A Systematic Approach for Identification of SOPTD Processes using a Relay Feedback with a Fractional Order Integrator

Rahul B. Gaikwad, Raghu Raja Pandiyan K, R. Ranganayakulu, G. Uday Bhaskar Babu

Department of Chemical Engineering, National Institute of Technology Warangal, Telangana, India – 506004, grahul6@student.nitw.ac.in, raghuraj@nitw.ac.in, rayalla.ranga@student.nitw.ac.in, udaybhaskar@nitw.ac.in

Abstract: In this work, a new systematic identification approach is proposed to obtain second-order plus time delay (SOPTD) models by relay feedback with hysteresis and fractional order integrator. A relay with hysteresis and fractional order integrator is used to generate sustained oscillation at process output for model identification. The addition of a fractional order integrator helps improve the position frequency point obtained by the Describing function (DF) method and thus leads to accurate model. The proposed approach has an additional degree of freedom for estimating parameters. In addition, the proposed relay test was performed in the presence of measurement noise. The proposed method was applied to overdamped, underdamped and critically damped transfer function models. The performance of the proposed approach is evaluated by comparing the Integral absolute error (IAE) criteria in the frequency domain, Nyquist plot, and step response. Compared with the literature method, the proposed approach reduced IAE for Overdamped, Overdamped, Underdamped, and critically damped processes by 77.68%, 68.34%, 98.57%, and 95.78%, respectively. The simulation results show that the proposed approach identifies satisfactory models compared to existing techniques.

Keywords: SOPTD; model identification; Fractional order integrator; Describing function (DF); Relay feedback with hysteresis

1 Introduction

Most of the process industries use Proportional Integral Derivative (PID) controllers and their tuning largely depends on identification of a good process model. The process model can be identified by open-loop and closed-loop methods. In the open-loop method, introduce an excitation in each input variable one at a time of the process to get the output responses. Then, the transfer function model is to be identified using the output responses. The open loop identification method is simple, but it has some drawbacks, i.e., sensitive to disturbances, more computational time, and sometime process output deviates from the set point. Closed-loop identification methods overcome the drawbacks of open loop identification. The relay feedback identification method based on the closed loop test has gained interest for tuning PID controllers because of its simplicity. Relay feedback is one of the promising tools for the identification of process models. The theory behind relay feedback identification is straightforward as a Ziegler-Nichols (Z-N) closed-loop test. The relay feedback method uses the relay instead of the controller (Figure 1); thus, the system generates sustained oscillations called a limit cycle. This limit cycle gives valuable process information, i.e. peak amplitude and frequency. Hence, by using the limit cycle information, the process model parameters are estimated.

Some chemical processes with higher order dynamics may not be satisfactorily described by first-order plus time delay (FOPTD) models but more accurately described by SOPTD models. The relay feedback technique was first introduced to tune the PID controller [1-3]. Using Laplace transforms used asymmetric relay for process identification in the frequency domain [4]. Developed the identification method using relay data and state-space approach to derive nonlinear equations for various lower and higher order process models [5]. The relay with hysteresis was used to generate a limit cycle at process output. Identification was carried out offline and online using the DF method [6-9]. The state-space method and relay with hysteresis identifies stable and unstable processes to estimate the unknown process model parameters [10] [11].

Time domain-based analytical expressions are emanated to assess the exact model parameters using a relay with hysteresis for non-minimum phase (NMP) processes [12]. Proposed a method based on Fourier series analysis, like a DF method using an ideal relay with a fractional order integral. A comparative study of different relay identification techniques has been conducted [13]. Nonlinear equations for non-zero set points were developed and identified as first and second-order process models [14]. DF method was used to determine the higher-order and NMP process models as first and second-order models [15]. The limit cycle information near the non-zero set point was used to derive mathematical equations for accurately identifying unknown plants [16]. Novel explicit expressions are proposed to identify stable, unstable and integrating first order plus dead time processes. An asymmetrical relay generates a smooth limit cycle at the output [17]. The frequency domain and state space approach was proposed for modeling and identifying non-minimum phase processes [18]. After the relay feedback experiment, a set of explicit expressions was derived for identifying unknown FOPTD and SOPTD models [19].

A new "shifting method" was introduced recently in the literature to estimate three points on Nyquist plot of an unknown process from limit cycle data generated by biased relay with hysteresis. Now, optimization technique is used to identify anisochronic and isochronic models by minimizing the error between identified model and actual model with reference to the three points [20] [21]. The shift method was extended by developing explicit formulas for identifying isochronic process model [22]. The shift method was modified by adding an integrator or time delay in relay feedback loop to identify stable, unstable, higher order, integrating and NMP processes [23-25]. There were two methods namely closed-loop test with proportional controller and unbiased ideal relay feedback test along with the usage of Lamber W function for calculating unknown process parameters [26].

Although the relay feedback technique for process identification has been widely addressed, much scope still exists to improve the developments. In particular, in this paper, we have investigated and contributed to the following: Simple analytical expressions based on the DF technique are derived for identifying the SOPTD transfer function models. A single relay with hysteresis and fractional order integrator is used in a closed loop to extract process information and reduce measurement noise's effect. The additional fractional order integrator helps improve the DF method's frequency point. The proposed approach provides flexibility in the degree of freedom and thus leads to more accurate models. Since measurement noise is a critical issue in process industries, the validity of the proposed method is illustrated in noisy environments. As relay with hysteresis and fractional order integrator reduces the effect of noise, the Fourier series-based curve fitting technique is appended to obtain noise-free process output. The accurate model was identified based on the minimum IAE. Furthermore, the effect of fractional order integrator on model parameters is studied.

This paper considered four examples of SOPTD process from works of literature. The results are compared based on the integral absolute error criteria in the frequency domain, Nyquist plot, and step response between the proposed model, actual process, and methods present in literature with and without noise. MATLAB Programming/Simulink environment used for all experiments. This paper is arranged in the following sections, the proposed method is given in Section 2, mathematical expressions of Process identification are derived in Section 2.1, the Simulation study is detailed in Section 3, and finally, conclusions are presented in Section 4.

2 Proposed Identification Approach

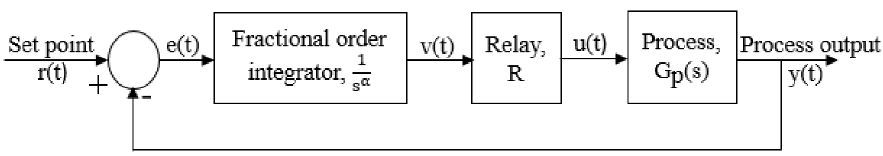

Figure 1
Block diagram of Relay feedback with FO integral

The scheme used for identification is shown in Figure 1, which consists of process $G_p(s)$, nonlinear element relay with hysteresis R and fractional order integrator. $u(t)$, $v(t)$ and $e(t)$ are the process input, relay input, and error respectively.

