

# Correcting Fine Structure of Surfaces by Genetic Algorithm

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*Abstract: Highlight lines are powerful in quality evaluation and disclosing errors of high quality surfaces. We propose a method that enables the designer to correct surfaces by adjusting their highlight lines. The adjustment of highlight line curves is carried out by replacing the defective parts with curves of the designer's intent. The corrected surface, which corresponds to the highlight line adjustment, is determined by a genetic algorithm (GA). The paper discusses genetic representation and fitness function developed for the specific problem and gives a usability analysis. The advantage of the method is its robustness and applicability to surfaces regardless of shape, and CAD representation. The method is meant to be applied in the final phase of shape modeling; its effectiveness is demonstrated by several industrial examples.*

*Keywords: surface errors; highlight lines; genetic algorithm*

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## 1 Introduction

The class of “high quality surfaces” is attributed to their smoothness, evenness and aesthetic appeal. The most important high quality surfaces in industry are those representing car, airplane and ship hulls, household appliances, etc. Their design involves not only functional criteria but also subjective ones, related to style and appearance. Creating methods and tools that support the work of a stylist is a challenging task in the areas in CAD and CAGD (Computer Aided Geometric Design).

Various visual display methods are available for the quality evaluation of surfaces. Some of these are also suitable for visualizing different representations of reflection status of the surfaces [8, 9, 15, 16]. In this way, designers can determine surface quality in accordance with their own aesthetic demands. One of the most sensitive indicators of surface quality is the highlight line display method. The

method is well applicable in disclosing minor errors and in fine tuning of surface shape that other methods fail to achieve.

**Related work.** The highlight-line display method was first suggested by Klaus-Peter Beier and Yifan Chen [1] but they didn't address surface modification. A method for smoothing free form surfaces by the adjustment of highlight lines was first developed by Klass and Kaufmann [13, 14]. The correlation between highlight lines and the defining parameters of the surfaces i.e. control points (CP) is established by a non-linear equation system, which is too time consuming to solve, and the results are not always good enough. The method developed by Zhang and Cheng [17] introduces a great number of simplifications to obtain a linear system of equation to modify control points through highlight lines. However, the highlight line cannot accurately follow the points specified by the designer and the method yields adequate results only in a small range of the errors.

The above methods try to handle the complex mathematic relation between the adjusted highlight line and the defining parameters of the corresponding surface. We propose a method that solves this problem by genetic algorithms (GA), which can find modified control points even in the absence of direct mathematical relations.

The proposed method of surface correction starts with the computation of highlight lines (Figure 1). Inspection of the surface quality is carried out by several light-source settings and surface orientations. Then the designer selects and corrects the defective highlight lines using the facilities of a CAD system [2]. This is followed by the automatic determination of the affected surface region and corresponding control points. Adjustment of the control points is carried out by a genetic algorithm.

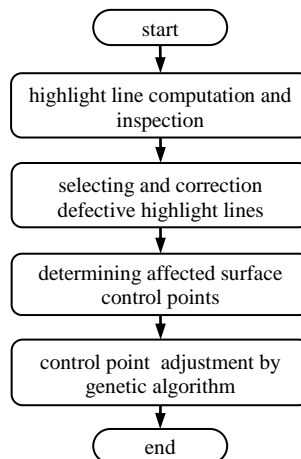


Figure 1

Block diagram of the surface correction method

## 2 Representation of Surfaces and Highlight Line Computation

Parametric representation of the free form surfaces in Bézier, B-spline or NURBS form [12] are widely used in CAD applications. In surface design they are used for the interactive creation and modification of shapes as well for converting physical prototypes, such as automotive clay models, into CAD models. Parametric representation defines the shape of the surface  $\mathbf{S}(u, v)$  by an array of control points  $\mathbf{P}_{ij}$  and the Bézier, B-spline or NURBS basis functions  $N_{ik}$  and,  $N_{jl}$  of order  $k$  as

$$\mathbf{S}(u, v) = \sum_{i=0}^m \sum_{j=0}^n \mathbf{P}_{ij} N_{ik}(u) N_{jl}(v) \quad (1)$$

The shape of the surface is mainly defined by the control points, knots and weights of the basis functions provide additional degree of freedom in design.

