The Statics of the Traditional Hungarian Composite Reflex Bow

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Abstract: The operation of the Hungarian bow raises several fascinating mechanical questions. To answer these questions a good number of experiments and calculations need to be made, moreover the mechanical model of the bow is needed to be prepared which appropriately confirm the results of experiments. Teachers in the Bánki Donát Mechanical Engineering College of Budapest Polytechnic set up a small laboratory in 1997 in order to study and measure the physical characteristics of traditional bows. The mechanical analysis of bows is based on the experiments gained in the laboratory and the results of measurements. The knowledge acquired about the mechanical model of bows facilitates not only the analysis of the traditional Hungarian bow, but also provides a good foundation for the comparison from the technical point of view of various composite reflex bows belonging to different historic ethnic groups.

1 Introduction

In the course of the history of mankind certain peoples and nationalities can always be traced to have risen and fallen and it is primarily the historians' task to research in the circumstances. According to historians, in many cases the immediate reason for certain peoples' rise was the ability to set up the best-organised and most disciplined army of their age, which was equipped with the most advanced weaponry.

In the history of Hungarians there was a period of at least one and a half centuries in which Hungarians had by far the most powerful army of their time. There is written evidence proving that princes or pretenders in western countries often requested Hungarians still living in Etelköz (i.e. the homeland of nomadic Hungarians in Asia) to support them with their tribes. Hungarians had a good reputation worldwide for their modern, well-organised and well-disciplined warfare, which bore a lot of resemblance to the Huns' army. Their most efficient weapon, the composite reflex bow, which was exclusively used by eastern nomadic tribes, was regarded as crucially decisive for every battle. After the

Hungarian tribes had occupied their new homeland in the Carpathian Basin, almost the whole of Europe paid taxes to the Hungarian principality' in return for the support of their invincible army. The taxes Hungarians imposed on western civilisations assured peace and quiet for them, on the other hand if they had failed to pay their taxes, the "roaming" Hungarian tribes soon appeared on the horizon claiming their share. Legend has it that the inhabitants of medieval Modena had been found in their church praying to God in the following manner: "...Almighty God, please save us from the arrows of Hungarians."

The ancient weapon called the reflex bow had been widely used for hunting and fighting by nomadic tribes in the steppes. While preserving its basic operational principal, the different tribes produced their own versions of the original weapon by developing new geometrical varieties. As a result, the Hungarian bow can be distinguished fairly easily from the Hun, Avarian, Mongolian, Chinese or Turkish bows.

In Hungary, the ethnographer Károly Cs. Sebestyén was the first who had identified the remains of the ancient Hungarian bow with the long flat bone blades which were similar to knives and had been found arranged in similar patterns in some of the graves from the time of the Hungarian Conquest of the Carpathian Basin. They have obviously meant an almost indecipherable riddle for archeologists. The bone blades covered and decorated the grip areas and their rigid ends, the horns of bows. Károly Cs. Sebestyén's articles had focused attention to the Hungarians' ancient weapon. It was Kálmán Jakus, a Physical Education teacher at Lónyai Street Reformist Academic Grammar School, though, who succeeded in manufacturing the first Hungarian bow. His primary purpose was to develop an efficient bow for sport. One of the most prominent of the next generation of developers was Dr. Gyula Fábián (1915-1985), a department head at the University of Agriculture in Gödöllő (present day Szent István University) who had carried out scientific research into the evolution of the Hungarian bow, moreover he had also been able to make a reconstruction of the traditional reflex bow. His reconstructions were also acknowledged by archeologists specialising in the given historic period. His attempts have been followed by more or less successful reconstructions of bows. In the past few decades new bows have appeared with some metal or fibreglass parts in their construction. They also contain some plastic, and therefore proved to be much stronger than the traditional constructions. Manufacturing bows which are exclusively made of natural materials is more time-consuming, requires more expertise, moreover the acquisition of special raw materials such as animal sinew, ox horn, special glue or resin etc. would make the whole process extremely difficult. Although the so-called "Hungarian Conquest period" bows available for sale these days are based on the functional and geometrical construction of their traditional Hungarian counterparts, it must be noted that their flexible bow arms are made of plastic containing glass fibre or carbon fibre.