If *r(t)*=0 then the error signal *e(t)=y(t)*. Consider the error as a sinusoidal function as given in eq. (1)

$$e(t) = A\,sin(\omega t) \tag{1}$$

Where A and ω are the Amplitude and fundamental frequency of the process output signal. The representation of eq. (1) which is used throughout the identification procedure is given by eq. (2)

$$e(t) = A\,sin(L) \tag{2}$$

Where $L = \omega t$ Fractional order integral follows the Riemann-Lowville (R-L) definition [27-30]. The Describing Function of relay with hysteresis changes with the order of fractional integral, which is fixed in the case of conventional relay. The fractional order integrator is a linear element and is defined (eq. 3) as

$$\frac{1}{s^{\alpha}} = \frac{1}{(j\omega)^{\alpha}} = e^{-j\frac{\pi}{2}\alpha} \tag{3}$$

Where α is the order of fractional integrator. The signal after passing through the fractional integrator shifts their phase by $\frac{\pi}{2}\alpha$. The output of fractional integrator $v(t)$ is

$$v(t) = A\,sin(L - \frac{\pi}{2}\alpha) \tag{4}$$

Then, the output of the relay is given by (5) [31].

$$u(L) = \begin{cases} -h & 0 < L < \theta_0 \\ +h & \theta_0 < L < \theta_0 + \pi \\ -h & \theta_0 + \pi < L < 2\pi \end{cases} \tag{5}$$

Where

$$\theta_0 = sin^{-1}\left(\frac{\varepsilon + \alpha\frac{\pi}{2}}{A}\right) \tag{6}$$

Where $\pm h$ indicates relay height or amplitude and $\pm\varepsilon$ is the hysteresis width. As describing function analysis provides the tool for frequency domain analysis of nonlinear system, the DF is obtained by considering the principle harmonics of relay output signal. Therefore, relay with hysteresis is approximated with gain as given in (7)

$$N = \frac{1}{\pi a}\int_0^{2\pi} u(L)(sin\,L + j\,cos\,L)dL \tag{7}$$

The Describing function of relay with hysteresis and fractional integrator is obtained by solving eq. (7) using eq. (5). The resulting describing function *N* is given by eq. (8)

$$N = \frac{4h(\sqrt{A^2 - \varepsilon^2} - j\varepsilon)}{\pi A^2}\, e^{-j\frac{\pi}{2}\alpha} \tag{8}$$

The condition to obtain sustained oscillations during identification is

$$NG_p(j\omega) = -1 \tag{9}$$

2.1 Identification Procedure for SOPTD Process

Consider the SOPTD model given in eq. (10)

$$G_p(s) = \frac{ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)} \tag{10}$$

Equation (11) gives the frequency domain representation of above equation with *s=jω* is

$$G_p(j\omega) = \frac{ke^{-\theta j\omega}}{(\tau_1 j\omega+1)(\tau_2 j\omega+1)} \tag{11}$$

The unknowns to be identified are: process gain *(k),* time constants (τ_1, τ_2) and time delay(θ). Substitute eq. (11) and eq. (8) in eq. (9) to obtain the condition for sustained oscillation

$$\frac{4hke^{-j\omega\theta}\left(\sqrt{A^2-\varepsilon^2}-j\varepsilon\right)}{\pi A^2(\tau_1 j\omega+1)(\tau_2 j\omega+1)} e^{j\frac{\pi}{2}\alpha} = -1 \tag{12}$$

Equate the magnitude and phase angles on both sides of eq. (12) to get the unknown parameters. The equation obtained by equating the magnitude is given in (13)

$$\frac{4hk}{\pi A\sqrt{(\tau_1{}^2\omega^2+1)}\sqrt{(\tau_2{}^2\omega^2+1)}} = 1 \tag{13}$$

The resulting equation (14) after simplifying eq. (13) in terms of $\tau_1$and τ_2 is

$$\tau_1 + \tau_2 = \sqrt{\frac{1}{\omega^2}\left[\left(\frac{4hk}{\pi A}\right)^2 - 1\right] + 2\tau_1\tau_2 - (\omega\tau_1\tau_2)^2} \tag{14}$$

Equation (15) is obtained by equating the phase angles in eq. (12)

$$-\theta\omega - tan^{-1}(\tau_1\omega) - tan^{-1}(\tau_2\omega) - tan^{-1}\left(\frac{\varepsilon}{\sqrt{A^2-\varepsilon^2}}\right) - \frac{\pi}{2}\alpha = -\pi \tag{15}$$

Rearranging the above equation for $\tau_1$and τ_2 results in eq. (16)

$$\tau_1\tau_2 = \frac{1}{\omega^2}\left[1 - \frac{\omega(\tau_1+\tau_2)}{tan\left[\phi-\omega\theta-tan^{-1}\left(\frac{\varepsilon}{\sqrt{A^2-\varepsilon^2}}\right)\right]}\right] \tag{16}$$

Where

$$\theta = t_2 - t_1 \tag{17}$$

$$\phi = \pi - \alpha\frac{\pi}{2} \tag{18}$$

2.2 The Systematic Approach for Identification of SOPTD Model Parameters is given as follows:

Step 1: Choose the parameters h and ε before performing the relay test.

Step 2: The parameter h is usually chosen as a symmetrical value. In the present work, it is fixed to ± 1 and ε is considered 2.5% of h.

Step 3: Perform relay test by choosing different values for α in the range 0.1-1.8.

Note: Any value of α beyond 2 results in an unstable response [32].

Step 4: Set $\alpha=0.1$, conduct relay experiment and note A, T and θ from the sustained oscillation along with IAE (eq. 19).

$$IAE = \int_0^{\omega_{cr}} \left| \frac{G_m(j\omega) - G(j\omega)}{G_m(j\omega)} \right| d\omega \tag{19}$$

Where ω_{cr} is the critical frequency of actual model, $G(j\omega)$ is the actual model and $G_m(j\omega)$ is the identified model.

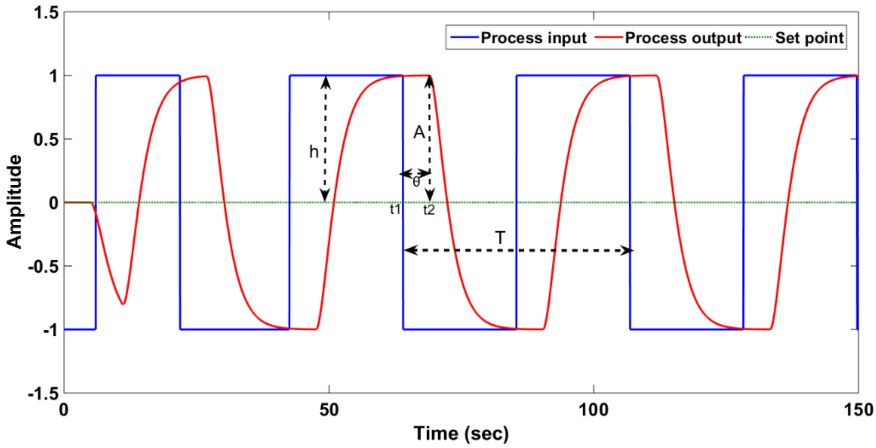

Figure 2

Process input-output signals

A sample input output signal diagram is shown in Figure 2, T is the critical time period of process output, t_1 is the time at which process input crosses set point $r(t)$ and t_2 is the time at peak amplitude of the process output.

