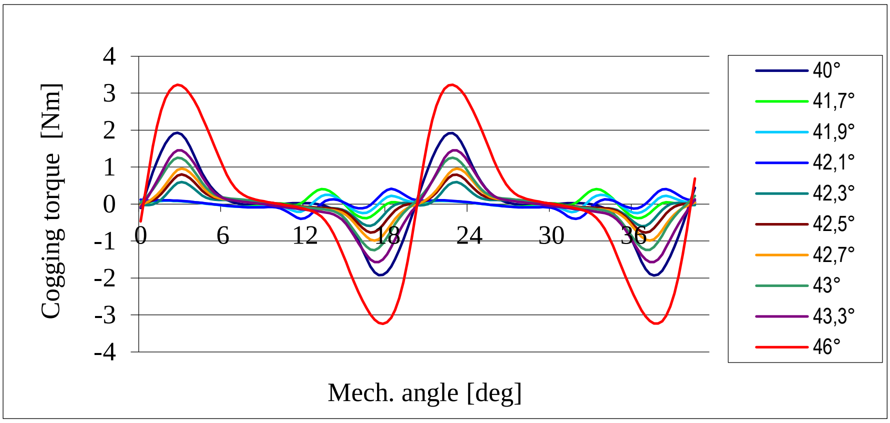

Figure 4

Magnet pole pitch dependence of cogging torque for a 6 pole 9 slot machine

If diagonal magnet is used and magnet pole width is modified, the analytic pole width scaling method may contain higher error level. The optimal pole width value can be tightened to a range of 1°. Another opportunity is the modification of the position of some magnets and/or to take advantage of the cogging compensation effect, for magnet pairs, not for identical magnets as in this point.

4 Pole Pairing Technique

In many slot number-pole number cases the magnet poles can be arranged such a way that the magnet pole self-cancelling method can arise for pole edges of different magnet poles.

This eliminating method means that the positive and negative peaks of the individual cogging waves are in phase for different pole pairs. For better understanding see Figure 5. The magnet pairs A-A and B-B are the self-eliminating pairs for cogging torque. By the actual rotor position one B magnet leaves a slot opening, the other B magnet arrives to a slot opening. The magnets marked by C and D are creating their own cogging torque graphs and their position was not changed. The sum cogging torque graph is only the sum of the graph of magnet C and D.

The identical cogging torque wave does not need to be modified, only the phase angle should be modified, so the analytic result will have very good accuracy. The optimal pole positions were calculated and such a rotor geometry was imported to the FEA software to validate the result. The difference between the optimal pole width value from analytic and FEA result was 0.2°. The final optimization was fast.

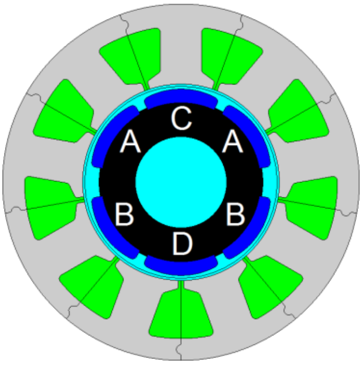

Figure 5

Magnet pole pairing technique applied for a 6 pole 9 slot machine

If the magnet arrangement on the rotor is symmetric, there is 120° mechanical symmetry for the rotor and stator. In this case the cogging torque of the 3 magnet groups is summarized in phase. This effect can be eliminated by the modified magnet arrangement.

The usual applied cogging torque reducing method is magnet pole skewing. The effects of pole skewing and the magnet pole pairing technique can be compared. The outer diameter of the analyzed machine is 80 mm, the stator stack length is 30 mm, rated torque is 3.5 Nm. The comparison of different rotor configurations as regular magnet pole arrangement, pole paired rotor configuration and skewed rotor was analyzed by FEM. The magnet pole shape (pole pitch, magnet material, geometry) was not changed.

Table 2

Comparison of cogging torque peak values for regular, skewed rotor and a rotor with modified magnet positioning

	cogging torque [Nm]	ripple torque [Nm]	rated torque [Nm]
limit	0.0175	0.105	3.50
regular rotor	0.0440	0.350	3.71
pole pairing	0.0064	0.097	3.68
skewed rotor	0.0040	0.320	3.42

The cogging torque of the regular rotor arrangement is higher than the limit. The skewed rotor configuration can pass the cogging torque limit but the ripple torque is higher than the limit. Unfortunately, skewing is only effective for one pulsating torque harmonic. Skewing also reduces the torque density of the machine.

The pole paired rotor configuration can pass both cogging and torque ripple limits and the torque density also remains at a high level. The partial compensation of the identical cogging waves reduces effectively the cogging torque.

Unfortunately, the pole pairing method is not usable for all pole number-slot number combinations. In some cases, the usage of the cancelling technique creates a high torque ripple ratio. The goal of the machine optimization is to achieve low level of cogging torque and torque ripple. In such cases, another pulsating torque reducing method should be used, which may also reduce the torque density of the machine. When the pole pairing method is applicable, it can be effectively used and the torque density of the machine remains higher than using one of the other cogging torque reducing methods.