Among Professor Gyula Fábián's disciples, Imre Puskás and Csaba Búza were the most outstanding. Árpád Ambrózy also needs to be remembered since he wrote a book about hunting archery in 1994. In this field Gábor Szőllősy should also be referred to as he was the first in Hungary to have done his doctorate in archery, moreover he has written several scientific articles and given a great number of lectures to express appreciation for the traditional Hungarian bow which is regarded as a significant product of ancient Hungarian craftsmanship as well as a brilliant "technological" achievement. He also puts great emphasis on the balanced relationship between the forces in humans and the bow. Today several manufacturers specialise in manufacturing the Hungarian bow, nevertheless Lajos Kassai's and Csaba Grózer's bows are by far the most popular.

The operation of the Hungarian bow along with the special backward shooting technique, which was so much favoured by our ancestors, raises several fascinating mechanical questions. To answer these questions a good number of experiments and calculations need to be made, moreover the mechanical model of the bow is needed to be prepared which may appropriately confirm the results of experiments. Teachers in the Bánki Donát Mechanical Engineering College of Budapest Polytechnic with the professional assistance of Dr. Gábor Szőllősy set up a small laboratory in 1997 in order to study and measure the physical characteristics of traditional Hungarian bows. The mechanical analysis of bows is based on the experiments gained in the laboratory and the results of measurements.

Our objective was to prepare the mechanical model of the Hungarian bow and then the preparation of a computer program which can be used for examinations about the geometrical optimalisation of the bow from the energetic point of view. The knowledge acquired about the mechanical model of bows facilitates not only the analysis of the traditional Hungarian bow, but also provides a good foundation for the comparison from the technical point of view of various composite reflex bows belonging to different historic ethnic groups.

2 The *Mechanical Model* of the Bow

Before starting any mechanical calculations, the geometrical features, more precisely the identifiable characteristic points of the bow need to be unambiguously defined, thus defining their position and co-ordinates with the minimum of measuring errors. It is a good idea to begin with the situation of the characteristic points and examine how they are related to each other. The measurements are related to each other as they form a measurement chain, therefore it makes checking easier. Minor mistakes might be made, though, if distances are measured instead of the characteristic angles, and then the figures calculated and concluded from the distances of angles are compared with each other.

The geometry of the drawn Hungarian bow is calculated and concluded as in Figure 1.

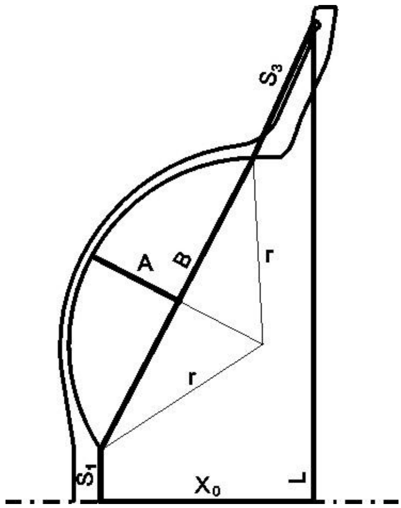

Figure 1
Geometry of the drawn Hungarian bow

in which

$2s_1$ – the length of the grip section (rigid part in the middle) (mm)

s_3 – the length of the axis of the rigid horn (mm)

$2L$ – the length of the string (mm)

A – the biggest distance between the flexible bow (bow arm) and its string (mm)

B – the length of the geometrical string of the flexible bow (mm)

x_0 – the distance between the string and the grip section, the height of the drawing of the bow (mm)

In order to make the calculations simpler, the following assumptions can be made:

- the bow arm forms a curved line,
- the bow is perfectly symmetrical,
- the cross section of the bow arm is constant,
- the connection between the bow arms and the rigid parts of the bow is like bracketing,

- the material of the bow arm is homogeneous and flexible,
- the grip and the horn are rigid,
- the effect of the pre-stretching of the bow is fully contained in the characteristics of the bow.

Later, after the mechanical model has been necessarily adjusted and made more precise, the assumptions above can be ignored.

The most important geometrical characteristics of the drawn condition of the bow are shown in Figure 2.