Step 5: The process gain (k) is usually predefined and here it is chosen to be equal to the gain present in the actual process.

Step 6: Identify τ_1 and τ_2 using equations (14) and (16) after substituting $h, \varepsilon, k, A, \theta,$ and ω. Where $\omega = \frac{2\pi}{T}$ obtained in steps 4 and 5.

Step 7: Now, the second order model parameters are identified for α=0.1

Step 8: Repeat steps 4 to 6 by varying α from 0.2 to 1.8 and identify the model parameters for each value of α.

Step 9: Finally, choose the optimum value for α and accurate second order model identified based on minimum IAE.

3 Simulation Results and Discussion

The simulations for model identification have been carried out on different second order systems Viz., underdamped, overdamped and critically damped systems. The identification of model parameters according to the novel systematic approach is delineated with the plots of model parameter variation with respect to fractional order (α) of the integrator. The value of α for which optimum model is identified is characterized through α versus IAE plots. Further, step response is observed to compare the exactness of identified model with the actual model.

3.1 Example 1

Consider the overdamped SOPTD model [4] given in eq. (20)

$$G_1(s) = \frac{e^{-2s}}{(10s+1)(s+1)} \tag{20}$$

The relay test is initiated by setting h=±1 and ε =±0.025. Now, the test is performed by considering α=0.1 and then the model parameters are identified according to the systematic procedure. The relay test is repeated for different values of α (0.2-1.8) and the second order model parameters are identified for each value of α. The trends of variation of the identified parameters τ_1, τ_2 and θ for each value of α is shown in Figure 3. It is observed that the variation of identified τ_1 and τ_2 are very close to actual values and are equal to the actual values at α =1.325. The delay also becomes equal to the actual θ at α =1.325. The best model is identified from the set of identified models based on minimum IAE for α =1.325, which is evident from Figure 4. The critical period and amplitude are obtained as T=67.80 and A=0.9275.

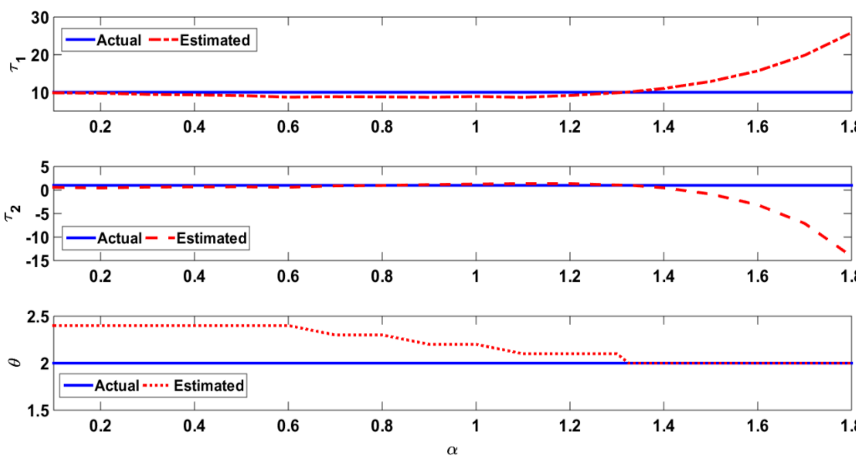

Figure 3
Trends of τ_1, τ_2 and θ for variation in α

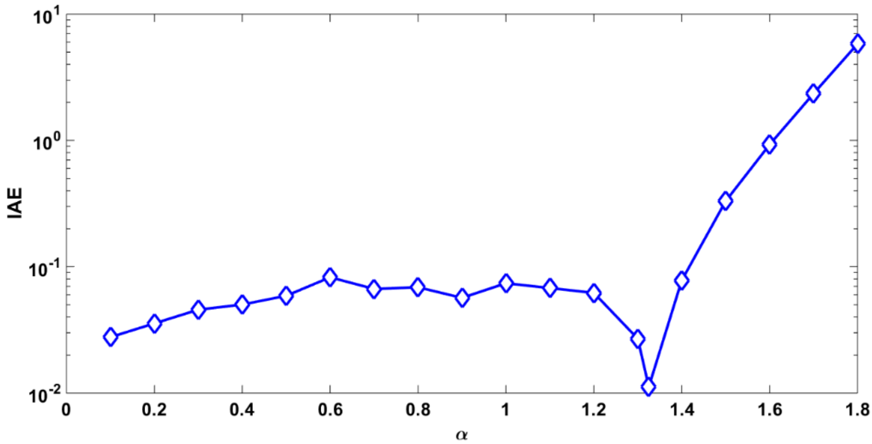

Figure 4

α vs IAE graph

The identified model, actual model and other models used for comparison are listed in Table 1 along with IAE. The model identified through proposed method gives low IAE compared to other models. The efficiency of the proposed identification method is proved under noisy environment in presence of measurement noise of 20 dB. The noise effect is achieved using a random additive noise with zero mean and 0.00013526 variance. The noisy process output and noise free limit cycle output obtained by curve fitting technique are as shown in Figure 5. The identified model with measurement noise is given in Table 1 and it proves that the proposed method is efficient with low IAE even under the influence of noise.

Table 1

Comparison of process models

Methods	Model	IAE
Actual Process	$\frac{e^{-2s}}{(10s+1)(s+1)}$	--
Proposed model	$\frac{e^{-2s}}{(10.061s+1)(1.054s+1)}$	0.0112
Proposed with measurement noise	$\frac{e^{-1.9s}}{(10.073s+1)(1.161s+1)}$	0.0161
Method (offline) in [7]	$\frac{e^{-3.12s}}{10.24s+1}$	0.0418
Method (online) in [7]	$\frac{e^{-3.15s}}{9.81s+1}$	0.0502
Method in [4]	$\frac{e^{-2.84s}}{11.98s+1}$	0.0601

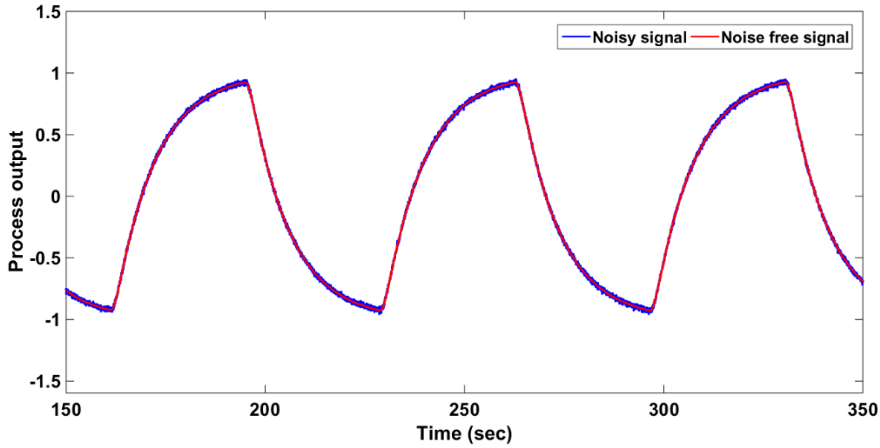

Figure 5
Noisy and noise free process output

The step response of the proposed model is shown in Figure 6. It is clear that the response with proposed model is close to the actual model compared to the other methods [7, 4]. It is observed from Figure 7 that the Nyquist plot of the proposed method is close to the actual model.

