Numerical Investigation of Glue Laminated Timber Beams considering Reliability-based Design

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Abstract: Structural models and their related parameters, are most often considered as deterministic, in numerical analysis. However, according to test results, one can see the existence of uncertainties, in most cases, due to various reasons, such as, natural variabilities and ignorance. Thus, dealing with uncertainty has gained massive attention, due to its importance in structural analysis and anticipating the performance of models. In fact, in some cases of special structure components, like glue laminated timber beams, it appears to be, that there is an absence of information concerning uncertainties. Therefore, the main objective of this study is to inspect uncertainties that facing designers and their role in glue laminated timber beams behavior, by considering different material parameters as random variables. In addition, four-point bending tests are conducted and finite element analysis is conducted, using ABAQUS software, to model the nonlinear behavior of GLT beams. For purposes of numerical model calibration, Hill yield criterion constitutive model is considered based on the obtained data from the experimental test. The results of this study provide a better outline for understanding the effect of uncertainties on glue laminated timber beams.

Keywords: Glue laminated timber beams; Hill yield criterion; Reliability-based design; Finite element analysis

1 Introduction

Timber has been greatly considered as a construction material since centuries in various structural projects, due to its benefits such as its low cost, well mechanical properties, durability, lightweight material, and its ability to respond to seismic and high wind events without occurring of critical failure [1-3]. Moreover, timber is considered as anisotropic material which shows various constitutive relationship in tension and compression which categorized according to the direction of grain by three distinct directions: longitudinal, radial, and tangential [4] [5]. A composite doubly-curved laminated shells were investigated in the study of Monge et al. [6]

by utilizing kinematic models, where a simply supported shell was subjected to various loads. Also, in the study of Le et al. [7], an analytical technique was adopted to expect the nonlinear buckling behavior of graphene-reinforced composite laminate shells considering thermal environment. Taking into consideration that one of the most structural timber product is glued laminated timber (GLT) which can be effectively used in cases curved shapes production.

Glue laminated beams could be either horizontally or vertically laminated. Also, these configurations can illustrate the fact that the horizontal method can produce curved members efficiently [8].

Glued laminated timber nowadays is considered as one of most common building material since it has shown efficient behavior and saving energy properties. Thus, glue laminated timber is widely used in industrial projects.

Recently, GLT has been utilized in uncovered applications like vaulted roofs and huge open spaces. Also, glulam is utilized when you look for a mixture of aesthetic and structural qualities. This incorporates a scope of engineering applications, such as glulam gallery which was built in Johor Bahru [9], an arched glulam timber bridge in Sheshan golf court of Shanghai in China [10] and Grandstand of Kulm Hotel in St. Moritz [11].

Different constitutive models have been developed during last decades to represent the nonlinear behavior of timber, which can be classified into three categories: elasto-plastic material models, elastic-damage models and combination of elasto-plastic material with damage model [12] [13]. Thus, finite element models have been considered recently to simulate the behavior of timber.

Currently, the Hill yield criterion and Tsai-Hill criterion are widely used to model the timber behavior. For instance, Xu et al. [14] presented in their study a nonlinear finite element model to simulate the strength of GLT. A constitutive elasto-plastic model of timber beam with openings was introduced in the study of Guan and Zhu [15].

The existence of randomness such as in material, loadings, and geometry properties that might reduce the strength of timber, lead engineers to deal with uncertainties. Thus, a stochastic models of timber structures are required for designs. In fact, with contrary to deterministic approach, the probabilistic method improves design reliability where it provides several benefits to designers such as improve sensitivity analysis and allow designers to determine the crucial parameters of uncertain models [16].

Numerical investigations have turned into a significant common methodology for analyzing various designing systems. Simulations are normally depicted as the means for supplanting the actual world according to a combination of theories as well as envisioned models of reality.

Monte Carlo simulation technique was created to be a probabilistic way to deal with complex deterministic complications as computers simply simulate a large number of exploratory tests which yield random results [17]. In the study of Corradi et al. [18], the uncertainty in strength of reinforced timber was modelled and various bending tests were carried to illustrate the effect of reinforcement on timber strength. The randomness of longitudinal strength of timber beam according to the existence of knots was analyzed by Czmoch [19] where Monte Carlo technique was adopted to find the statistics of a specified timber elements. Jenkel et al. [20] investigated and analyzed two timber structures by considering stochastically models of material parameters.