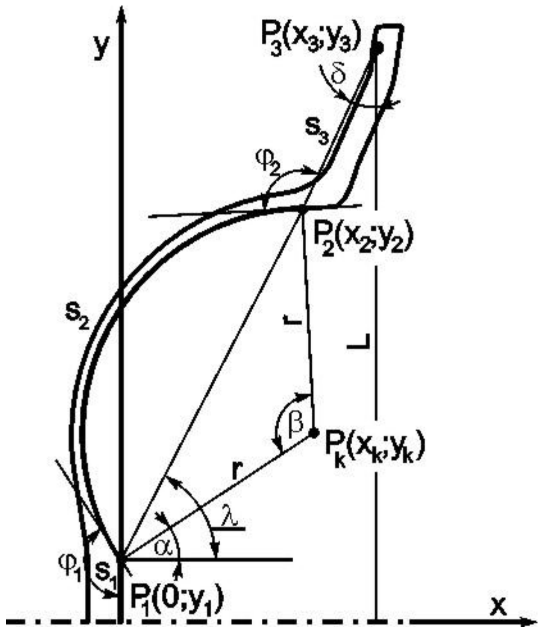

Figure 2

Geometrical characteristics of the drawn condition of the bow

The calculation of the radius of the flexible curved line:

$$r = \frac{A}{2} + \frac{B^2}{8A}\,, \tag{1}$$

then the equation about the central angle of the curved line:

$$\sin\frac{\beta}{2} = \frac{4AB}{4A^2 + B^2}\,. \tag{2}$$

With the knowledge of the results of the equations above, the length of the flexible curved line can be calculated as follows:

$$s_2 = r\beta \,. \tag{3}$$

Considering basic geometry, the horizontal projection of the lenght of the rigid horn (s_3) can be calculated as follows:

$$x_{S3} = \frac{ab - \sqrt{a^2b^2 - (1+b^2)\cdot(a^2 - s_3^2)}}{1+b^2}, \tag{4}$$

in which $a = \dfrac{x_0^2 + (L-s_1)^2 + s_3^2 - B^2}{2\cdot(L-s_1)}$, (5)

and $b = \dfrac{x_0}{L-s_1}$. (6)

With the knowledge of x_{S3}, the λ angle between the string of the flexible bow and the horizontal x axis can be concluded as follows:

$$\cos\lambda = \frac{x_0 - x_{S3}}{B} \,. \tag{7}$$

This way the α angle is calculated:

$$\alpha = \frac{\beta}{2} + \lambda - \frac{\pi}{2} \,. \tag{8}$$

The flexible bow fixed to the grip section shall be regarded as rigid, therefore the φ_1 angle between them is considered constant. Therefore its calculation:

$$\varphi_1 = \frac{3}{2}\pi - \frac{\beta}{2} - \lambda \,. \tag{9}$$

The coordinates of the characteristic P_1, P_2, P_3 and P_k points as indicated in Figure 2.

$$x_1 = 0 \,; \qquad y_1 = s_1 \,, \tag{10}$$

$$x_2 = B\cdot\cos\lambda \,; \qquad y_2 = s_1 + B\cdot\sin\lambda \,, \tag{11}$$

$$x_3 = x_2 + s_3\cdot\sin\delta \,; \qquad y_3 = L \,, \tag{12}$$

$$x_k = r\cdot\cos\alpha \,; \qquad y_k = s_1 + r\cdot\sin\alpha \,, \tag{13}$$

in which the δ angle between the rigid horn and string can be calculated from the angle function below as follows:

$$\cos\delta = \frac{L - y_2}{s_3} . \tag{14}$$

Finally, the calculation of the φ_2 angle, which is characteristic of the rigid context of the flexible curved line and the rigid horn, therefore can be regarded as a constant figure:

$$\varphi_2 = \alpha - \beta + \delta + \pi . \tag{15}$$

In order to check the geometrical figures, it is recommended to also check the figures of the bow with measuring (by measuring distance and angle), and to draw and construct a picture of the bow. The mechanical calculations about the bow can only be made if correct geometrical characteristics are available.

3 The Statics of the Drawn Bow

The basic static figures of the mechanical calculations of the traditional Hungarian composite reflex bows is the product [Nmm^2] of multiplying the F_x drawing force [N], the flexibility modulus of the material of the bow (I) and the secondary momentum of the cross-section of the bow (E).

In order to minimize the errors in the calculation due to the above-mentioned assumptions, a correctional function needs to be applied which modifies the IE product of multificaton according to the size of the deformation and to what extent the bow is drawn. The correctional equation shall be defined with the discrepancy between the results of measurements and calculations.

The transformation of the flexible curve caused by the H and F forces as well as M momentum can be calculated in the ζ-η system of coordinates with the application of the basic rules in stress analysis. The curve is regarded as a flat curve and a braced holder. Based on the theoretical considerations of the above-mentioned, the transformations, i.e. the change of the Ψ angle and the u, v movements shall be calculated as follows (based on Muttnyánszky 1981) in equation 16 a-c.