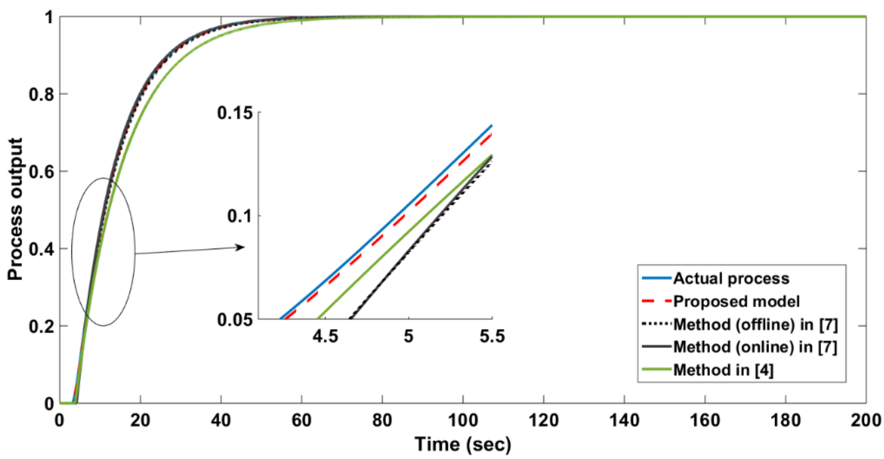

Figure 6
Step responses of the proposed model, actual process, and methods present in literature

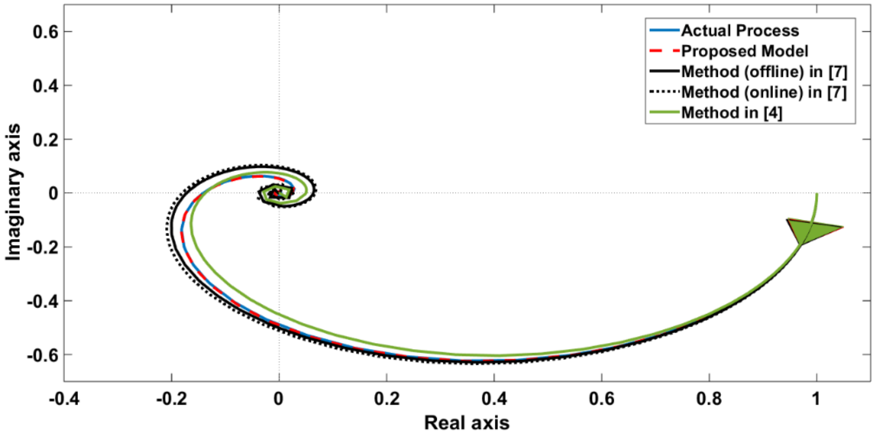

Figure 7
Nyquist plot

3.2 Example 2

The overdamped SOPTD process [9] used for identification is given in (21)

$$G_2(s) = \frac{e^{-4s}}{(10s+1)(2s+1)} \tag{21}$$

The values of h=±1 and ε =±0.025 are chosen to perform relay test. The model is identified according to the systematic approach given in section 2.1. The trends of variation of the identified parameters τ_1, τ_2 and θ for each value of α is shown in Figure 8. The error between identified model and actual model (IAE) for each value of α is illustrated in Figure 9. The best model parameters are identified based on minimum IAE at α =1.15, which is evident from Figures 8 & 9. The critical period and amplitude are obtained as T=68.2 and A=0.9205. The model identified according to the proposed method and existing model are listed in Table 2.

The model identified in presence of measurement noise (random noise with zero mean and 0.00013526 variance) is given in Table 2. The identified model under these circumstances is very close to actual model with minimum error (0.0063 lower than noise free model 0.0069) which is evident from Table 2 and Figure 10. It is also observed from the step response (Figure 11) and Nyquist plot (Figure 12) that the proposed model is in the close proximity of actual model compared to other models [9].

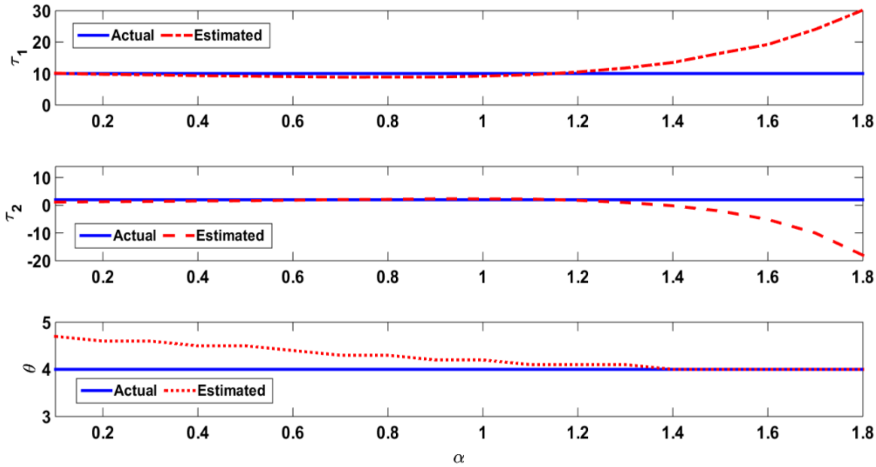

Figure 8

Trends of τ_1, τ_2 and θ for variation in α

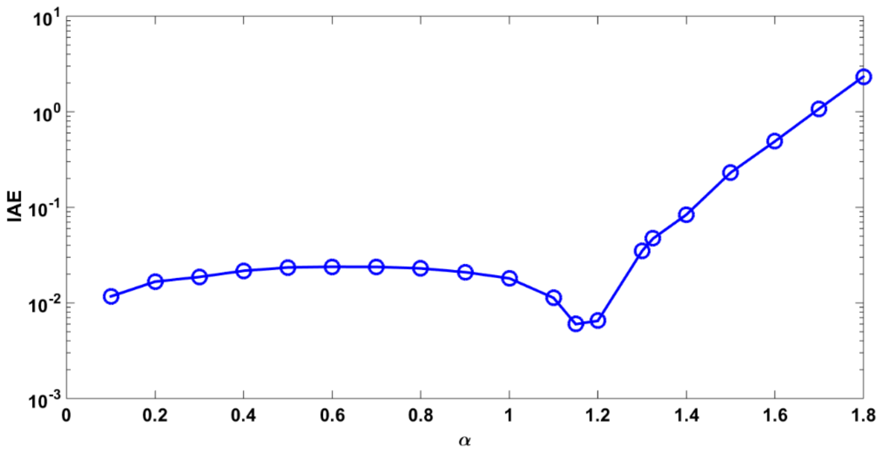

Figure 9

α vs IAE graph

Table 2

Comparison of process models

Methods	Model	IAE
Actual Process	$\frac{e^{-4s}}{(10s+1)(2s+1)}$	--
Proposed model	$\frac{e^{-4.1s}}{(10.012s+1)(2.032s+1)}$	0.0069
Proposed with noise	$\frac{e^{-4.1s}}{(10.023s+1)(2.0156s+1)}$	0.0063

