

Improvement of the Power Transmission of Distribution Feeders by Fixed Capacitor Banks

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Abstract: The aim of this paper is the presentation of a new analytical formulation of the reactive energy compensation on distribution lines, which are characterised by their radial configuration. It will be devoted to the determination of the sizes and locations of a given number of fixed capacitor banks placed on a non-homogeneous radial line with a non-constant voltage. For the network solution, it is required to know the voltage at each node and at the capacitors location. On the basis of what has been just said and due to the radial type of the line, an iterative method called voltage drop method will be applied. The voltage rms values and phase-angles at all the nodes and on the capacitor banks will be calculated. The mathematical models of the current distributions are made considering the line active and reactive power losses. In the reactive energy optimisation process and for the power and energy loss reductions, we have used new models. The latter take into account the effect of all the capacitors in the calculation of the loss reductions due to a particular one. The results obtained then, are compared to those of authors having previously worked on the subject.

Keywords: shunt capacitors, voltage drop, radial line, optimal capacitor sizes, optimal capacitor locations

1 Introduction

The transit of a strong reactive component of the current in an electrical line causes power losses, voltage drop and thus a reduction of the line power transmission. Compared to transmission lines, the distribution ones have a low voltage and high current. The RI^2 loss (up of 13%) in distribution systems is than significantly high. To improve the line power transmission and to avoid turning to investments in new distribution lines, the power utilities are firstly forced to reduce the losses in distribution systems. To achieve power and energy loss reductions as well as voltage correction, shunt capacitor banks are widely used.

The capacitor installation techniques can be classified in several groups among of which we note; the analytical methods [1]-[7] which are easy to understand and gives detailed mathematical models, the numerical methods [8]-[11] which are iterative and with or without constraints, the heuristic methods [12]-[14] developed through intuition and experience. They are a fast practical way which leads to a near optimal solution and reduce the search space. Artificial intelligence based methods [15]-[20]. Based on the natural evolution and the annealing of solids, these methods can also be a combination of a set of methods (hybrid methods). In this class of methods we note: genetic algorithms [15]-[17]; fuzzy logic and genetic algorithms [18]; simulated annealing [19]-[20].

Our interest in this paper relates to the analytical methods. Several methods have been used to conduct the reactive energy compensation in an optimal way i.e., to have less power and energy losses. If the early analytical works [1]-[4] constituted an important step in the modelling of the optimisation of the reactive energy compensation, they remain however non-realistic because of the number of assumptions that they considered. Some of these assumptions are uniform line, uniform load and constant voltage along the line. Later works based on heuristic search [12]-[14], although they do not make any simplifying assumptions, tackle the problem by initially identifying the possible nodes candidate to carry capacitor banks, then determining their optimal sizes. The problem is thus reduced to capacitor sizes optimisation. The advantage of this method lies in the fact of being easy and very practical especially for the feeders with laterals. But, the capacitor locations are the more indicated but not the optimal ones. Moreover, for more than one capacitor, the effect of the batteries the ones on the others does not appear clearly in the objective function for a possible mathematical analysis of the problem. It appears implicitly in the reactive current updating when we perform the load flow. References [5]-[7], in our point of view, present best and complete mathematical models for leading the reactive energy optimisation. Indeed, this method allows the electrical energy suppliers to choose either to optimise only one parameter (capacitors location or size) or both of them at the same time.

Nevertheless, owing to the fact that the authors in [5]-[7] have defined the power and energy loss reductions due to a given capacitor as dependent only on the

powers of those located at its downstream and owing to fact that the formulation of the capacitor current [7] is not homogenous, we present in this paper new expressions for the power and energy loss reductions as well as the equations whose results give the capacitor optimal sizes and locations. However, we firstly define the objective function as well as a simple manner to calculate the nodes voltage and the voltage at the capacitor locations.