The goal of this study is to examine the effect of reliability indices on the GLT beams using probabilistic finite element analysis. Thus, the novelty of the article is about considering the probabilistic design in order to make the model more reliable and safer in which the designer must deal with the existence of uncertainties to make the method more practical. Moreover, this paper presents the results of an experimental program of 4-point bending tests of GLT beams. A written code is used to pursue the required goal by considering the reliability index as a limit when the material parameters for both tension and compression sides of timber are considered as random variables. Moreover, Monte Carlo sampling method has been adopted to calculate the reliability indices depending on statistics of material parameters. Finally, determination of corresponding load, displacement, and mean stresses values.

The rest of this paper is structured as: Section 2 introduce the used model of timber material behavior while Section 3 illustrates the reliability-based analysis. Furthermore, Section 4 introduces the experimental program and numerical model validation. Finally, Sections 5 and 6 represent the discussion of the results and conclusions respectively.

2 Behavior of Timber Material

The behavior of timber in compression parallel to grain exhibits some strain relaxing as a rule, it tends to be roughly viewed as elastic perfectly plastic. The fundamental anisotropic yield criterion which can be applied in evaluating timber is the Hill yield criterion.

The Hill yield criterion can be used in the numerical analysis of wooden elements. The theory is based on the generalization of the Huber-Mises-Hencky hypothesis for anisotropic materials where a connection is allowed between the anisotropic directions and material strengths. This can be adopted in modelling of materials, such as metals in rolling processes, which display partial orthotropic behavior or composite materials.

In case of using this criterion along the isotropic hardening option, the yield function is expressed by [21]:

$$f(\sigma) = \sqrt{(\sigma)^T \cdot [M] \cdot (\sigma)} - \sigma_0{}^{(\bar{\varepsilon}^p)} \tag{1}$$

where σ_0 is the reference yield stress, ε^p is the equivalent plastic strain and $[M]$ represents mass matrix. While in case of use it with kinematic hardening option, the yield function will have the following expression:

$$f(\sigma) = \sqrt{\big((\sigma) - (\alpha)\big)^T \cdot [M] \cdot \big((\sigma) - (\alpha)\big)} - \sigma_0 \tag{2}$$

where α is the yield surface translation vector. The Hill yield stress potential within a coordinate system which is aligned with anisotropy coordinate system can be formulated as following:

$$\begin{aligned} f(\sigma, \sigma_y) = F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L{\sigma_{23}}^2 \\ + 2M{\sigma_{31}}^2 + 2N{\sigma_{12}}^2 - {\sigma_y}^2 = 0 \end{aligned} \tag{3}$$

where F, G, H, L, M and N are constants calculated experimentally [21] of the material in various orientations.

$$F = \frac{1}{2} \cdot \left(\frac{1}{R_{22}^2} + \frac{1}{R_{33}^2} - \frac{1}{R_{11}^2}\right) \tag{4}$$

$$G = \frac{1}{2} \cdot \left(\frac{1}{R_{33}^2} + \frac{1}{R_{11}^2} - \frac{1}{R_{22}^2}\right) \tag{5}$$

$$H = \frac{1}{2} \cdot \left(\frac{1}{R_{11}^2} + \frac{1}{R_{22}^2} - \frac{1}{R_{33}^2}\right) \tag{6}$$

$$L = \frac{3}{2 \cdot R_{23}^2} \tag{7}$$

$$M = \frac{3}{2 \cdot R_{13}^2} \tag{8}$$

$$N = \frac{3}{2 \cdot R_{12}^2} \tag{9}$$

where $R_{i;j}$ are anisotropic yield stress ratios.

Furthermore, this criterion can be used in modelling wood and processed wood products, fiber matrix composites, zirconium alloys and titanium alloys.