A highlight line is created on the surface by the reflection of a linear light-source of infinite length. The highlight line consists of a set of highlight points. They are points on the surface where the corresponding surface normal and the light-source intersect each other; that is, the perpendicular distance between them is zero.

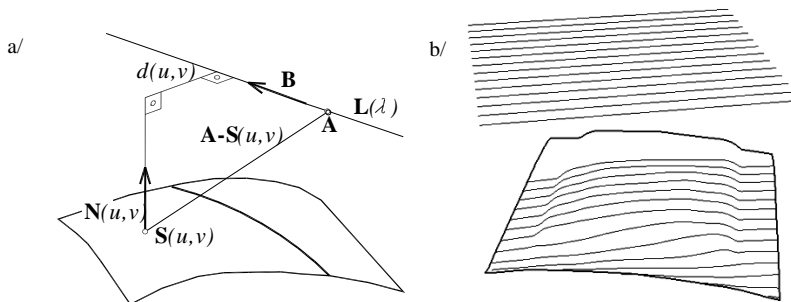


Figure 2

Distance interpreted between surface normal and the light-source (a) Highlight lines of a surface (b)

The line of the light-source can be described as  $\mathbf{L}(\lambda) = \mathbf{A} + \mathbf{B}\lambda$  where  $\mathbf{A}$  is a point on  $\mathbf{L}(\lambda)$ , and  $\mathbf{B}$  is a vector defining the direction of the line (Figure 2a). The signed perpendicular distance  $d(u, v)$  between the normal  $\mathbf{N}(u, v)$  at a surface point  $\mathbf{S}(u, v)$  and the linear light-source is:

$$d(u, v) = \frac{[\mathbf{B} \times \mathbf{N}(u, v)] \cdot [\mathbf{A} - \mathbf{S}(u, v)]}{\|\mathbf{B} \times \mathbf{N}(u, v)\|} \quad (2)$$

For a point on the highlight line  $d(u, v) = 0$  holds, which must be solved for the control points of  $\mathbf{S}(u, v)$ . To calculate highlight lines, this relation must be computed with high accuracy. We have developed a robust method for computing highlight lines, which is described in detail in [6].

For the detailed investigation of surfaces, an appropriate set of coplanar parallel linear light-sources is necessary (See Figure 2b). As a result, a corresponding set of highlight lines is obtained; their distribution density and shape is a firm and sensitive indicator of surface quality.

### **3 Fundamentals of Genetic Algorithm**

Genetic algorithms were introduced by J. H. Holland [11]. The basic idea is to apply the Darwinian mechanism of evolution in finding optimal solution to complex or non-linear problems. Solutions are represented by chromosomes, composed of genes that contain variable parameters of the solution. The chromosomes form a population and they are evaluated according to predefined criteria called fitness, which quantifies the optimality of the solution they represent. Chromosomes of the next generations are created by genetic operators. The basic operators include selection, crossover and mutation. The algorithm runs until stop criteria is met; that is, until an acceptable solution is found, or a maximal number of generations is reached. A number of articles have been published dealing with the design of GA [3, 4, 5, 10].

The parameters of effective GA including the applied operator types depend on the particular problem. Their selection and adjustment must be analyzed and tested carefully. Special attention must be made to the fitness function. It should be composed of terms closely related to the objective of the search. The effectiveness of GAs is often characterized by achieving precise results quickly and reliably in a wide parameter range of the particular problem.

### **4 Genetic Algorithm in Surface Correction**

Our goal is to correct the shape of surfaces by means of their reflection characteristics through the shape and distribution of their highlight lines. The objective of GA is to adjust the parameters of the surfaces, resulting in a new surface shape that produces the desired highlight lines.

The efficiency of the proposed method was tested on several industrial surfaces that greatly differ from each other in size, shape and degree of detected shape irregularity. This latter is defined in terms of magnitude, extent and complexity of the shape error. Special attention was paid to tuning the genetic process, in order to arrive at a fast and stable process that at the same time reveals the desirable technical solution.

In this paper we outline the GA we developed by providing its main characteristics and give insight into the construction of three, problem specific elements – the genetic representation, the fitness function and the stop condition. A more detailed description of our GA can be found in [7].