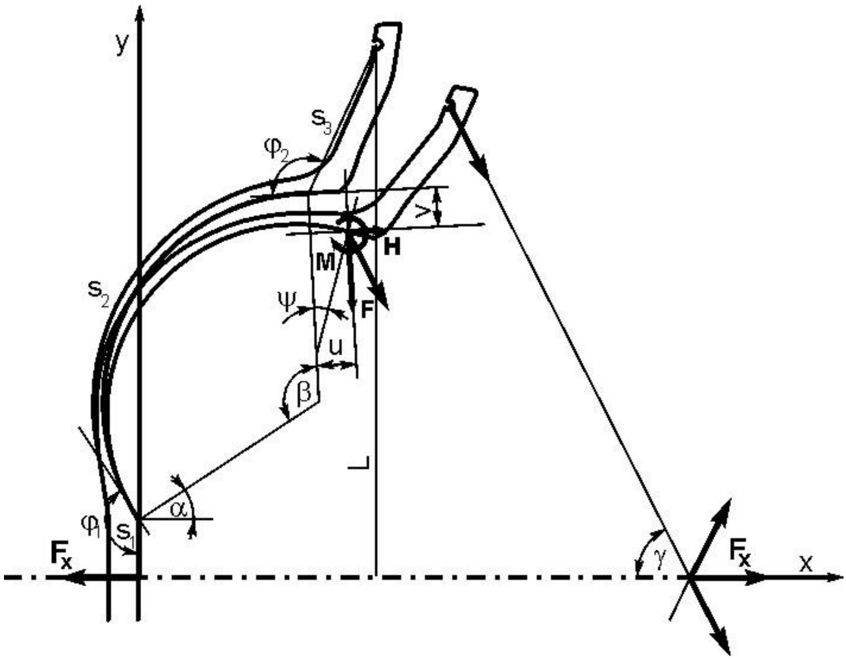

Figure 3
The theoretical constructed figure of a drawn bow

$$\psi = \frac{r}{IE}\left(a_1 M + a_2 rF + a_3 rH\right),$$
$$u = \frac{r^2}{IE}\left(b_1 M + b_2 rF + b_3 rH\right), \qquad (16\text{ a-c})$$
$$v = \frac{r^2}{IE}\left(c_1 M + c_2 rF + c_3 rH\right),$$

in which

$$a_1 = \beta,$$
$$a_2 = c_1 = \sin(\beta) - \beta\cos(\beta),$$
$$a_3 = b_1 = \beta\sin(\beta) + \cos(\beta) - 1,$$
$$b_2 = c_3 = 0{,}5 - \left[1{,}5\cos(\beta) + \beta\sin(\beta) - 1\right]\cos(\beta), \qquad (17\text{ a-f})$$
$$b_3 = \beta\left[0{,}5 + \sin^2(\beta)\right] + (1{,}5\cos(\beta) - 2)\sin(\beta),$$
$$c_2 = \beta\left[0{,}5 + \cos^2(\beta)\right] - 1{,}5\sin(\beta)\cos(\beta).$$

The transformation of the drawn bow is significant, therefore these transformations strongly affect the forces, the situation, direction and size of the momentum of the forces causing deformation. Because of this situation the geometry and the play of power forces of the bow shall be determined with iteration, i.e. the method of gradual approach.

The first step of iteration is to modify ζ_2 and η_2 as interpreted in the ζ-η. system of coordinates and defined as (P_2) that is the common point of the x_2 and y_2 coordinates of the flexible bow arm and the horn, with *u, v* and ψ transformation figures, which can most easily be read from a constructed figure. Further on, the new coordinates are defined with the knowledge of the calculated transformation during iteration:

$$\xi_2^* = \xi_2 + u, \qquad \eta_2^* = \eta_2 - v. \tag{18 a-b}$$

The familiar transformation of coordinates is used for the calculations between the ζ-η system of coordinates which are revolved with x-y and angle α of coordinates.