3 Reliability-based Analysis

The fundamental concept of reliability analysis can be presented, by assuming X_R which indicates the non-negative limit for X_S, thus the failure could be estimated through $X_R \leq X_S$. Supposing that X_R and X_S independent random variables having probability density functions $f_R\ (X_R\ )$ and $f_R\ (X_S)$. The probability of failure P_f could be estimated according to following expression [22]:

$$P_f = P[X_R \leq X_S] = \iint_{X_R \leq X_S} f_R\ (X_R)\, f_S\ (X_S) dX_R dX_S \tag{10}$$

A possible definition of the previous formulation can be given in terms of the so-called limit-state function which is identified by:

$$g(X_R, X_S) = X_R - X_S \tag{11}$$

Taking into consideration that $g \leq 0$ identifies the failure domain D_f. Thus, P_f is expressed by:

$$P_f = F_g(0) \tag{12}$$

Additionally, P_f can be determined as:

$$P_f = \int_{g(X_R,X_S)\leq 0} f(X)dX = \int_{D_f} f(X)dX \tag{13}$$

Monte-Carlo sampling method involves realizations generating x of the random vector X upon their probability joint density function $f_X(x)$ and examining if failure occurs or not according to a given realization. The failure probability can be estimated according to the ratio of total number of points within the failure domain to the entire number of generated points. The number of points in the failure domain with respect to the total number of generated points is an estimator of the probability of failure. This concept could be formulated by initiating an indicator function of D_f[23]:

$$\chi_{D_f}(x) = \begin{Bmatrix} 1 & \quad if\ \ x \in\ D_f \\ 0 & \quad if\ \ x \notin\ D_f \end{Bmatrix} \tag{14}$$

Then equation (13) can be rewritten as:

$$P_f = \int_{-\infty}^{+\infty} \ .\ .\ .\ \int_{-\infty}^{+\infty} \chi_{D_f}(x) f_X(x) dx \tag{15}$$

Consequently, function $\chi_{D_f}(X)$ is a random variable having two points distribution:

$$\mathbb{P}\left[\ \chi_{D_f}(X) = 1\right] = P_f \tag{16}$$

$$\mathbb{P}\left[\ \chi_{D_f}(X) = 0\right] = 1 - P_f \tag{17}$$

where $P_f = \mathbb{P}[\ X \in\ D_f\ ]$. The mean value and variance of $\chi_{D_f}(X)$ are expressed by:

$$\mathbb{E}\left[\chi_{D_f}(X)\right] = 1 \cdot P_f + 0 \cdot \left(1 - P_f\right) = P_f \tag{18}$$

$$\mathbb{V}ar\left[\chi_{D_f}(X)\right] = \mathbb{E}\left[\chi_{D_f}^2(X)\right] - (\mathbb{E}\left[\chi_{D_f}(X)\right])^2 = P_f - P_f^2 = P_f(1 - P_f) \tag{19}$$

In Monte-Carlo sampling method, to determine P_f, the following estimator of mean value is applied:

$$\hat{\mathbb{E}}\left[\chi_{D_f}(X)\right] = \frac{1}{Z}\sum_{z=1}^{Z}\chi_{D_f}(X^{(z)}) = \hat{P}_f \tag{20}$$

Where $X^{(z)}$ are random independent vectors (where $z = 1, \dots, Z$) accompanied with probability density functions which can be determined by $f_X(x)$. Due to the uncertainties, the material properties are defined as a random variables and it follows the Gaussian distribution with mean value $\mathbb{E}$ and variance $\mathbb{V}ar$. Accordingly, the mean value and the variance of the estimator can be simply determined as the following:

$$\mathbb{E}[\hat{P}_f] = \frac{1}{Z}\sum_{z=1}^{Z}\mathbb{E}\left[\chi_{D_f}(X^{(z)})\right] = \frac{1}{Z}ZP_f = P_f \tag{21}$$

$$\mathbb{V}ar[\hat{P}_f] = \frac{1}{Z^2}\sum_{z=1}^{Z}\mathbb{V}ar\left[\chi_{D_f}(X^{(z)})\right] = \frac{1}{Z^2}ZP_f(1-P_f) = \frac{1}{Z}P_f(1-P_f) \tag{22}$$

Therefore, the reliability constraint can be illustrated by considering the reliability index β as:

$$\beta_{target} - \beta_{calc} \leq 0 \tag{23}$$

To calculate β_{target} and β_{calc}, the following equations are used:

$$\beta_{target} = -\Phi^{-1}(P_{f,target}) \tag{24}$$

$$\beta_{calc} = -\Phi^{-1}(P_{f,calc}) \tag{25}$$

4 Numerical Model Validation

4.1 Experimental Program

In this research, four-point bending tests are conducted similar to de Jesus [24]. Based on this experimental work, three beams of $2500\ mm$ long GLT beams having a cross-sectional area of $100 \times 240\ mm$ are tested. Adhesion test of timber was performed before the start of the tests. Commercial glulam available beams are used, the properties of which were defined by the producer. Table 1 represents the experimental properties of the used materials. Furthermore, the layout of the laboratory experimental test is shown in Figure 1.

