State Convergence-based Control of a Multi-Master-Single-Slave Non-linear Teleoperation System

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Abstract: This paper presents the design of a state convergence based control scheme, for a multi-master-single-slave nonlinear teleoperation system. The control objective is that the slave follows the weighted motion of the master systems, in free motion, and the master systems receive the scaled force feedback, while the slave system is in contact with the environment. To achieve the desired objectives, extended state convergence architecture is modified and appropriate control gains are chosen following a Lyapunov based stability analysis. MATLAB simulations considering a two-degree-of-freedom tri-master-single-slave nonlinear teleoperation system are provided to show the validity of the proposed scheme.

Keywords: state convergence method; multilateral teleo-peration system; nonlinear dynamics; MATLAB/Simulink

1 Introduction

Teleoperation refers to the control of a remote process and forms an important class of robotics due to its wide range of applications [1]. Typical units of a teleoperation system are a human operator, master manipulator, communication channel, slave manipulator and the environment. The working of the system is such that the human operator initiates the task through the use of a master manipulator which is installed at the local site. This task-related information is then transmitted over the communication channel to the remote environment where the slave manipulator performs the desired task. At the same time, slave manipulator provides a feedback, which is representative of the remote environment conditions, to the human operator through the master manipulator who then can guide the slave manipulator by adapting to the environment conditions. This form of teleoperation system is known as bilateral, as the information flows both ways as opposed to the unilateral version where the flow of information is from master to slave side only. Clearly, the bilateral scheme is more reliable because of the presence of the feedback connection which provides the operator with a feel of environment. On the other hand, the presence of the feedback connection in a bilateral system poses a great challenge mainly because of the time delay in the communication channel and the uncertainties of different units complicate the problem further. A variety of control laws have been proposed in literature to stabilize the bilateral systems against the time delays of the communication channel while providing the satisfactory task performance [2]-[4]. The groundbreaking works on the use of transmission line theory and the wave variables in bilateral teleoperation systems form the basis of many other algorithms [5], [6]. Such a class of algorithms is collectively referred to as passivity paradigm for the bilateral control of teleoperation systems and by far, is the most popular choice for designing bilateral systems due to their strong robustness to time delays [7]. The other types of bilateral algorithms include sliding mode control [8], $H\infty$ control [9], Lyapunov-Krasovskii functional based control [10], disturbance observer based control [11], adaptive control [12], intelligent control [13]-[15] and the state convergence based control [16], [17].

In recent years, another class of teleoperation system, known as multilateral teleoperation systems, has emerged due to the need of performing the remote tasks in a cooperative fashion. In order to maintain the stability and to ensure the satisfactory task performance in such systems, various control algorithms have also been proposed. Small gain theorem based approach is presented in [21] for a multi-master-single-slave teleoperation system where the slave system is influenced by the master systems according to a weighting criterion and a force feedback is provided equally to all the master systems. Based on the theory of adaptive control and wave variables, the control of an uncertain single-master-multi-slave teleoperation system is discussed in [22] where each slave is made to follow the master commands in the presence of time varying delays. A disturbance observer-based scheme is presented in [23] to control a multi-master-single-slave teleoperation system with different degrees-of-freedom. In this method, reaction force is estimated and a modal transformation is introduced to accomplish the position and force tracking tasks. A Lyapunov-based approach is presented in [24], [25] to design a multilateral controller for dual-master-single-slave nonlinear teleoperation system. The approximation ability of fuzzy logic control has been used in [26] to design an adaptive controller for uncertain dual-master-dual-slave teleoperation system.

This paper presents the design of a time-delayed multi-master-single-slave nonlinear teleoperation system based on the method of state convergence. In our earlier work [27], we have presented an extended state convergence control architecture where *k*-master systems can cooperatively control *l*-slave systems. However, this extended state convergence method is only applicable to linear teleoperation systems when the communication channel offers no time delays. These two limitations have been addressed in this paper by considering the nonlinear dynamics of master/slave systems and constant asymmetric communication time delays. A Lyapunov-based stability analysis is presented and control gains of the extended state convergence method are selected to ensure the stability of the multilateral system against the communication time delays and to achieve the zero tracking error of the slave system. In order to validate the proposed scheme, MATLAB simulations are performed on a tri-master-single-slave nonlinear teleoperation system.