## 4.1 Structure of Genes and Chromosomes

Free form surfaces are determined by a number of parameters. However, the most effective parameter for surface modification is the control point  $\mathbf{P}_{i,j}$ . In genetic representation those control points are included that have influence on the surface region that the designer wants to optimize. They can be computed from the basis functions corresponding to particular control points. Their strength of influence is represented by the constant  $b_{i,j}$ , which is calculated by integrating the basis functions over the region of interest. A gene  $g_\gamma$  consist of control point modification  $\Delta\mathbf{P}_{i,j}$  and the constant  $b_{i,j}$  applied to the corresponding  $\mathbf{P}_{i,j}$ :

$$g_\gamma = \Delta\mathbf{P}_{i,j}(x, y, z), b_{i,j}. \quad (3)$$

where  $x$ ,  $y$  and  $z$  are Cartesian co-ordinates of  $\Delta\mathbf{P}_{i,j}$ , while  $\gamma$  is the identifier of genes within a chromosome. The chromosome of a surface has the following structure:  $c_\beta = (g_1 \dots g_\gamma \dots g_J)$ , where  $\beta$  identifies the chromosome in the population and  $J$  is the number of genes in the chromosomes.

## 4.2 Search Space

The search space of the GA is the region in the space where the control points may vary. It is the union of search spaces of chromosomes  $r_\beta$ , and is defined by

$$r_\beta = s \cdot b_{i,j}. \quad (4)$$

The scaling factor  $s$  is estimated from the maximal necessary improvement of the highlight lines. The search space limits the change of control point positions in the generation of initial population and in mutation.

## 4.3 Fitness Function

The developed fitness function contains geometric deviation between actual and desired highlight lines. It consists of two components: accuracy and shape similarity. Accuracy is based on the distance, while shape similarity on the angle difference of tangent vectors between corresponding highlight points. Denote  $h_i^{des}$  the desired, and  $h_i^{cur}$  the highlight line, created during the search and  $d_i(t_k)$  the deviation between corresponding highlight points at different  $t$  parameters of highlight lines. Then, the distance error component of the fitness function is:

$$f_{\text{dist}} = \sum_{i=1}^l \left( \sum_{k=1}^{n_i} \left( d_i(t_k) - \frac{1}{n_i} \cdot \sum_{k=1}^{n_i} d_i(t_k) \right)^2 \cdot \frac{1}{n_i} \right) \quad (5)$$

where  $d_i(t_k) = |h_i^{cur}(t_k) - h_i^{des}(t_k)|$  while  $n_i$  denotes the number of examined highlight points. Variable  $l$  indicates the number of highlight lines. The angle

difference of the error component  $f_{ang}$  is calculated in same manner, except the deviation is composed as follows:

$$d_i(t_k) = \arccos \left( \frac{\mathbf{h}_i^{des}(t_k) \cdot \mathbf{h}_i^{cur}(t_k)}{\|\mathbf{h}_i^{des}(t_k)\| \|\mathbf{h}_i^{cur}(t_k)\|} \right) \quad (6)$$

We evaluated the fitness components regarding their efficiency of correcting highlight lines. We found that the distance error component promotes the creation of accurate highlight lines, but their shape similarity is often poor. The tangency error component behaves in the opposite way; it promotes producing highlight lines with good shape similarity, but at the expense of their accuracy. We eliminated the disadvantages by weighting so that the distance component dominates in the beginning, while the tangency component is prevalent in the latter phase of the search. This is realized by the following fitness function:

$$f = f_{dist}(w_{dist}^0 - \Delta w) + f_{ang}(1 - w_{dist}^0 + \Delta w) \quad (7)$$

where  $w_{dist}^0$  is the weight of distance error component of the initial population, and  $\Delta w$  is the weight value change during the search.

Fine adjustment of fitness function was performed by choosing an appropriate weighting strategy. They were applied to several test surfaces with different shape errors. In each case a rank concerning the number of generations to reach the stop condition was established.

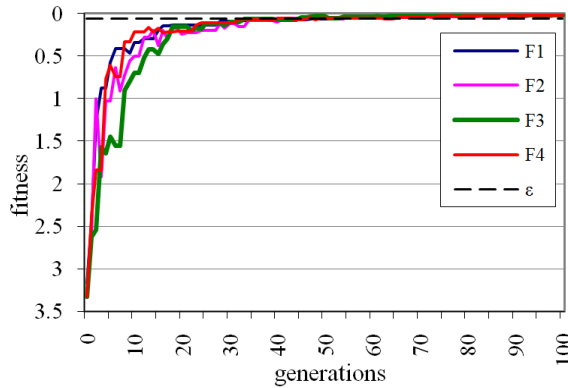


Figure 3

Weight change strategies tracked in fitness development

In Figure 3, comparison of four strategies applied to a particular case is seen. The final rank of a weighting strategy is determined by the average rank calculated concerning all test surfaces. In Table 1 the parameters and the calculation rules of the best performing strategies are shown. It can be seen that the best rank – the best weighting strategy – is obtained by strategy F3.