Table 1

Material properties

Material	***Flexural strength [N/mm²]***	***Compression strength [N/mm²]***	***Tensile strength [N/mm²]***	***Shear strength [N/mm²]***	***Elastic modulus [N/mm²]***
Timber	$f_{m,k} = 50.0$	$f_{c,0,k} = 29.0$ $f_{c,90,k} = 3.2$	$f_{t.0.k} = 30.0$ $f_{t90.k} = 0.4$	$f_{v.k} = 4.0$	$E_{0,mean} = 9400$ $E_{90,mean} = 390$

Figure 1
Layout of the experiment

4.2 Material and Methods

Finite element analysis is considered to model the nonlinear behavior of GLT beams. 8-node solid element (C3D8) is considered for modeling the laminated beams. While the contact between lamellas is assumed perfectly bonded. At points of loading, steel bearing plates with dimensions of length = $150\ mm$, width = $100\ mm$ and thickness = $30\ mm$ are installed to avoid local failure which might be caused by crushing.

The geometry, loading and supporting conditions of the beams are presented in Figure 2, where the whole beams are tested under monotonic loading up to failure, with two concentrated load acting on the top of the beams.

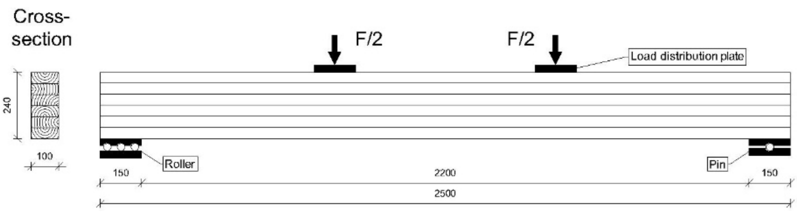
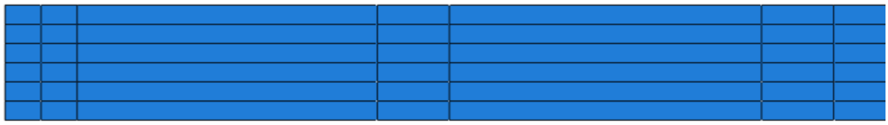
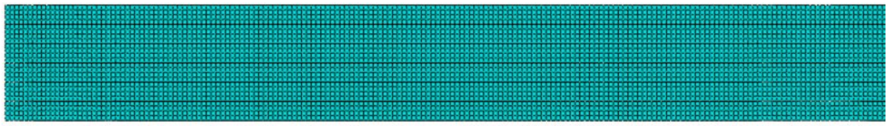


Figure 2
Beams geometry and loading condition

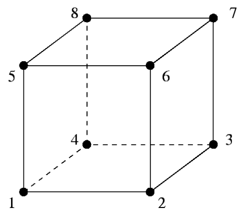
The boundary conditions are roller as a left support to produce rotation and horizontal movement and pin set as the right support to produce rotation. Two vertical concentrated loads are given on the distribution plates which are placed at the top of the beam and these applied loads are distributed by the coupling effect. Furthermore, a fine mesh is considered to obtain results with sufficient accuracy, where the number of total elements is approximately 60000. Figure 3 shows the considered numerical model, while Tables 2 and 3 show the numerical specimen's properties.