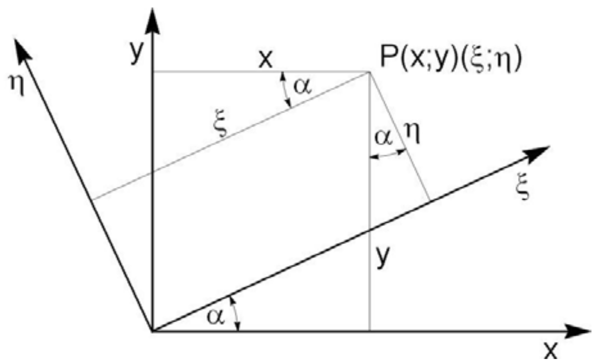

Figure 4

The equation for the calculation from the x-y system of coordinates to the ζ-η system:

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. \tag{19}$$

And here is the reverse of the equation, i.e. the calculation is transferred from the ζ-η system of coordinates to the x-y system:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} \xi \\ \eta \end{bmatrix}. \tag{20}$$

With the knowledge of ζ_2^* and η_2^*, x_2^* and y_2^* as the new coordinates of P_2 shall be calculated with the application of the above-mentioned transformation of coordinates. After all P_3 as the new position of the common point of the bow horn and the string can be calculated as follows:

$$\begin{aligned} x_3^* &= x_2^* + s_3 \cdot \cos(3\pi/2 + \alpha - \beta - \varphi_2 - \psi) = x_2^* + s_3 \cdot \sin(\alpha - \beta - \varphi_2 - \psi), \\ y_3^* &= y_2^* + s_3 \cdot \sin(3\pi/2 + \alpha - \beta - \varphi_2 - \psi) = y_2^* - s_3 \cdot \cos(\alpha - \beta - \varphi_2 - \psi). \end{aligned} \qquad (21\text{ a-b})$$

Therefore the half angle of the string:

$$\gamma = \arcsin\left(\frac{y_3^*}{L}\right), \qquad (22)$$

and then the x_F coordinate of the introduction of force of the F_x pulling force:

$$x_F = x_3^* + \sqrt{L^2 - y_3^{*2}} \ . \qquad (23)$$

The calculation of the pulling force in the string:

$$F_1 = \frac{F_x}{2 \cdot \cos(\gamma)} \cdot \qquad (24)$$

The same equation in a vector form:

$$\mathbf{F}_1 = \begin{bmatrix} F_1 \cos(\gamma) \\ -F_1 \sin(\gamma) \end{bmatrix}. \qquad (25)$$

The position vector between the P_2 and P_3 points:

$$\mathbf{r}_{23} = \begin{bmatrix} x_3^* - x_2^* \\ y_3^* - y_2^* \end{bmatrix}. \qquad (26)$$

The vector of M turning momentum from transferring force F_1 from P_3 point to point P_2:

$$\mathbf{M} = \mathbf{r}_{23} x \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & 0 \\ F_1 \cos(\gamma) & -F_1 \sin(\gamma) & 0 \end{vmatrix} = -F_1 [r_x \sin(\gamma) + r_y \cos(\gamma)] \cdot \mathbf{k}$$

$$(27)$$

from which the size of momentum shall be calculated as follows:

$$M = F_1[(x_3^* - x_2^*)\sin(\gamma) + (y_3^* - y_2^*)\cos(\gamma)]\,. \tag{28}$$

The forces loading the braced circularly curved holder:

$$\begin{aligned} H &= F_1 \sin(\beta - \alpha - \gamma), \\ F &= F_1 \cos(\beta - \alpha - \gamma). \end{aligned} \tag{29 a-b}$$

After all the new figures of the ψ angle change and the u and v movements can be calculated with the (16 a-c) equations, then with the knowledge of this we can formulate the new coordinates of the common P_2 point of the flexible bow arm and the horn, while the iteration can be continued until the calculation has reached the appropriate margin of error.

4 The Energetical Analysis of the Measurements of the Bow

By means of the model, the parameters of the change of certain geometrical measurements on the energy accumulated in the bow, as the most typical characteristic of the application of the bow, can be analysed. When a bow is drawn, flexible energy accumulates in its structure, which gets mainly transferred to the arrow during shooting, while causing it to move. The characteristics of the bow are concluded from the relationship between the extent of the tension and the force that is necessary for it. See the characteristic equation below:

$$F = f(x)\,, \tag{30}$$

in which Fx is the x direction force belonging to x distance (extension). The flexible energy accumulated in the bow can be calculated as follows:

$$E = \int_0^x f(x)\cdot \mathrm{d}x\,. \tag{31}$$

In the following part of this study the consequences of the individual alterations of each of the four geometrical characteristics of the Hungarian composite reflex bow shall be discussed. For the sake of better comparison, the maximum of the pulling force is defined as 200 N in every case, which as a matter of fact results in a deformation of different extent in the cases of bows of different sizes despite the fact that the cross section of the flexible bow arm (secondary momentum) and the material (flexibility modulus) have not changed. Regarding the characteristics of a real bow as standard, the measures are modified by -40, -20, +20 and +40%, then the characteristic curves are defined followed by the formulation of the energy of the bow.