Table 1  
Best performing weighting strategies

| Id. | parameter(s)                 | calculation rule   | rank  |
|-----|------------------------------|--|-------|
| F1  | $w_{dist}^0 = 0.75$          | $\Delta w = 0$   | 4.25  |
| F2  | $w_{dist}^0 = 0.8, a = 0.75$ | $\Delta w = 0$ if $\tau < a\tau_{max}$ else $\Delta w = 2w_{dist}^0 - 1$               | 2.625 |
| F3  | $w_{dist}^0 = 0.75$          | $\Delta w = (2w_{dist}^0 - 1) \cdot (1 - (c_{var}^0 - c_{var}^{\tau-1}))$              | 1.25  |
| F4  | $w_{dist}^0 = 1, a = 0.15$   | $\Delta w = 0$ if $c_{var}^0 - c_{var}^{\tau-1} < a$ else $\Delta w = 2w_{dist}^0 - 1$ | 2.375 |

Where  $a$  coefficient that defines the swap point of component weights  
 $\tau$  index of the current generation  
 $\tau_{max}$  allowed maximal number of generations  
 $c_{var}^0$  chromosome variability of the initial population  
 $c_{var}^{\tau-1}$  chromosome variability of the previous generation

#### 4.4 Stop Condition

The source of the search stop condition is the allowable residual error, which is based on the comparison of the original and the redesigned highlight lines. Let  $\mu_{stop}$  denote the stop condition

$$\mu_{stop} = \begin{cases} \text{true} & \text{if } f^\tau / f^0 \leq \varepsilon \text{ or } \tau = \tau_{max} \\ \text{false} & \text{otherwise} \end{cases} \quad (8)$$

Where  $\varepsilon$  is the allowable residual error.

$f^\tau$  fitness of the best chromosome at generation number  $\tau$

$f^0$  fitness of the initial chromosome.

In case the search fails to meet the stop condition, it is stopped at the maximum number of generations  $\tau_{max}$ . During the analysis, we used the following stop conditions:  $\varepsilon = 0.05$ ,  $\tau_{max} = 100$ .

## 5 Analysis and Results

In practice, different sizes, extents and complexity of shape errors occur in different design conditions that may have influence on the quality and preciseness of the resulting surface. We investigated this relation extensively. In this paper we give details of one investigation; we prove that the same final surface is achieved regardless of the size of the necessary corrections.

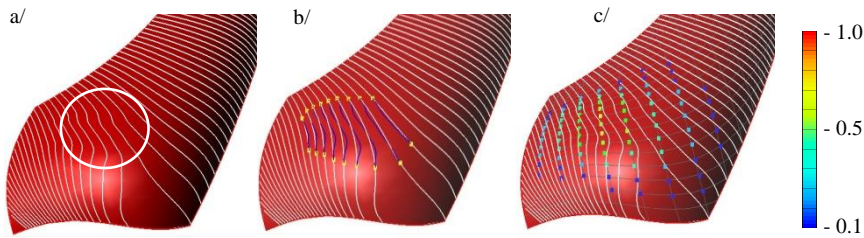


Figure 4

Disclosing defective highlight lines (a) Redesign of highlight lines (b) Affected CP-s (c)

Figure 4a displays the defective highlight lines of a car body part. Irregularities occur in the indicated region. The domain of highlight lines to be redesigned is marked by the designer (parts between bullets in Figure 4b). The affected control points are selected automatically. The different colors of control points in Figure 4c indicate the values of  $b_{ij}$ . Genetic search is performed with the above genetic operators and the fitness function. GA runs until the user-defined stop criterion is fulfilled. The same surfaces with different quality, i.e. different error sizes in the highlight lines were taken into consideration. The results of the analysis are presented in Table 2.