(a)



(b)



(c)

Figure 3
Considered numerical model: **(a)** assembly of the model **(b)** finite element mesh of the model **(c)** the considered elements for FEA

Table 2
Material properties for numerical modelling of timber (compression side)

Elasticity			Plasticity		
E_1=9400 MPa	E_2=0.53 GPa	E_3= 0.53 GPa	σ_{yield}= $f_{c,0,k}$ = 29.0 MPa		
G_{12}=0.72 GPa	G_{13}=0.24 GPa	G_{23}=0.24 GPa	R_{11}=5.800	R_{22}=0.640	R_{33}=0.640
ν_{12} = 0.40	ν_{13}=0.40	ν_{23} = 0.40	R_{12}=1.386	R_{13}=1.386	R_{23}=1.386

Table 3
Material properties for numerical modelling of timber (tension side)

Elasticity			Plasticity		
E_1=9400 MPa	E_2=0.39 GPa	E_3= 0.39 GPa	σ_{yield}= $f_{t,0,k}$ = 30.0 MPa		
G_{12}=0.72 GPa	G_{13}=0.24 GPa	G_{23}=0.24 GPa	R_{11}=6.000	R_{22}=0.080	R_{33}=0.080
ν_{12} = 0.40	ν_{13}=0.40	ν_{23} = 0.40	R_{12}=1.386	R_{13}=1.386	R_{23}=1.386

Figure 4 show the deflections of the middle cross section of the validated model compared to the average experimental test.

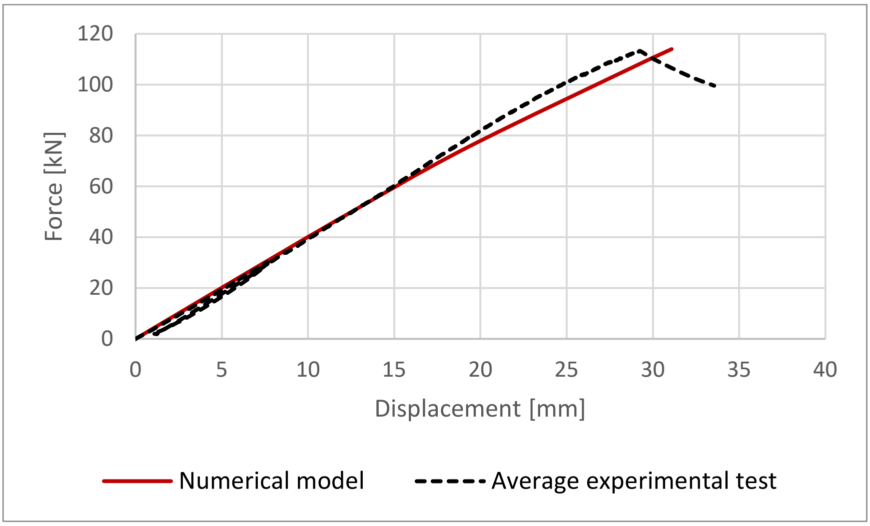


Figure 4
Comparison of experimental and numerical results

5 Results and Discussion

In order to validate the numerical model, finite element software (ABAQUS) is used based on the collected data from experimental tests. Afterward, a code is written and connected to ABAQUS to begin the analysis considering the reliability index as a limit and timber properties as random variables with mean values and standard deviations. Table 4 represents the considered random variables for timber material.

Table 4
Considered random variables of material properties

Parameter	$f_{c,0,k}$ N/mm^2	$f_{c,90,k}$ N/mm^2	$f_{v.k}$ N/mm^2	$f_{t.0.k}$ N/mm^2	$f_{t,90,k}$ N/mm^2	E_0 N/mm^2	E_{90} N/mm^2	G GPa	v
Mean	29.00	3.20	4.00	30.00	0.40	9400	390	0.72	0.40
Std Dev.	5%								

The Monte Carlo simulation technique is used to analyze the samples according to various material properties to examine the effect of reliability index on the behavior of the timber beam by assuming the number of sample point ($Z = 3 \times 10^6$). For illustration, Table 5 shows results of three different considered reliability index.

The effect of reliability index (β) as it works as a limit is obvious on the results alongside the variation of the material properties to show the corresponding loads (F) and displacements (U). Furthermore, adopting small values of β will produce higher loads, thus greater values of displacements too, so a small value β will involve more applied loads to be achieved. However, assuming the material properties with a variance, will cause the operation to produce random material properties, for each cycle and this explains how the role of uncertainties, is considered in this paper.