Method (offline) in [9]	$\dfrac{e^{-4s}}{(8.9312s+1)(2.1515s+1)}$	0.0206
Method (online) in [9]	$\dfrac{e^{-4.1s}}{(8.855s+1)(2.171s+1)}$	0.0218

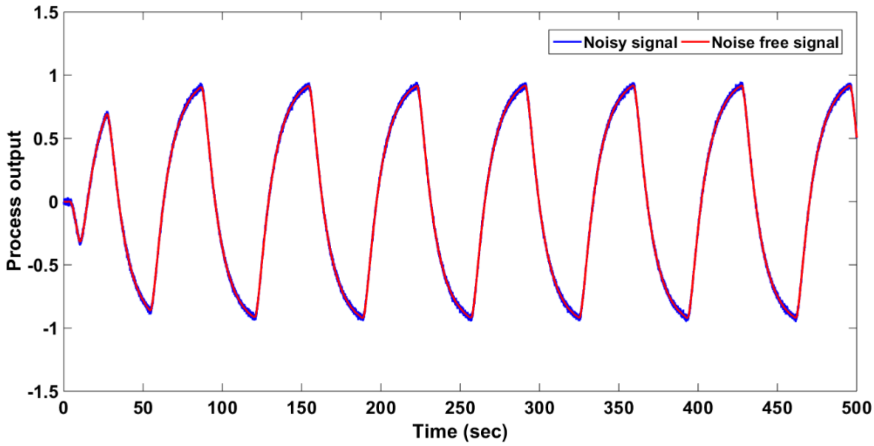

Figure 10
Noisy and noise free process output

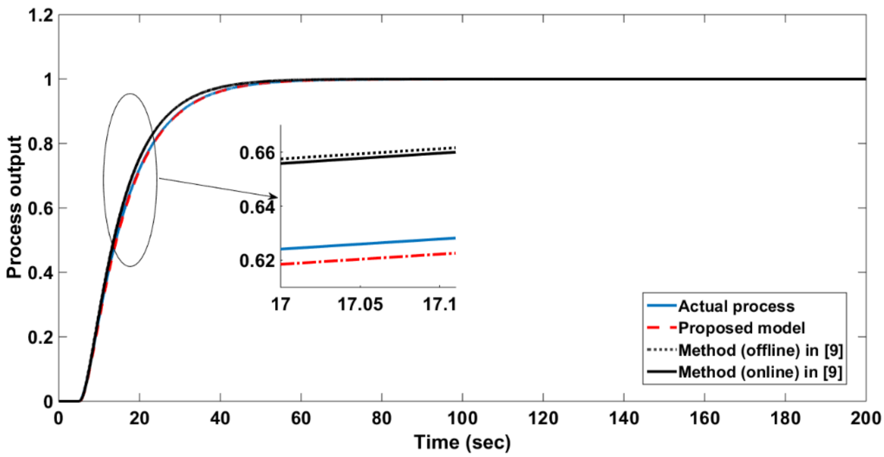

Figure 11
Step responses of the proposed model, actual process, and methods present in literature

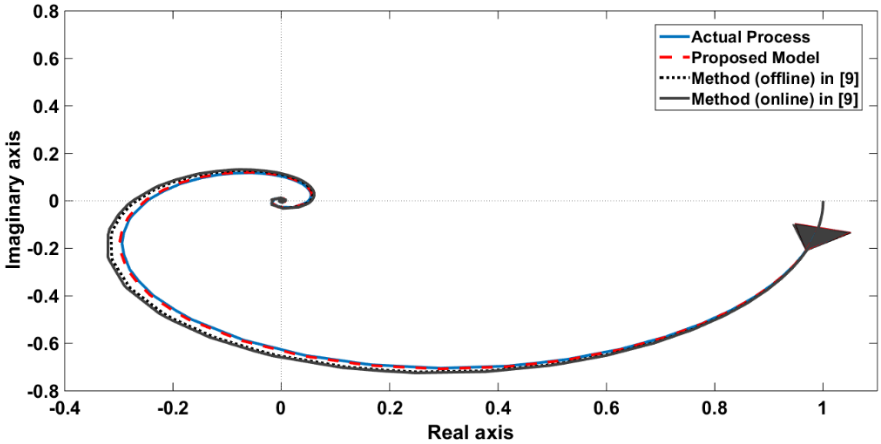

Figure 12
Nyquist plot

3.3 Example 3

The underdamped SOPTD process [2] used for simulation is in (22)

$$G_3(s) = \frac{e^{-s}}{9s^2+2.4s+1} \tag{22}$$

The model identification as per the proposed method is done with the settings: h=±1 and ε =±0.025. The models are identified by varying α from 0.1 to 1.8 following the systematic approach. The identifications results in complex values for the parameters τ_1 and τ_2 as the process is an underdamped system but a real value for θ. Hence, the trends (Figure 13) are plotted between $\tau_1\tau_2$ and $\tau_1 + \tau_2$ with respect to variation in α. The best model is identified at α=1.15 (see α versus IAE in Fig. 14) and the corresponding model is given in Table 3 along with the error. The critical period and amplitude are identified as T=23.5 and A=1.651. It is observed that the proposed model is near the actual model with minimum error compared to method in [2]. The model in presence of random noise is also identified (Table 3) and it is a bit far from the actual model (illustrated in Figure 15) with a slightly high error. The exactness of the identified model to the actual model is also evident from step response and Nyquist plot shown in Figures 16 and 17.