2 Objective Function

To optimise the reactive energy compensation, the definition of an objective function is essential. This objective function called also ‘economic savings’ depends on the power and energy loss reductions due to all the installed capacitors and their costs. This function noted ‘ S ’, is defined by [5]-[7]:

$$S = k_p \Delta P + k_e \Delta E - k_{cf} \sum_{k=1}^n Q_{ck} \quad (1)$$

By using the capacitor voltage and current, the expression (1) becomes:

$$S = k_p \Delta P + k_e \Delta E - k_{cf} \sum_{k=1}^n \frac{V_{ck} I_{cck}}{\cos \varphi_{ck}} \quad (2)$$

Expression (2) shows that ‘ S ’ is dependent on the capacitors current and voltage. It depends also on the locations of the capacitors which appear explicitly in the expression of the power and energy loss reductions given below. These expressions are for a balanced radial main feeder on which ‘ n ’ capacitors numbered, for the calculation suitability, from the end of the line to the substation end (Figure 1), as it follows:

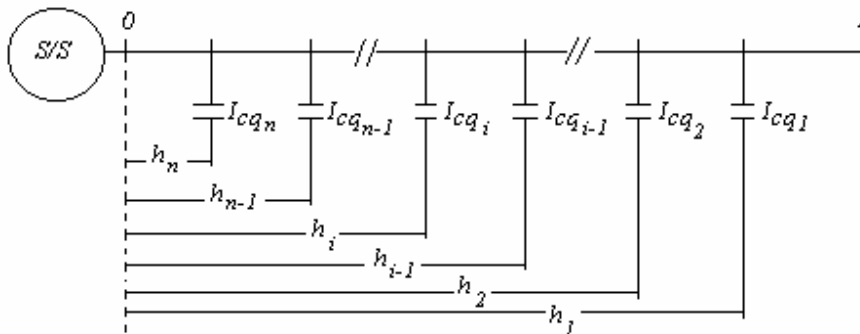


Figure 1

Notation of the capacitors current and location

2.1 Power Loss Reduction

For a balanced three phase radial feeder and using the power loss reduction due to each capacitor, the total loss reduction can be written as it follows:

$$\Delta P = 3 \sum_{i=1}^n \Delta P_i \quad (3)$$

ΔP_i : is then the power loss reduction due to the i^{th} capacitor. It is given by:

$$\Delta P_{i=2} = 2 R I_{cqi} \int_0^{h_i} \left(I_s(t) F_q(x) - \sum_{k=1}^{i-1} I_{cqk} \right) dx - 2 R I_{cqi} \sum_{k=i+1}^n I_{cqk} h_k - R h_i I_{cqi}^2 \quad (4)$$

2.2 Energy Loss Reduction

As for the power loss reduction, the energy loss reduction for this radial line is given by:

$$\Delta E = 3 \sum_{i=1}^n \Delta E_i \quad (5)$$

The capacitors being of fixed type and their in service duration is T then, ΔE_i is equal to the integral between 0 and T of ΔP_i and it admits the following expression:

$$\Delta E_i = 2 R I_{cqi} T L_f \int_0^{h_i} I_s F_q(x) dx - 2 R I_{cqi} h_i T \sum_{k=1}^{i-1} I_{cqk} - 2 R T I_{cqi} \sum_{k=i+1}^n I_{cqk} h_k - R T h_i I_{cqi}^2 \quad (6)$$

3 Voltages Calculation

The load and battery currents are voltage-dependent thus, the calculation of the complex voltages is necessary. However, distribution networks being characterized by a high ratio R/X and a radial configuration, it is not recommended to use conventional load flow methods such as Gauss-Seidel or Newton decoupled which are essentially developed for transmission or strongly meshed networks. Applied for distribution networks, they can encounter convergence problems. In this case and for the voltage calculation, we suggest a method called voltage drop method or a backward and forward method. In the line backward sweep, the branch currents, the active and reactive powers and the power losses are calculated. After which, the forward sweep is carried out to determine the nodes

voltage and phase-angle. For a branch 'i', the complex voltage of its receiving end is defined equal to that of its sending end 'i - 1' decreased by the branch voltage drop located between the two considered nodes. For the node 'i' (Figure 2), the complex voltage is written as it follows:

$$\bar{V}_i = \bar{V}_{i-1} - (r_i + jx_i) [F_{di} - j(F_{qi} - F_{ci})]$$

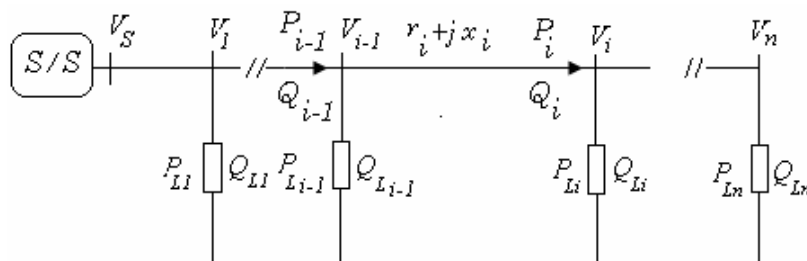


Figure 2

Descriptive diagram of the line

Where the d and q components of the i^{th} branch current and the current due to the capacitors located downstream this branch, are given by:

$$\begin{cases} F_{di} = \frac{P_i \cos \varphi_i + Q_i \sin \varphi_i}{V_i} \\ F_{qi} = \frac{Q_i \cos \varphi_i - P_i \sin \varphi_i}{V_i} \\ F_{ci} = \sum_{k=1}^i I_{ck} \end{cases} \quad (7)$$

The active and reactive powers injected into the node 'i' are given by:

$$\begin{cases} P_i = P_{i+1} + P_{Li} + P_{loss\ i+1} \\ Q_i = Q_{i+1} + Q_{Li} + Q_{loss\ i+1} \end{cases} \quad (8)$$

The line active and reactive power losses are:

$$\begin{cases} P_{loss\ i+1} = r_{i+1} \frac{P_{i+1}^2 + Q_{i+1}^2}{V_{i+1}^2} \\ Q_{loss\ i+1} = x_{i+1} \frac{P_{i+1}^2 + Q_{i+1}^2}{V_{i+1}^2} \end{cases} \quad (9)$$

The d and q voltage-components, using the uniform normalised line concept, are:

$$\begin{cases} V_{di} = V_{di-1} - RL_{uni} F_{di} - x_{ni} L_{uni} F_{qi} + x_{ni} L_{uni} F_{ci} \\ V_{qi} = V_{qi-1} - x_{ni} L_{uni} F_{di} + RL_{uni} F_{qi} - RL_{uni} F_{ci} \end{cases} \quad (10)$$

Once the d and q components calculated, the voltage rms value and phase-angle of node ' i ' are obtained by:

$$\begin{aligned} V_i &= \sqrt{V_{di}^2 + V_{qi}^2} \\ \varphi_i &= \arctan g \frac{V_{qi}}{V_{di}} \end{aligned} \quad (11)$$

The complex voltage of the k^{th} capacitor is equal to that of bus ' i ' if it is located on it. If the capacitor location is between the buses ' $i - 1$ ' and ' i ', its d and q components are given by:

$$\begin{cases} V_{cdk} = V_{di-1} - R(L_{nni} - h_k) F_{di} + X_{ni}(L_{nni} - h_k) F_{qi} + X_{ni}(L_{nni} - h_k) F_{ci} \\ V_{cqk} = V_{qi-1} - X_{ni}(L_{nni} - h_k) F_{di} + R(L_{nni} - h_k) F_{qi} + R(L_{nni} - h_k) F_{ci} \end{cases} \quad (12)$$

From d and q components we get:

$$\begin{aligned} V_{ck} &= \sqrt{V_{cdk}^2 + V_{cqk}^2} \\ \varphi_{ck} &= \arctan g \frac{V_{cqk}}{V_{cdk}} \end{aligned} \quad (13)$$

To determine both voltage magnitude and phase-angle, one will initialise all the complex voltages to that existing at the substation end (reference bus) and calculate initially the current distributions according to (7) by going up the line (backward sweep). Then d and q voltage components, in agreement with the expressions (10) for nodes and (12) for capacitors, are calculated by going down the line (forward sweep). The method being iterative, the computing process will be stopped only if the results converge. As a convergence test, we have adopted a per-unit difference of voltages of two successive iterations equal or less than 0.0001.