The basic characteristics of the bow which are based on the measurements of the Hungarian bow known from archeological findings as well as the bow that was available for our research (these figures are later referred to as „standard" characteristics of the bow):

$2s_1 =$ 112 mm

$s_3 =$ 226 mm

$2L =$ 1260 mm

$A =$ 93 mm

$B =$ 367 mm

$x_0 =$ 148 mm

First of all the characteristic curve of the bow is formulated with measuring and calculations:

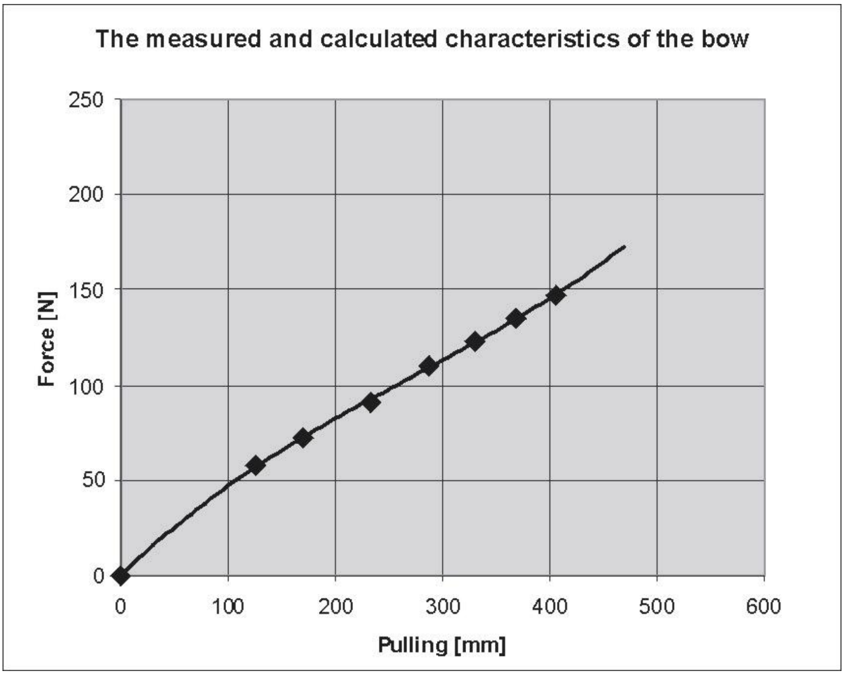

Figure 5
Characteristic curve of the bow

The applied equation for correction:

$$k = 100/(-0{,}000001 \cdot \Delta x^3 + 0{,}0008 \cdot \Delta x^2 + 0{,}097 \cdot \Delta x), \tag{32}$$

in which *Δx* means the proportion of the draw in mm:

$$\Delta x = x_F - x_0 \,. \tag{33}$$

When the *IE* product of multiplication is multiplied by the *k* correctional factor, the error concluded from the assumptions shall decrease.

The following characteristic curve shown in the next figure is the result if the s_1 size of the grip section is changed.