2 Modeling of Multilateral Nonlinear Teleoperation System

We consider a nonlinear multilateral teleoperation system which is comprised of n-degrees-of-freedom p-master and single-slave manipulators as:

$$M_m^j\left(q_m^j\right)\ddot{q}_m^j + C_m^j\left(q_m^j, \dot{q}_m^j\right)\dot{q}_m^j + g_m^j\left(q_m^j\right) = \tau_m^j + F_h^j, \forall j = 1, 2, ..., p \tag{1}$$

$$M_s\left(q_s\right)\ddot{q}_s + C_s\left(q_s, \dot{q}_s\right)\dot{q}_s + g_s\left(q_s\right) = \tau_s - F_e \tag{2}$$

Where $\left(M_m^j, M_s\right) \in \mathbb{R}^{n\times n}$, $\left(C_m^j, C_s\right) \in \mathbb{R}^{n\times n}$ and $\left(g_m^j, g_s\right) \in \mathbb{R}^{n\times 1}$ represent inertia matrices, coriolis/centrifugal matrices and gravity vectors for master/slave systems respectively. Also, $\left(q_m^j, q_s\right) \in \mathbb{R}^{n\times 1}$, $\left(\dot{q}_m^j, \dot{q}_s\right) \in \mathbb{R}^{n\times 1}$, $\left(\ddot{q}_m^j, \ddot{q}_s\right) \in \mathbb{R}^{n\times 1}$, $\left(\tau_m^j, \tau_s\right) \in \mathbb{R}^{n\times 1}$ and $\left(F_h^j, F_e\right) \in \mathbb{R}^{n\times 1}$ represent the joint variables of master/slave manipulators namely position, velocity, acceleration, torque and external force signals, respectively. In the sequel, the following properties of the master/slave manipulators (1)-(2) will be utilized in proving the stability of the closed loop teleoperation system:

Property 1: The inertia matrices are symmetric, positive definite and bounded i.e., there exists positive constants β_l and β_u such that $0 < \beta_l I < M(q) < \beta_u I \le \infty$.

Property 2: A skew-symmetric relation exists between the inertia and coriolis/centrifugal matrices such that $x^T\left(\dot{M}(q) - 2C\left(q, \dot{q}\right)\right)x = 0, \forall x \in \mathbb{R}^n$.

Property 3: The coriolis/centrifugal force vectors are bounded i.e., there exists positive constant β_f such that $\left\|C\left(q, \dot{q}\right)\dot{q}\right\| \le \beta_f \left\|\dot{q}\right\|$.

Property 4: If the joint variables $\dot{q}$ and $\ddot{q}$ are bounded, then the time derivative of coriolis/centrifugal matrices is also bounded.

In addition to the above properties, we make the following assumptions:

Assumption 1: The gravity force vectors for the master/slave manipulators are assumed to be known.

Assumption 2: The operators are assumed to be passive i.e., there exist positive constants $\rho_m^j, j = 1,2,..., p$ such that $-\rho_m^j < \int_0^{t_f} -F_h^j \dot{q}_m^j \, dt$. Also, the environment is assumed to be passive and is modeled by a spring-damper system i.e., $F_e = K_e q_s + B_e \dot{q}_s$ where $K_e \in \mathbb{R}^{n\times n}$ and $B_e \in \mathbb{R}^{n\times n}$ are positive definite diagonal matrices.

In addition to the above properties and assumptions, we will use the following lemmas in proving the stability of the multilateral teleoperation formed by (1), (2):

Lemma 1: For any vector signals $x, y \in \mathbb{R}^n$ and scalar $\gamma > 0$, time delay T and positive definite matrix $K \in \mathbb{R}^{n\times n}$, the following inequality holds over the time interval [0, t_f]: $-2\int_0^{t_f} \dot{x}^T K \int_0^T \dot{y}(t-\sigma) d\sigma dt \le \gamma \int_0^{t_f} \dot{x}^T Kx dt + \frac{T^2}{\gamma}\int_0^{t_f} \dot{y}^T Ky dt$

Lemma 2: For the vector signal $x \in \mathbb{R}^n$ and the time delay T, the following inequality holds: $x(t-T) - x(t) = \int_0^T \dot{x}(t-\sigma) d\sigma \le T^{1/2} \left\|\dot{x}\right\|_2$