4 Optimisation of the Reactive Energy

Making the objective function maximum is equivalent to finding the batteries size and location which satisfy the following system:

$$\begin{cases} \partial S / \partial I_{cqi} = 0 \\ \partial S / \partial h_i = 0 \end{cases} \quad (14)$$

The solution strategy suggested is an iterative procedure being based on the solution of each equation of the system (14) for the two following major reasons:

- The solution facility which the iterative method offers in this case.
- Each equation of the system (14) taken separately, constitutes alone, a problem the importance of which is proven. Indeed, it happens that the interest of the electrical energy suppliers, for considerations which are peculiar to them, relates only to one of the two parameters independently of the other. Owing to the fact that one solves each equation separately, the access to the solution of only one of the two problems is than possible.

4.1 Optimisation of the Sizes

The substitution of «S» by its expression (2) in the first equation of the system (14) and after reorganizing the equation, we end up with the following contracted matrix expression (see the appendix for more details).

$$\tilde{H} \tilde{I}_{cq} = \tilde{B} \quad (15)$$

Where:

- \tilde{H} : is an $n.n$ matrix called matrix locations the elements of which are such that:

$$h_{ij} = \begin{cases} h_i & \text{if } i = j \\ 2h_j & \text{if } i < j \\ 2h_i & \text{if } i > j \end{cases} \quad (16)$$

- \tilde{I}_{cq} : is a $1.n$ matrix called capacitors size matrix the transposed of which is:

$$\tilde{I}_{cq}^t = [I_{cq1}, I_{cq2}, \dots, I_{cqn}]$$

- \tilde{B} : is a $1.n$ matrix the elements of which are such that:

$$B_i = \frac{k_p + k_e T L f}{k_p + k_e T} \int_0^{h_i} I_s F_q(x) dx - \frac{k_{cf} V_{ci}}{2R(k_p + k_e T) \cos \varphi_{ci}}$$

Obtaining the optimal sizes passes by the resolution of the matrix equation (15) which gives the capacitor current I_{cck} . The reduced optimal sizes of the capacitors (Q_{ck}) are then deduced from:

$$Q_{ck} = \frac{V_{ck} I_{cck}}{\cos \varphi_{ck}} \quad (17)$$

4.2 Optimisation of the Locations

Just like for the sizes of the batteries, the resolution of the second equation of the system (14) and after the equation reorganisation, leads to (see appendix for details):

$$F_q(h_i) = \frac{2(k_p + k_e T)}{I_s(k_p + k_e T L_f)} \sum_{k=1}^{i-1} I_{c q k} + \frac{k_p + k_e T}{2 I_s(k_p + k_e T L_f)} I_{c q i} \quad (18)$$

Knowing $F_q(h_i)$, the per-unit optimal locations and consequently their real values are then deduced from the graph of the reactive current distribution function $F_q(x)$ shown in *Figure 3*. However, and to make the locations determination automatic, a program was envisaged for this purpose.

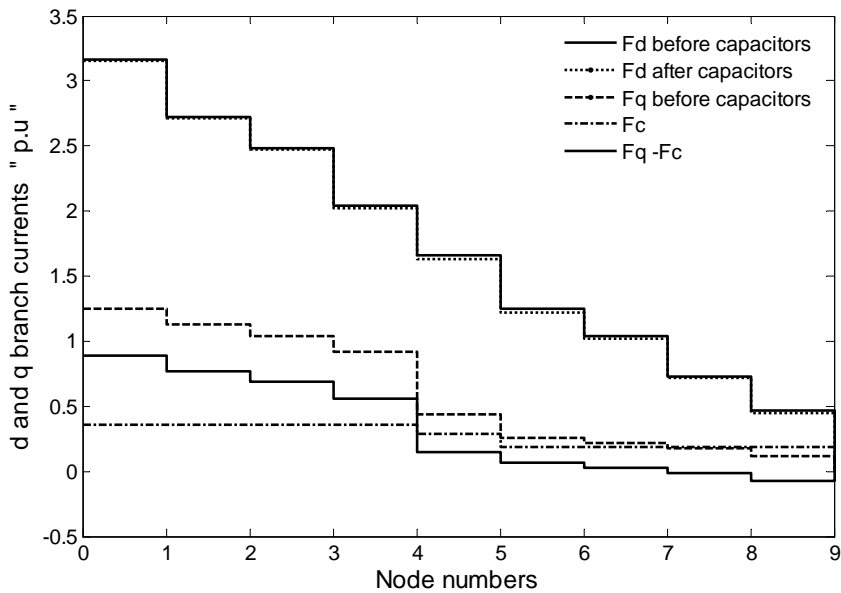


Figure 3
d and q branch current distributions before and after compensation

4.3 Optimisation of Two Parameters

To optimise the locations and the sizes of the batteries at the same time, an iterative procedure is planned. It calls upon the programs developed to determine each of the two parameters separately. The execution of this iterative method requires the knowledge of an initial solution. Arbitrary values are then assigned to the two required parameters. The determination of the optimal locations and sizes will be done, according to the following algorithm:

- Step 1 Read the line data.*
- Step 2 Read the arbitrary capacitors size and location.*
- Step 3 Initialise the tensions of the various bus bars and on the capacitors.*
- Step 4 Uniform and normalise the line and the loads.*
- Step 5 Calculate the normalised currents I_{cqk} due to the capacitors from (17).*
- Step 6 While the convergence is not reached perform the following steps:*
- a) Calculate the current distribution functions according to first and second expressions of (7).*
 - b) Calculate the bus voltages and the capacitors voltages bus from (11) to (13).*
 - c) While the capacitors location and size are not identical to the precedent ones, carry out the following steps:*
 - i) Calculate the optimal locations according to (18).*
 - ii) Calculate the optimal currents due to the capacitors according to (15).*
 - iii) Calculate the relative powers of the capacitors according to (17).*
 - iv) Calculate the cost reductions according to (1).*
 - d) Else continue.*
- Step 7 Else continue.*
- Step 8 Return to the real dimensions.*
- Step 9 Write the results.*

5 Application

As an application example and in order to be able to undertake a comparative study, we have considered the physically existing distribution line given by [5-7]. It's a non-homogeneous distribution line of medium voltage having nine sections of five wire-sizes. The line loads are non-uniform and are concentrated at the end of each section. As a base voltage we have adopted the voltage at the sub-station end (23 kV) which is also regarded as the angles origin. As a base power we have considered an apparent power of 4186 kVA. This value is equal to the sum of all the reactive loads.

As in [5-7], the three capacitors size and location optimisation problem is considered where: the load factor L_f is equal to 0.45; the annual cost of the kW is

$k_p = 168\$/\text{kWh}$; the annual cost of the kWh is $k_e = 0.015 \text{ \$/kWh}$ and the annual cost of the installed kVAr is $k_{cf} = 4.9 \text{ \$/3 phasekvar}$. A 14.3% annual fixed charge rate is applied for capacitor cost. The obtained results for the optimal sizes and locations are consigned in *Table 1*. Some others interesting results are also given. The voltage rms values and phase-angle after the optimisation of the reactive energy are consigned in *Table 2* and those of the effect of the shunt battery current definition in *Table 3*.

Table 1
Results of the capacitors size and location optimisation

N° of the capacitor	1	2	3	ΔP (kW)	ΔP in [7]	ΔE (kWh)	S (\\$)	number iterations
Initial sizes (kVAr)	300	600	800	152.71	111.00	83818	19328	3
Initial locations (mile)	13.27	07.32	3.02					
Optimal Locations (p.u)	1.0000	0.2248	0.1074					
Locations in [7] (p.u)	1.0000	0.2248	0.1074					
Optimal Locations (mile)	16.27	06.32	4.02					
Optimal Sizes (p.u)	0.1628	0.0964	0.0637					
Optimal Sizes (kVAr)	681.48	403.62	266.73					
capacitors Currents (p.u)	0.1877	0.1040	0.0669					
capacitors Sizes in [7] (kvar)	464	1070	2961					
$Q_{eq[7]}/Q_{eq}$ Ratios	0.68	2.65	11.10					

Table 2
Bus voltage rms values and phase-angles after capacitors installation

Bus	V (p.u)	φ (rd)	$V_{[7]}$	$\varphi_{[7]}$	Bus	V (p.u)	φ (rd)	$V_{[7]}$	$\varphi_{[7]}$
0	1.0000	0.0000	1.0000	0.0000	5	0.9249	-0.0726	0.9441	-0.0831
1	0.9941	-0.0095	0.9967	-0.0102	6	0.9163	-0.0811	0.9350	-0.0906
2	0.9853	-0.0212	0.9917	-0.0232	7	0.9003	-0.0926	0.9183	-0.1001
3	0.9648	-0.0419	0.9790	-0.0481	8	0.8753	-0.1135	0.8912	-0.1169
4	0.9513	-0.0495	0.9695	-0.0600	9	0.8600	-0.1319	0.8732	-0.1311