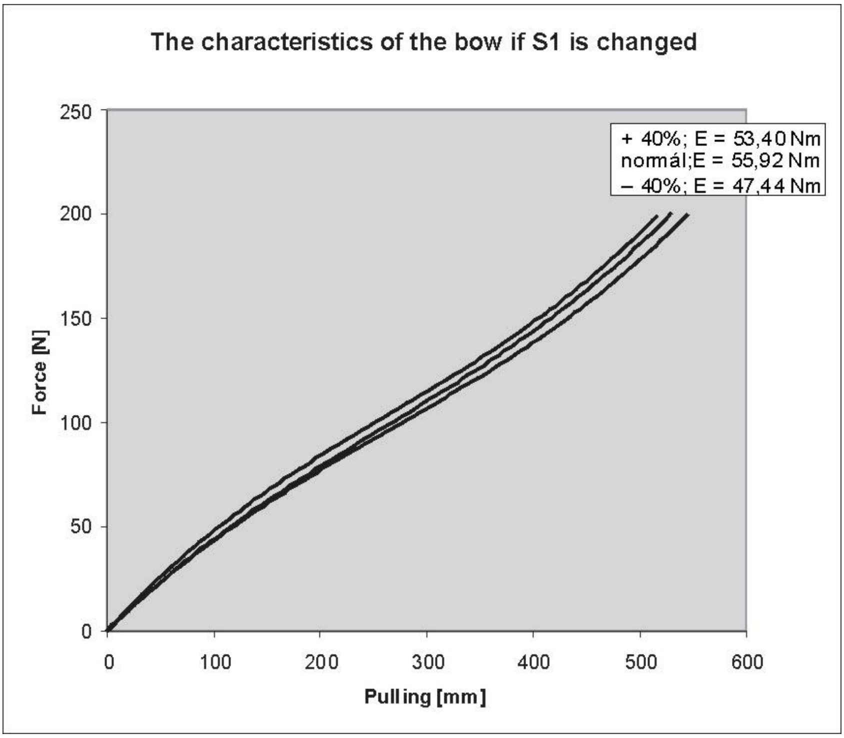

Figure 6

As it can clearly be seen in Figure 6, the alteration of the size of s_1 does not practically affect the static characteristics of the bow.

„A" measurement indicates the longest distance between the geometrical string of the curved bow arm and the bow arm itself. If „A" is changed, the following characteristic curves can be drawn as in Figure 7.

It can be seen well in Figure 7 that the characteristics of the bow hardly change if the curve is increased, however, if the curve is decreased, it results in a substantial modification of the characteristics. From the energetic point of view, the 20% decrease in the curve of the bow marked „normal" may result in some improvement.

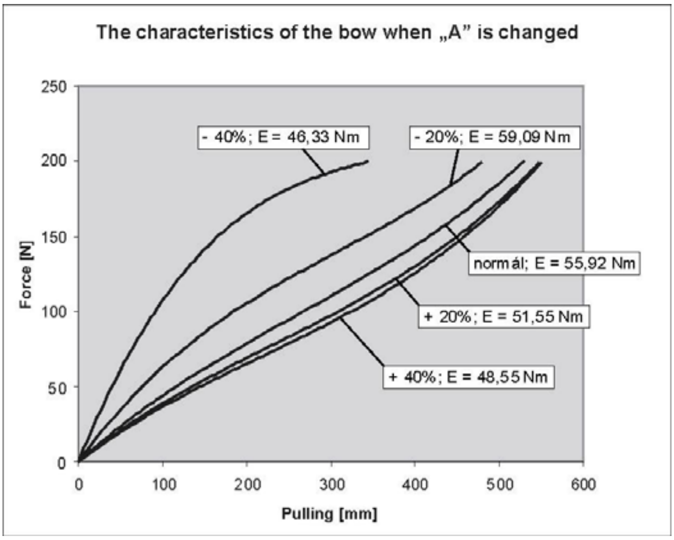

Figure 7

Changing the „B" measurements, i.e. the length of the longest string of the flexible bow arm may result in the following characteristic curves:

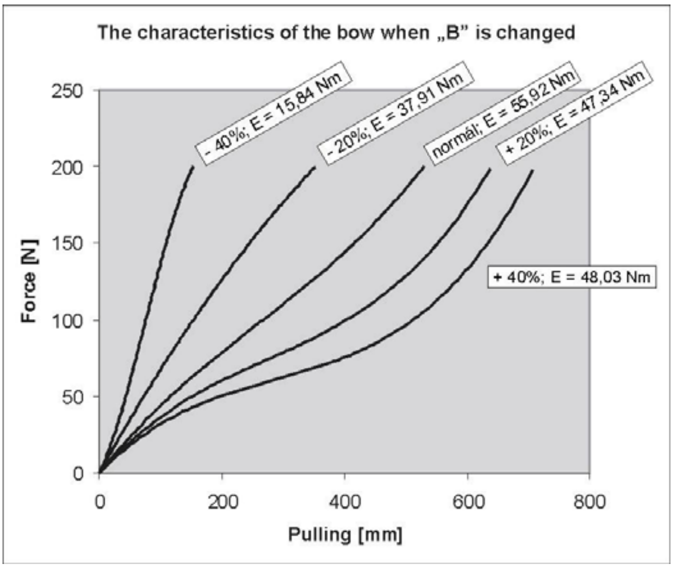

Figure 8

It can clearly be established if we look at the curves that the length of the flexible bow arm makes a substantial impact on the energy stored in the bow, therefore it influences the quality and the efficiency of the bow.

If the length of the so-called horn of the Hungarian bow (s_3 measurement) is modified, the characteristic curve will change according to Figure 9.