Table 3
Comparison of process models

Methods	Model	IAE
Actual process	$\frac{e^{-s}}{9s^2+2.4s+1}$	--
Proposed model	$\frac{e^{-s}}{8.622s^2+2.487s+1}$	0.0042

Proposed model with noise	$\frac{e^{-0.9s}}{8.138s^2+2.532s+1}$	0.0330
Method in [2]	$\frac{1.0e^{-3.35s}}{3.96s+1}$	0.2948

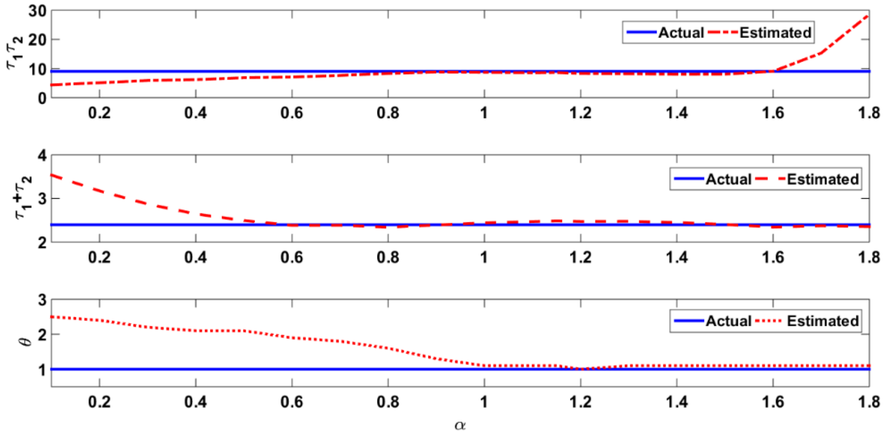

Figure 13
Trends of τ_1, τ_2 and θ for variation in α

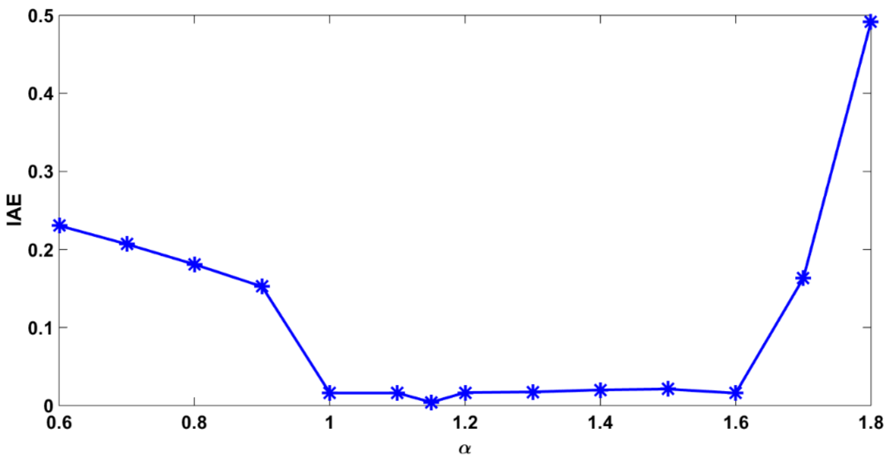

Figure 14
α vs IAE graph

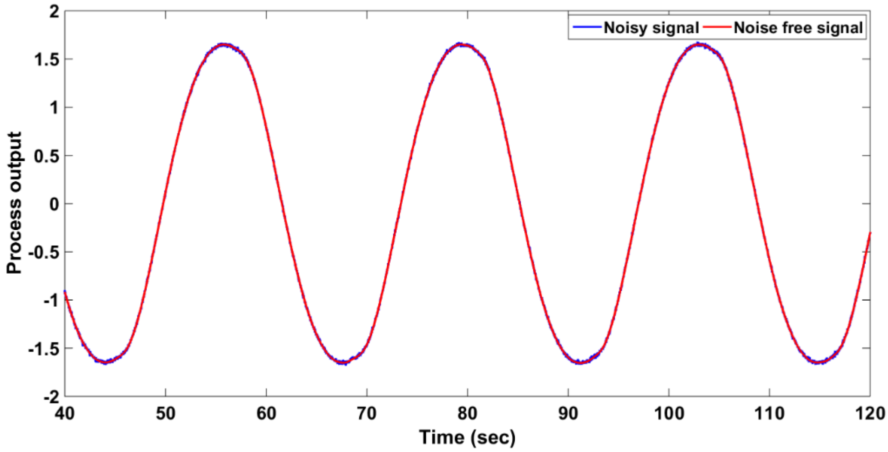

Figure 15
Noisy and noise free process output

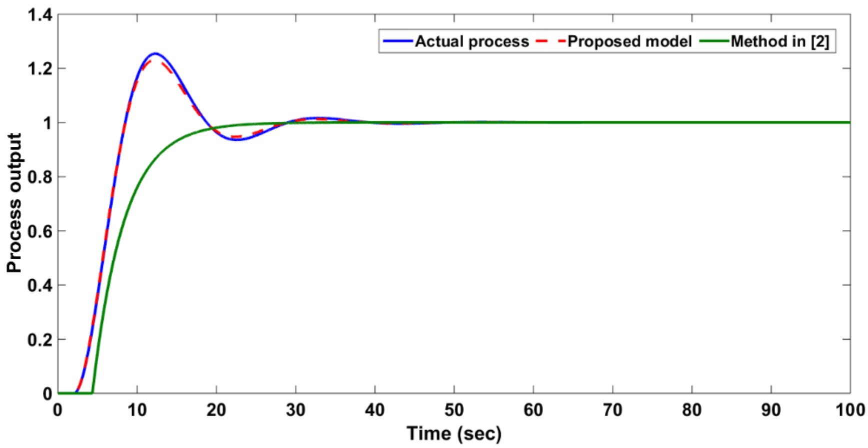

Figure 16
Step responses of the proposed model, actual process, and methods present in literature

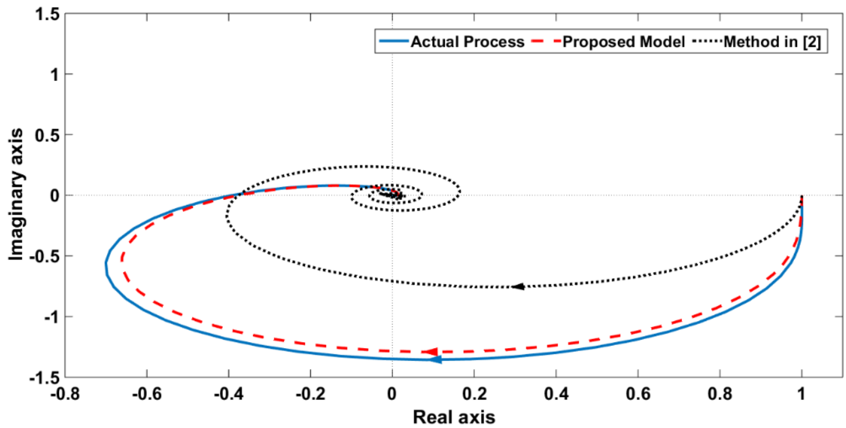

Figure 17
Nyquist plot

3.4 Example 4

Consider the following (eq. 23) critically damped process [9]