Table 3
Effect of the current battery definition

Battery	I_{cq} (p.u)	Q_{cq} (p.u)	$Q_{cq [7]}$ (p.u)	$Q_{cq[7]} / Q_{cq}$	Deviation (%)
1	0.1877	0.1628	0.2168	1.33	33
2	0.1040	0.0964	0.1127	1.17	17
3	0.0669	0.0637	0.0704	1.11	11

Conclusion

As consequences of the changes in the expressions (3) for ΔP_i and (4) for ΔE_i , we note that:

The non-diagonal terms of the matrix locations are multiplied by two, if the capacitors size optimisation is of interest see expression (16). The first term of the right-hand side of the equation (18) giving $F_q(h_i)$ is multiplied by two, during the capacitors location optimisation.

From the results point of view, if the optimal locations (nodes: 9, 5 and 4) of the capacitors are the same to those given in [7], their sizes are completely different, see *Table 1*. The ratios of the capacitors size given in [7] to those which we have obtained, vary from 0.68 to 11.10 (see *Table 1*). The optimal choice of capacitors location and size, taking into account the line power losses, conduct to a power loss reduction equal to 152.71 kW, an improvement in the voltage profile (see *Table 2*) and a decreasing in the reactive current distribution $F_q(x)$ (see *Figure 3*). The cost reduction is then equal to 19328 \$ and would be better in our case. Indeed, if we count the total number of kVAr installed, it is equal to 1351.83 kVAr in our case and 4495 kVAr in [7]. Reported to the total reactive power (4186 kVAr), the ratio is of 32.29% in our case and 107.4% in [7]. This last value i.e. 107.4% means that the total requested reactive energy is satisfied by an external contribution and violates the maximum limit of the reactive energy to be compensated. In addition to the power loss reduction due the optimal capacitors placement, the improvement of the voltage, reduce the d component of the branch current (see *Figure 3*) and thus an additional reduction in the power losses. The power loss reduction due to this component of the branch current is equal to 20.28 kW and consequently, a supplementary cost reduction of 3407 \$. The total reductions of the power losses and the cost are respectively of 172.99 kW and 22735 \$.

Note that the optimal sizes of the capacitors obtained in our case are not standard ones. To overcome this, we suggest moving each non-standard size to that of smaller standard size or larger standard one and then choose those whose economic saving is the best.

If we consider that the optimal reactive currents are the same in both, reference [7] and our proposed method (*Table 3*), the difference in the capacitor power definition (Equation 17) leads to a deviation between 11% and 33%. This deviation increases as one move away from the sub-station end or as voltage magnitude decreases.

Nomenclature

- k_p (k_e): is the annual unit price of the kW (kWh).
- k_{cf} : is the unit price of the three phase kVAr installed.
- V_i (φ_i): is the voltage rms value (phase-angle) at bus i .
- V_{ck} (φ_{ck}): is the k^{th} battery voltage rms value (phase-angle).
- X_{ni} : is the per-unit normalised reactance of the i^{th} line section.
- R : is the per unit resistance of the normalised uniform line.
- h_k : is the normalised uniform location of the k^{th} battery.
- Q_k : is the k^{th} battery size.
- $I_s(t)$: is the time-dependent reactive current at the substation end.
- T : is the in service duration of the batteries which are of fixed type. Its per-unit value is equal to 1.
- L_{uni} : is the normalised uniform length of the branch ' i '.
- L_{nni} : is the total normalised uniform length (from the reference node to the node ' i ').