5 Reduction of Sensitivity for Air Gap Eccentricity

Permanent magnet synchronous machines act are usually sensitive for manufacturing misalignments. The most frequent mechanical misalignment cases for mass-production are air gap eccentricity and magnet positioning error. Magnet positioning error creates a slot number order cogging torque harmonic, air gap eccentricity creates pole number order cogging torque harmonic.

The magnet positioning error can be eliminated by special rotor design: If internal permanent magnet (IPM) rotor design is chosen, the magnets are fixed in the slots of the rotor lamination. The air gap eccentricity problem still has to be investigated.

Air gap eccentricity may arise in various cases: The electric machine is built up of many parts, which are, for rotor side, the shaft, rotor lamination stack and magnets and for stator side, stator housing, laminated stator stack, end-shields (for both sides or only one side) and bearings.

Each of the many parts, have manufacturing tolerances. If the tolerances are too wide-ranging, the parts may need to be grouped or the air gap eccentricity of the machine may vary too much. If the manufacturing tolerances are too strict the scrap ratio may become too high. It is a common scenario to check the allowable air gap eccentricity level of the machine and also to apply the result for defining the tolerances for each part. This is a must, because it is cheaper to select the scrapped main parts then to scrap the complete machine if its cogging torque is higher than the prescribed limit.

A comprehensive finite element analysis was created for checking the sensitivity of the cogging torque for air gap eccentricity on a common PMSM machine pole-slot number combinations. The analysis was made for same size machines: The outside diameter was 150 mm, the stator stack length was 100 mm and the mechanical air gap was 1mm. Three cases were analyzed: The regular case with 0mm, then a 0.2 mm and a 0.4 mm air gap eccentricity. The results can be seen in Figure 6 below.

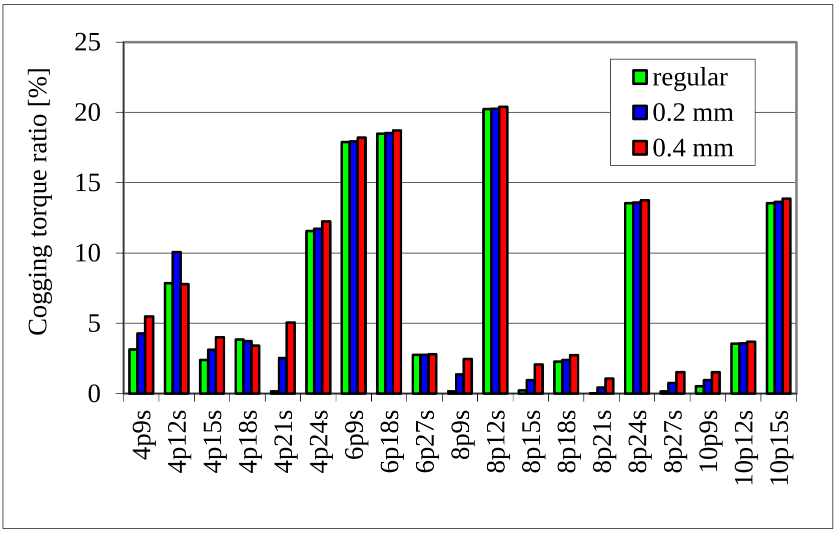

Figure 6

Comprehensive analysis for sensitivity of cogging torque for air gap eccentricity

As it can be seen, the machines can be divided into two groups: one group has low regular cogging torque values which increases rapidly for air gap eccentricity and the other group where the rated cogging torque graph has a high amplitude ,cogging torque graph but the sensitivity for air gap eccentricity is low.

The results can be explained by the hybrid method. The resultant cogging torque graph of the machine can be calculated by the summing of the identical cogging torque graphs.

The shape of the identical cogging torque graph is a function of air gap length. The shape and width of the graph remains the same, but the amplitude changes: higher air gap lengths results in lower cogging torques. The behavior of the identical cogging torque graph for different air gap length cases can be calculated by FEA. This function ($c(g_m)$) is usually defined by the saturation level of the magnetic circuit of the machine. If the saturation level is high, the graph is hyperbolic, if it is low, the $c(g_m)$ graph is almost linear. Also, the shape of the $c(g_m)$ graph is the function of the regular air gap length: if the regular air gap is small, the magnetic circuit become more sensitive.

For a defined air gap eccentricity case, the actual air gap length value for each slot can be calculated. The scale factor can be defined for the identical cogging torque graphs for each slot opening by using the $c(g_m)$ graph. The resultant cogging torque graph can be defined in (4):

$$f_{cogg} = \sum_{m=1}^{Z} c(g_m) \cdot f_{s\_360\_m} \tag{4}$$

The mechanical offset angle between slot dependent $f_{s\_360\_m}$ graphs is γ. If the pole number-slot number combination is relative prime, the offset angle between each $f_{s\_360\_m}$ graph is different. If the pole number-slot number combination has a common divider, there will be some $f_{s\_360\_m}$ graphs in the same γ phase, their number is the function of the common divider.

Let's check (4) and Figure 7. Normally, when air gap eccentricity is not present, the scale factor for $c(g_m)$ would be 1.