$$G_4(s) = \frac{e^{-0.01s}}{(2s+1)^2} \tag{23}$$

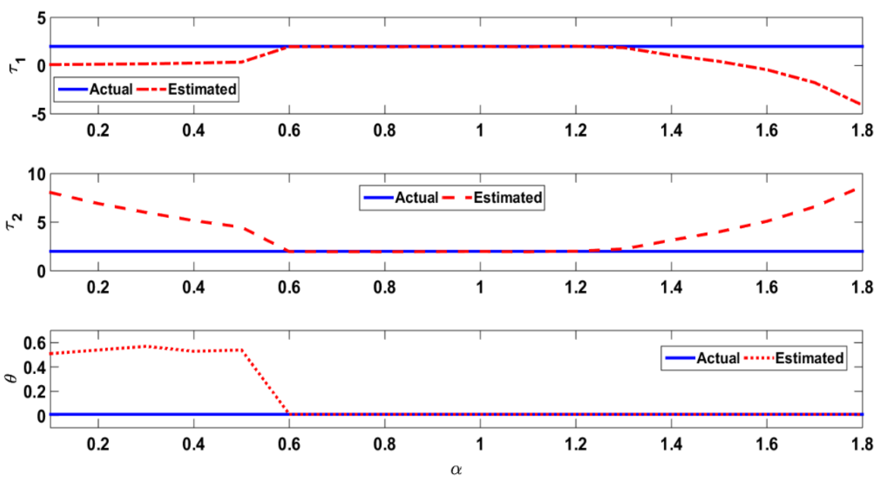

Figure 18
Trends of τ_1, τ_2 and θ for variation in α

The model identification is carried out according to the systematic approach with h=±1 and ε =±0.025. The variation of the identified parameters τ_1, τ_2 and θ for α is shown in Figure 18 and the IAE versus α plot is illustrated in Figure 19. It is observed that the model parameters are optimum at α =1.15 with minimum IAE.

The corresponding critical period and amplitude are T=16.6 and A=0.8459. It is interesting to note that the model parameters are equal to actual values for a wide range of α which is evident from Figure 18. The proposed model identified according to the systematic approach along with other models is presented in Table 4. There is a slight rise in the error (Table 4) between the models identified in presence of noise compared to actual model (Figure 20). Figure 21 and Figure 22 illustrate that the proposed model is close to actual one for step change in the input and for variation in frequency.

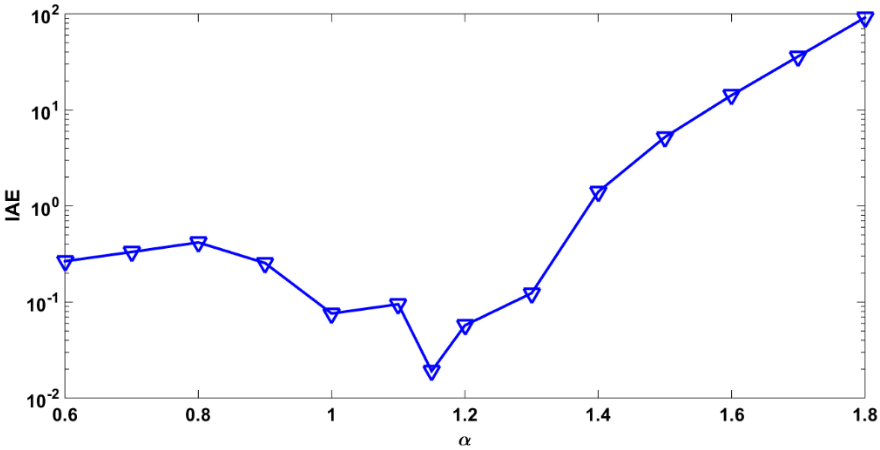

Figure 19

α vs IAE graph

Table 4

Comparison of process models

Methods	Model	IAE
Actual Process	$\frac{e^{-0.01s}}{(2s+1)^2}$	--
Proposed model	$\frac{e^{-0.01s}}{(2.002s+1)^2}$	0.0190
Proposed with noise	$\frac{e^{-0.013s}}{(2.035s+1)^2}$	0.388
Method in [9]	$\frac{1.0084e^{-0.01s}}{(1.9962s+1)^2}$	0.118
Method in [3]	$\frac{0.8709e^{-0.013s}}{(1.897s+1)^2}$	0.465

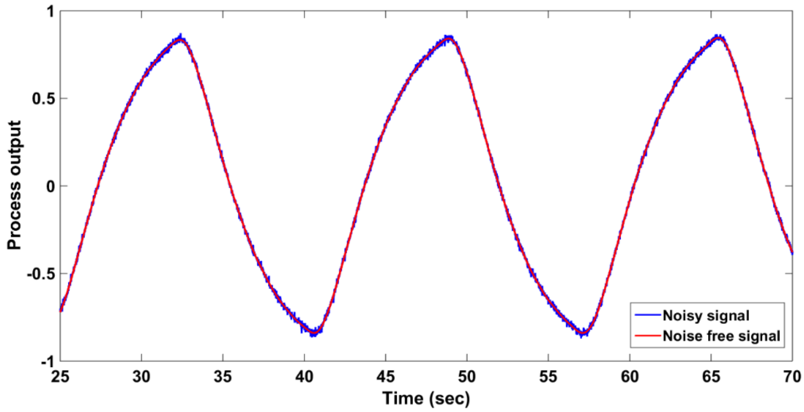

Figure 20
Noisy and noise free process output

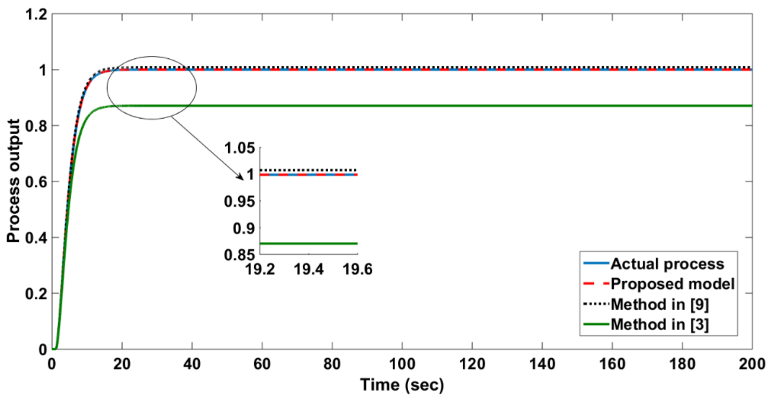

Figure 21
Step responses of the proposed model, actual process, and methods present in literature

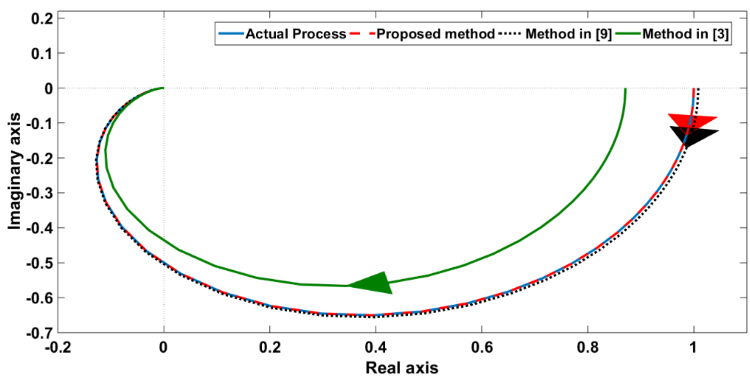

Figure 22
Nyquist plot

Conclusion

This paper proposes a new systematic identification approach for SOPTD processes using relay feedback with hysteresis and fractional order integrator. The proposed approach provides flexibility in the degree of freedom with the addition of fractional integrator and thus leads to more accurate SOPTD models. The proposed approach has an additional degree of freedom for estimating parameters, i.e., fractional order integrator. The describing function method developed the expressions to estimate the accurate model parameters. The addition of a fractional integrator helps improve the position frequency point obtained by the DF method. The proposed approach is found to be efficient under realistic conditions by estimating the model parameters in presence of measurement noise. To get the noise effect, white Gaussian noise is added to the process output, and the noisy limit cycle is processed through the curve fitting technique to obtain a clean signal. The proposed approach was applied to overdamped, underdamped, and critically damped SOPTD transfer function models and the performance is evaluated by comparing the absolute error criteria in the frequency domain, Nyquist plot, and step response. The proposed approach reduced IAE for Overdamped, Underdamped, and critically damped processes by 77.68%, 98.57%, and 95.78%, respectively compared to literature methods.