It is perhaps surprising what important role the horn plays in storing the energy in the bow. The horn is rigid and its deformation can be ignored, nevertheless it is a substantial characteristic element when it comes to the geometry and the functioning of the bow. The horn is responsible for the increase of the pulling length of the bow, which increases not only the velocity of the arrow and the energy that can be transmitted to the arrow, but it has also lead to a smaller size bow, which is one of the most outstanding features of reflex bows, as it is only the little size of the bow that makes horsing archery possible, therefore this characteristic had contributed greatly to the irresistable fighting manner of the conquering ancient Hungarians.

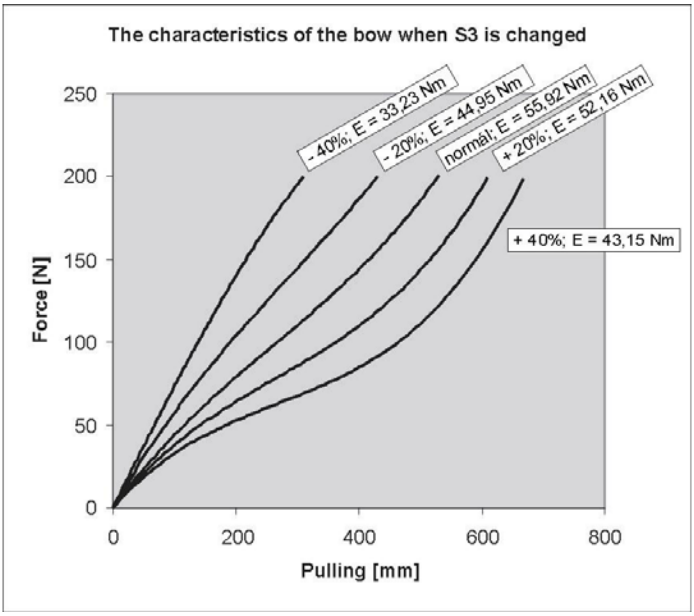

Figure 9
Change of characteristic curve

Finally Figure 10 summarizes the effect of the four altered geometrical characteristics of the Hungarian bow on the energy accumulated in the bow when it is affected by F=200 N force.

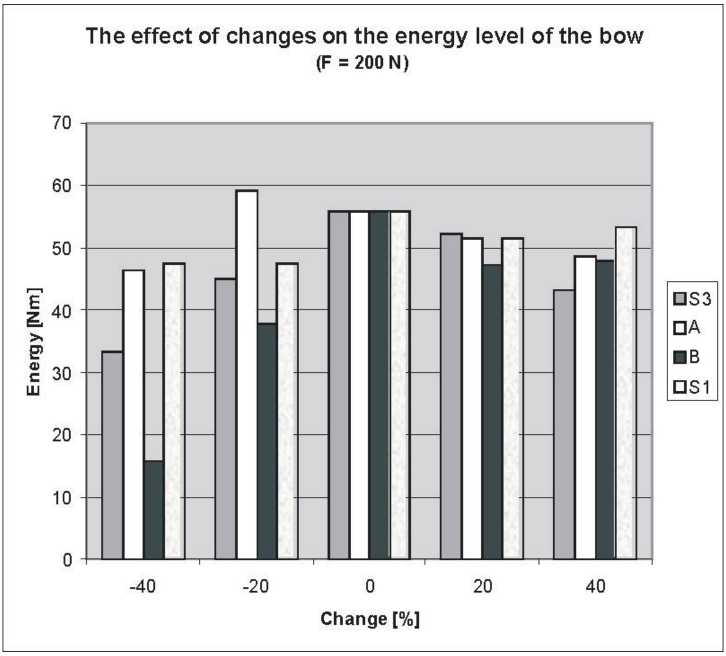

Figure 10

Conclusion

Based on the energetical analysis of the bow several conclusions can be drawn. It may be found surprising what important roles are played by the horns regarding the energy accumulated in the bow. Although the horn is rigid and its deformation can practically be ignored, it is still a relevant element in the geometry and the operation of the bow. Due to the horn the length of the extention of the bow substantially increases, which increases not only the acceleration path of the arrow and the transmittable energy but also results in the development of a small bow which means one of the most ingenious characteristics of reflexive bows since this small size makes their usage available for horse archery, moreover this characteristic contributed to the irresistable fighting manner of old Hungarians at the time of the Hungarian Conquest.

Finally the graph summarising the results of the various calculations shows that the analysed Hungarian bow has almost ideal measurements, which must have been the result of our ancestors' long experiments with the proportions of the bow throughout several centuries.

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