3 Modified Extended State Convergence Architecture

The authors recently proposed an extended-state convergence architecture [27] for *k*-master-*l*-slave delay-free linear teleoperation system, which can be modeled on state space. The aim of the present study is to explore the applicability of the extended state convergence architecture for nonlinear multilateral teleoperation system in the presence of asymmetric constant communication delays. For simplicity, a multi-master-single-slave nonlinear teleoperation system is considered in this paper which can be observed in literature. Further, the extended state convergence architecture is slightly modified by eliminating the gain terms G_{ij} which are responsible for direct transmission of operators' forces to slave systems. However, all other control gain terms are kept the same. Since the gravity force vectors are assumed to be known, they are included in torque inputs and will therefore become part of the extended state convergence architecture. The modified state convergence architecture is shown in Figure 1 and various parameters defining the architecture are described below:

F_h^j: It represents the force exerted by the j^{th} operator onto the j^{th} master manipulator.

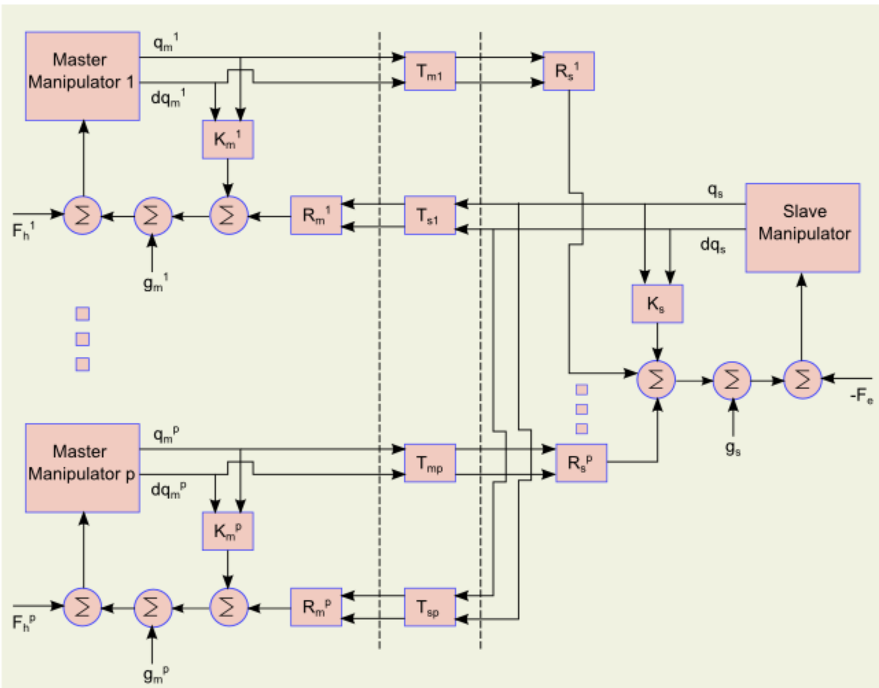


Figure 1

Modified extended state convergence architecture for multi-master-single-slave teleoperation

$K_m^j = \begin{bmatrix} K_{m1}^j & K_{m2}^j \end{bmatrix}$: It represents the stabilizing feedback gain for the j^{th} master manipulator. $K_{m1}^j \in \mathbb{R}^{n\times n}$ and $K_{m2}^j \in \mathbb{R}^{n\times n}$ are the state feedback gains for the j^{th} master's position and velocity signals respectively.

$K_s = \begin{bmatrix} K_{s1} & K_{s2} \end{bmatrix}$: It represents the stabilizing feedback gain for the slave manipulator. $K_{s1} \in \mathbb{R}^{n\times n}$ and $K_{s2} \in \mathbb{R}^{n\times n}$ are the state feedback gains for the slave's position and velocity signals respectively.

T_{mj}: It represents the constant time delay in the communication link connecting the j^{th} master system to the slave system.

T_{sj}: It represents the constant time delay in the communication link connecting the slave system to the j^{th} master system.

$R_s^j = \begin{bmatrix} R_{s1}^j & R_{s2}^j \end{bmatrix}$: It represents the influence of the motion signals generated by the j^{th} master manipulator (when operated by the jth human operator) into the slave manipulator. The matrices $R_{s1}^j \in \mathbb{R}^{n\times n}$ and $R_{s2}^j \in \mathbb{R}^{n\times n}$ weights the j^{th} master manipulator's position and velocity signals respectively.