References

[1] K. J. Astrom, T. Hagglund: Automatic Tuning of Simple Regulators with Specifications on Phase and Amplitude Margins, Automatica, Vol. 20, No. 5, 1984, pp. 645-651

[2] C. L. Chen: A simple method for online identification and controller tuning, AIChE Journal, Vol. 35, No. 12, 1989, pp. 2037-2039

[3] T. Thyagarajan, CC. Yu: Improved auto tuning using shape factor from relay feedback, Industrial & Engineering Chemistry Research, Vol. 42, No. 20, 2003, pp. 4425-4440

[4] S. Vivek, M. Chidambaram: Identification using single symmetrical relay feedback test, Computers and Chemical Engineering, Vol. 29, No. 7, 2005, pp. 1625-1630

[5] S. Majhi: Relay based identification of processes with time delay, Journal of Process Control, Vol. 17, No. 2, 2007, pp. 93-101

[6] U. Mehata, S. Majhi. :Estimation of process model parameters based on half limit cycle data, Journal system science & engineering, Vol. 17, No. 2, 2008, pp. 13-21

[7] R. Bajarangbali, S. Majhi: Relay Based Identification of Systems, International Journal of Scientific & Engineering Research, Vol. 3, No. 6, 2012, pp. 1-4

[8] R. Bajarangbali, S. Majhi: Identification of underdamped process dynamics, System Science & Control Engineering, Vol. 2, No. 1, 2014, pp. 541-548

[9] R. Bajarangbali, S. Majhi: Estimation of First and Second Order Process Model Parameters, The National Academy of Sciences, Vol. 88, No. 4, 2017, pp. 557-563

[10] R. Bajarangbali, S. Majhi, S. Pandey: Identification of FOPDT and SOPDT process dynamics using closed loop test, ISA Transactions, Vol. 53, No. 4, 2014, pp. 1223-1231

[11] R. Bajarangbali, S. Majhi: Identification of integrating and critically damped systems with time delay, Control Theory & Technology, Vol. 13, No. 1, 2015, pp. 29-36

[12] R. Bajarangbali, S. Majhi: Identification of non-minimum phase processes with time delay in the presence of measurement noise, ISA Transactions, Vol. 57, 2015, pp. 245-253

[13] Li. Zhuo, Chun Yin, Yang Quan, Chen, Jiaguo Liu: Process Identification Using Relay Feedback with a Fractional Order Integrator, Proceedings of the 19th World Congress The International Federation of Automatic Control Cape Town, South Africa, 2014, pp. 2010-2015

[14] P. Ghorai, S. Majhi, S. Pandey: Modeling and Identification of Real-Time Processes Based on Nonzero Set point Auto tuning Test, Journal of Dynamic Systems, Measurement, and Control, Vol. 139, No. 2, 2017, pp. 1-8

[15] P. Ghorai, S. Majhi, S. Pandey: A real-time approach for dead-time plant transfer function modeling based on relay auto tuning, International Journal of Dynamics & Control, Vol. 6, No. 3, 2018, pp. 950-960

[16] S. Pandey, S. Majhi, P. Ghorai: A new modelling and identification scheme for time-delay systems with experimental investigation: a relay feedback approach, international journal of systems science, Vol. 48, No. 9, 2017, pp. 1932-1940

[17] S. Pandey, S. Majhi: Limit cycle-based exact estimation of FOPDT process parameters under input/output disturbances: a state-space approach, International Journal of Systems Science, Vol. 48, No. 1, 2017, pp. 118-128

[18] S. Pandey, S. Majhi: Relay-based identification scheme for processes with non-minimum phase and time delay, IET Control Theory Appl., Vol. 13, No. 15, 2019, pp. 2507-2519

[19] S. Pandey, S. Majhi: Limit cycle based identification of time delay SISO processes, IFAC Journal of Systems and Control, Vol. 48, No. 1, 2018, pp. 118-128

[20] M. Hofreiter: Shifting method for relay feedback identification. IFAC-Papers Online, Vol. 49, No. 12, 2016, pp. 1933-1938

[21] M. Hofreiter: Biased-relay feedback identification for time delay systems, IFAC-Papers Online, Vol. 50, No. 1, 2017, pp. 14620-14625

[22] M. Hofreiter: Alternative identification method using biased relay feedback, IFAC-Papers Online, Vol. 51, No. 11, 2018, pp. 891-896

[23] M. Hofreiter: Relay feedback identification with additional integral IFAC-Papers Online, Vol. 52, No.13, 2019, pp. 66-71

[24] M. Hofreiter: Relay Feedback Identification with Shifting Filter for PID Control, IFAC Papers Online, Vol. 53, No. 2, 2020, pp. 10701-10706

[25] M. Hofreiter: Generalized Relay Shifting Method for System Identification, IFAC Papers Online, Vol. 54, No. 1, 2021, pp. 498-503

[26] R. Gerov, T. V. Jovanovic, Z. Jovanovic: Parameter Estimation Methods for the FOPDT Model, using the Lambert W Function, Acta Polytechnica Hungarica, Vol. 18, No. 9, 2021, pp. 141-159

[27] I. Podlubny: Fractional differential equations, Mathematics in Science & Engineering, Academic Press, New York, 1999

[28] E. C. De Oliveira, J. A. Tenreiro Machado: A review of definitions for fractional derivatives and integral, Mathematical Problems in Engineering, Vol. 2014, 2014, p. 6

[29] J. Munkhammar: Riemann-Liouville fractional derivatives and the Taylor-Riemann series, Department of Mathematics, Uppsala University, Sweden, 2004

[30] J. Liouville: Mémoire sur quelques Quéstions de Géometrie et de Mécanique, et sur un nouveau genre de Calcul pour résoudre ces Quéstions. Journal de l'école Polytechnique, Vol. 3, 1832, pp. 71-162

[31] M. Chidambaram, V. Sathe: Relay Auto tuning for Identification and Control, Cambridge University Press, 2014

[32] R. Caponetto, G. Maione, A. Pisano, M. R. Rapai´c and E. Usai: Analysis and shaping of the self-sustained oscillations in relay controlled fractional order systems, Fractional Calculus and Applied Analysis, Vol. 16, No. 1, 2013, pp. 93-108