Table 5

Results of beam analysis

β	$f_{c,0,k}$ N/mm^2	$f_{c,90,k}$ N/mm^2	$f_{v,k}$ N/mm^2	$f_{t,0,k}$ N/mm^2	$f_{t,90,k}$ N/mm^2	E_0 N/mm^2	E_{90} N/mm^2	G GPa	v	U mm	F kN
4.83	27.44	3.35	4.02	31.40	0.41	9308	384	0.71	0.42	22.07	88
4.28	30.19	3.42	3.78	30.06	0.38	8478	386	0.68	0.41	23.16	90
3.32	26.02	3.14	3.83	26.60	0.45	9271	411	0.71	0.39	23.39	92

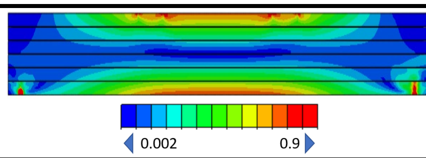
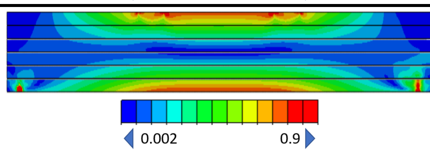
Moreover, the variation of material properties which showed in Table 4, indicates that the performance of the applied 5% standard deviation on those values and the results are changed accordingly where the material properties are directly affecting the load and displacement values alongside with the inserted β values.

Another comparison is made to show the stress distribution of the model according to three different β values as shown in table 6. Besides, Table 7 shows the deterministic results which obtained by applying the ultimate load (Fu). Also, the mean stress value and the corresponding displacement (U) are presented.

As three different models having different β values were considered, it can be noticed that the value of mean stress increases as β decreases, also the yielding stresses distribution which presented by the red color are more intense in the deterministic case than the probabilistic case.

Table 6

Stress distribution according to probabilistic analysis

β	F (kN)	Displacement (mm)	Mean stress (MPa)	Stress distribution
4.83	88	22.07	12.04	
4.28	90	23.16	12.35	

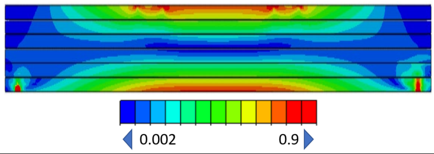
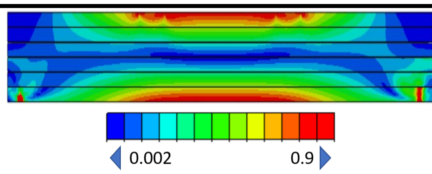
3.32	92	23.39	12.64	

Table 7
Stress distribution of according to deterministic analysis

F (kN)	Displacement (mm)	Mean stress (MPa)	Stress distribution
114	31.09	13.77	

From the obtained results of introducing the probabilistic analysis, it can be noticed that that the values of mean stresses in case of deterministic design are higher than which are obtained in probabilistic design. Hence, we can say that the β is working as a bound for a safe design controlling the yielding status in the model, where the intensity of the stress is graded from maximum red to minimum blue as seen in tables.

Conclusions

In this paper, the effect of considering the reliability index, for glue laminated timber beams, is examined, by using deterministic and reliability-based finite element analysis. Justification of the numerical model has been proposed using the Hill yield criterion constitutive model after recording the necessary data according to the experimental tests. In addition, a written code was utilized to achieve the aim of considering the reliability index as a limit while material properties of timber are considered as random variables with mean values and standard deviation for each parameter. Hence, according to what have been mentioned previously, the following key points are noted:

1) The effect of considering $\boldsymbol{\beta}$ is obvious on the results as it works as a limit together with the varied material properties to provide the corresponding loads and displacements.

2) It can be noticed according to the obtained results that as $\boldsymbol{\beta}$ decreases, the corresponding values of mean stresses increases.

3) Choosing smaller values of $\boldsymbol{\beta}$ will produce greater loads, thus greater displacements.

4) The considered standard deviation and mean values of timber properties causes random varied results since the timber properties are directly affecting the load and displacement values collaborating with the inserted $\boldsymbol{\beta}$ values.
5) Yield stress distributions are less intensive in the probabilistic cases in comparison to the deterministic cases, thus we can say that $\boldsymbol{\beta}$ works as a bound parameter controlling the yield state in the model.

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