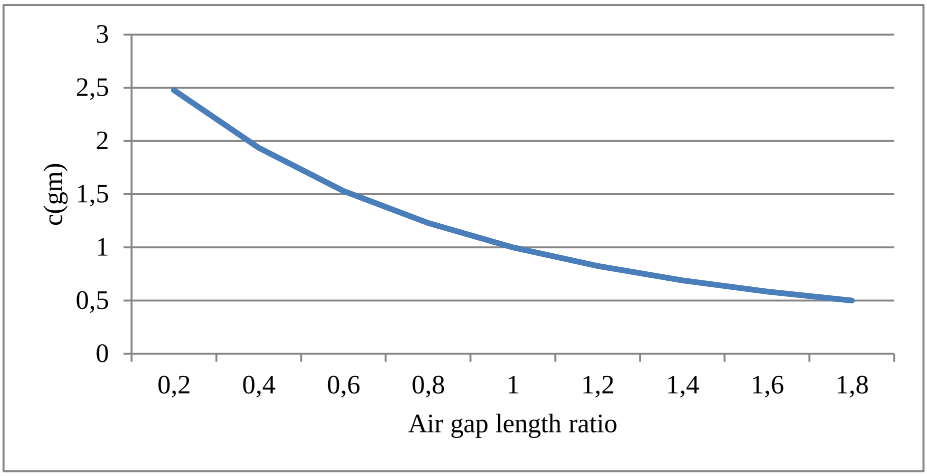

Figure 7

The scale factor graph for identical cogging graph for different air gap length values

If air gap eccentricity is present, the $f_{s\_360\_m}$ graphs should be scaled and then summed. The $c(g_m)$ graph increases heavily for smaller air gap values, that's why the cogging torque amplitude greatly increases for the case of machines having relative prime pole number-slot number combination.

Table 3

γ offset angle for identical cogging graphs for different slots for 10p12s and 8p9s machine

slot.No	1	2	3	4	5	6	7	8	9	10	11	12
γ -10p12s	0°	6°	12°	18°	24°	30°	36°	42°	48°	54°	60°	66°
	0°	6°	12°	18°	24°	30°	0°	6°	12°	18°	24°	30°
γ -8p9s	0°	5°	10°	15°	20°	25°	30°	35°	40°			

For example, a 10 pole - 12 slot machine, has a common divider, it is 2. In this case the $f_{s\_360\_m}$ graphs can be classified in two groups: slot 1-6 and slot 7-12. The γ angle for each slot is 6°, the α angle is 36°. The offset between slot 1 and 7 is 36°, which means one magnet pole shift, so these cogging waves are in the same phase.

In regular case without air gap eccentricity, equation (3) can be written as:

$$\mathrm{f_{cogg}} = s \cdot \sum_{\mathrm{m}=1}^{\mathrm{Z}/s} \mathrm{f_{s\_360\_m}}(\mathrm{x} + \mathrm{m} \cdot \gamma) \tag{5}$$

where s is the common divider between slot number and pole number, and also the number of the mechanical symmetry axes for the machine. For the 10p12s machine $s = 2$. If air gap eccentricity is present, (5) can be expressed as:

$$\mathrm{f_{cogg}} = \sum_{\mathrm{m}=1}^{\mathrm{Z}/2} c(g_m) \cdot \mathrm{f_{s\_360\_m}}(\mathrm{x} + \mathrm{m} \cdot \gamma) + \sum_{\mathrm{m}=\frac{Z}{2}+1}^{\mathrm{Z}} c(g_m) \cdot \mathrm{f_{s\_360\_m}}(\mathrm{x} + \mathrm{m} \cdot \gamma) \tag{6}$$

The air gap length for slot 1-6 is higher than for the regular value, for slot 7-12 the air gap length is lower than the regular value.

Due to mechanical symmetry, slot 1-7, 2-8 etc. are placed in opposition and the difference from regular air gap for the slot pairs is the same but with opposite sign. The air gap dependent $c(g_m)$ scale factor for (4) can be defined for each slot. For the case of the 10p12s machine, if the $c(g_m)$ graph would be linear, the sum of the scale factor for the higher and lower side $f_{s\_360\_m}$ graphs (slot 1-6, 7-2, etc) would remain 2, the positive and negative deviation from 1 for each slot pair would be the same.

For the case of 8p9s machine the phase offset between the $f_{s\_360\_m}$ is different, the previous compensating effect is not present.

If the value of s is greater than 1, eccentricity compensation is present, but the regular cogging torque peak value is higher because the number of $f_{s\_360\_m}$ graphs that are added in the same phase, is more than 1. If $s = 1$, the sensitivity for air gap eccentricity in the aspect of cogging torque is higher but the regular cogging torque level is lower.

Conclusions

This paper presents a new cogging torque analysis method which is called a “hybrid method”. It can be used to easily understand the nature of cogging torque. The cogging torque of the machine can be thought of as the summary of several graphs. The hybrid method can be used in many cases to reduce the cogging torque of the machine or reduce the sensitivity for air gap eccentricity in such a way to be applied in a mass production scenario, without the reduction of the rated torque for the machine.

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