Appendix

1 Optimisation of the sizes

The optimal sizes of the batteries are obtained by the resolution of the system of equations $\partial S / \partial I_{cqi} = 0$. To do it, the derivatives of the power and energy loss reductions are required. We then obtain:

$$\partial \Delta P / \partial I_{cqi} = \sum_{i=1}^n \partial \Delta P_i / \partial I_{cqi}$$

Where $\partial\Delta P_i/\partial I_{cqi}$ is equal to:

$$\frac{\partial\Delta P_i}{\partial I_{cqi}} = \begin{cases} -2RI_{cqi}h_i & \text{if } i > j \\ 2R \int_0^{h_j} I_s F_q(x) dx - 2Rh_i \sum_{k=1}^{i-1} I_{cqi} - 2R \sum_{k=i}^n h_k I_{cqi} & \text{if } i=j \\ 0 & \\ -2RI_{cqi}h_i & \text{if } i < j \end{cases} \quad (\text{A1})$$

From where the expression of $\partial\Delta P/\partial I_{cqi}$:

$$\frac{\partial\Delta P}{\partial I_{cqi}} = 3 \left(2R \int_0^{h_j} I_s F_q(x) dx - 4Rh_j \sum_{k=1}^{j-1} I_{cqi} - 4R \sum_{k=j+1}^n h_k I_{cqi} - 2Rh_j I_{cqi} \right)$$

Making the same reasoning we obtain for $\partial\Delta E/\partial I_{cqi}$:

$$\frac{\partial\Delta E}{\partial I_{cqi}} = 3 \left(2RTL_f \int_0^{h_j} I_s F_q(x) dx - 4RT h_j \sum_{k=1}^{j-1} I_{cqi} - 4RT \sum_{k=j+1}^n h_k I_{cqi} - 2RT h_j I_{cqi} \right) \quad (\text{A2})$$

Finally we obtain for $\partial S/\partial I_{cqi}=0$, once a certain number of arrangements operated:

$$2h_j \sum_{k=1}^{j-1} I_{cqi} + 2 \sum_{k=j+1}^n h_k I_{cqi} + h_j I_{cqi} = \frac{k_p + k_e TL_f}{k_p + k_e T} \int_0^{h_j} I_s F_q(x) dx - \frac{k_{cf}}{2R(k_p + k_e T)} \quad (\text{A3})$$

Where: $\sum_{k=1}^{j-1} I_{cqi} = 0$ for $j=1$ and $\sum_{k=j+1}^n I_{cqi} = 0$ for $j=n$.

(A3) can be written in the matrix form as it follows:

$$\tilde{H} \tilde{I}_{cq} = \tilde{B} \quad (\text{A4})$$

Where, \tilde{H} , \tilde{I}_{cq} and \tilde{B} are as defined in the section 4.1.

2 Optimisation of the locations

For the different derivatives to the locations, we obtain:

$$\frac{\partial \Delta P_i}{\partial h_j} = \begin{cases} 2R I_{cqi} I_s F_q(h_i) - 2R I_{cqi} \sum_{k=1}^{i-1} I_{cqk} - R I_{cqi}^2 & \text{if } j = i \\ 0 & \text{if } i \rangle j \\ -2R I_{cqi} I_{cqi} & \text{if } i \langle j \end{cases}$$

$$\frac{\partial \Delta E_i}{\partial h_j} = \begin{cases} 2R I_{cqi} T L_f I_s F_q(h_i) - 2R T I_{cqi} \sum_{k=1}^{i-1} I_{cqk} - R T I_{cqi}^2 & \text{if } j = i \\ 0 & \text{if } i \rangle j \\ -2R T I_{cqi} I_{cqi} & \text{if } i \langle j \end{cases}$$

And then:

$$\frac{\partial \Delta P}{\partial h_j} = 3(2R I_{cqi} I_s F_q(h_j) - 4R I_{cqi} \sum_{k=1}^{j-1} I_{cqk} - R I_{cqi}^2) \quad (\text{A5})$$

$$\frac{\partial \Delta E}{\partial h_j} = 3(2R T L_f I_{cqi} I_s F_q(h_j) - 4R T I_{cqi} \sum_{k=1}^{j-1} I_{cqk} - R T I_{cqi}^2) \quad (\text{A6})$$

At last $\partial S / \partial h_j = 0$ gives after having ordered it:

$$F_q(h_j) = \frac{2(k_p + k_e T)}{(k_p + k_e T L_f) I_s} \sum_{k=1}^{j-1} I_{cqk} + \frac{k_p + k_e T}{2(k_p + k_e T L_f) I_s} I_{cqi} \quad (\text{A7})$$

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