Table 2

Data to measure the efficiency and robustness of the algorithm

|   | size_1 | size_2 |
|---|--------|--------|
| average distance between desired and the defective highlight lines [mm] | 0.57   | 1.54   |
| average distance between desired and the corrected highlight lines [mm] | 0.028  | 0.031  |
| improvement [%]   | 95     | 98     |
| average distance between highlight lines of corrected surfaces [mm]     | 0.017  |        |
| iterations (number of generations past)                                 | 72     | 84     |

The initial error defined as the average distance between the desired and the defective highlight lines are highly different (more than 100%). The average distance between the desired and the corrected highlight lines shows how the initial error was eliminated by an improvement of 95% and 98% respectively. The average distance between the highlight lines of the corrected surfaces (0.017 mm) indicates that the algorithm converged to the same resulting surface in both case. This can be also verified by the distance map defined between the corrected surfaces (Figure 5). It can be easily recognized that the difference between them is insignificant.

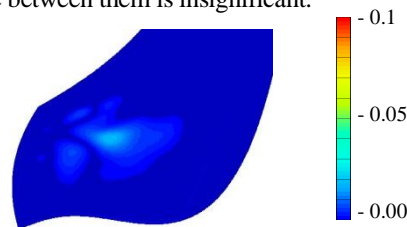


Figure 5

Difference map between resulting surfaces



To measure the computational costs of solving the problem, we used the required number of generations. In the case of bigger error it took 84 generations, while in case of smaller error 72 generations, to reach the stop criteria. This means that the algorithm needed only 17% more computation for a surface error that caused 170% growth in the error of highlight lines.

In the following we give practical examples visually demonstrating the efficiency of our method in Figures 6, 7 and 8. The rendered pictures of the surfaces (a. in figures) seem to be free of errors. The fine structure of the surface is disclosed by highlight lines, and the effective regions are indicated by circles (b. in figures). The highlight lines of the corrected surfaces are shown in c. figures. Their shape, smoothness, coherence and distribution show that the surfaces are smooth and error free.

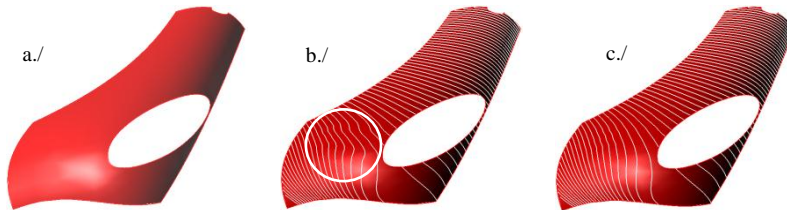


Figure 6

Surface of a car-body element before (a. and b.) and after correction (c.)

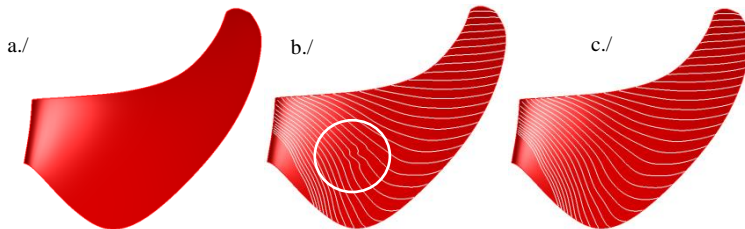


Figure 7

Surface of a propeller shovel before (a. and b.) and after correction (c.)

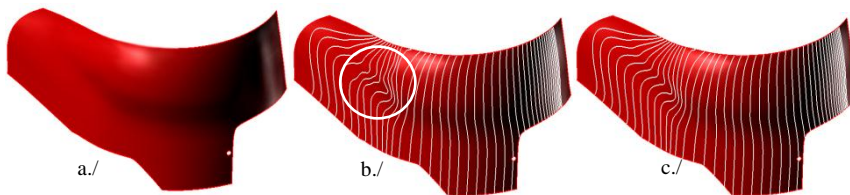


Figure 8

Surface of a car-body element before (a., b.) and after correction (c.)

## Conclusion

A robust and intuitive method for correcting surface errors by highlight lines was presented. Control point modification is achieved through genetic algorithm, circumventing the computation of highly non-linear relations between control

points and highlight lines. Our method is applicable to surfaces of any shape and any kind of CAD representations. It can be successfully applied to a wide range of highlight line and surface errors. The increase of computational cost is much smaller than the corresponding error size growth.

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