$R_m^j = \begin{bmatrix} R_{m1}^j & R_{m2}^j \end{bmatrix}$: It represents the effect of slave's motion into the j^{th} master manipulator. The slave's position and velocity signals are weighted by the matrices $R_{m1}^j \in \mathbb{R}^{n\times n}$ and $R_{m2}^j \in \mathbb{R}^{n\times n}$ to influence the j^{th} master manipulator's motion.

4 Lyapunov-based Stability Analysis and Control Design

Using the modified extended state convergence architecture of figure 1, we intend to achieve the following control objectives:

Control Objective 1: The slave manipulator's motion is the weighted effect of the masters' manipulators' motion i.e.,

$$\lim_{t\to\infty}\left(q_s(t) - \sum_{j=1}^{p} \alpha_j q_m^j(t) \right) = 0 \tag{3}$$

Where α_j is a weighting factor which is used to scale the j^{th} master manipulator's motion and obeys the property: $\sum_{j=1}^{p} \alpha_j = 1$.

Control Objective 2: The static force reflected onto the j^{th} operator is a function of the environmental force and the other operator's applied torques i.e.,

$$F_h^j = f_j\left(F_e, F_h^{i,i\neq j,i=1,2,\dots,p}\right) \tag{4}$$

To achieve the above objectives and to show the system stability, state convergence-based closed loop multi-master-single-slave nonlinear teleoperation system will be analyzed using a Lyapunov-Krasovskii functional technique. Towards this end, we first write the control inputs for the master/slave systems using Figure 1 as:

$$\tau_m^j = g_m^j\left(q_m^j\right) + K_{m1}^j q_m^j + K_{m2}^j \dot{q}_m^j + R_{m1}^j q_s\left(t-T_{sj}\right) + R_{m2}^j \dot{q}_s\left(t-T_{sj}\right), \forall j = 1,2,\dots,p \tag{5}$$

$$\tau_s = g_s\left(q_s\right) + K_{s1}q_s + K_{s2}\dot{q}_s + \sum_{j=1}^{p} R_{s1}^j q_m^j\left(t-T_{mj}\right) + \sum_{j=1}^{p} R_{s2}^j \dot{q}_m^j\left(t-T_{mj}\right) \tag{6}$$

By plugging (5) in (1) and (6) in (2) and using the assumptions 2 and 3, the closed loop master/slave systems can be given as:

$$M_m^j \ddot{q}_m^j + C_m^j \dot{q}_m^j = K_{m1}^j q_m^j + K_{m2}^j \dot{q}_m^j + R_{m1}^j q_s\left(t-T_{sj}\right) + R_{m2}^j \dot{q}_s\left(t-T_{sj}\right) + F_h^j, \forall j = 1,2,\dots,p \tag{7}$$

$$M_s \ddot{q}_s + C_s \dot{q}_s = K_{s1}q_s + K_{s2}\dot{q}_s + \sum_{j=1}^{p} R_{s1}^j q_m^j\left(t-T_{mj}\right) + \sum_{j=1}^{p} R_{s2}^j \dot{q}_m^j\left(t-T_{mj}\right) - F_e \tag{8}$$

4.1 Position Coordination Behavior

We now show the stability analysis of the closed loop teleoperation system formed by (7) and (8) by introducing the following theorem:

Theorem 4.1: By selecting the control gains of the multi-master-single-slave teleoperation system (7), (8) as in (9) and on the satisfaction of the $p+1$ inequalities as in (10), the stability of the closed loop teleoperation system of (7), (8) can be demonstrated as in (11).

$$\begin{aligned} K_{m1}^j = K_{s1} = -K, K_{m2}^j = K_{s2} = -3K_1, \forall j = 1,2,\dots,p \\ R_{m1}^j = R_{s1}^j = \alpha_j K, R_{m2}^j = R_{s2}^j = 2\alpha_j K_1, \forall j = 1,2,\dots,p \end{aligned} \tag{9}$$

Where $K \in \mathbb{R}^{n\times n}$ and $K_1 \in \mathbb{R}^{n\times n}$ are positive definite diagonal matrices.

$$\begin{aligned}&\left(3-2\alpha_j\right)K_1-\frac{\alpha_j\gamma_{sj}}{2}K-\frac{\alpha_j T_{mj}^2}{2\gamma_{mj}}K>0,\forall j=1,2,...,p\\&K_1-\sum_{j=1}^{p}\frac{\alpha_j\gamma_{mj}}{2}K-\sum_{j=1}^{p}\frac{\alpha_j T_{sj}^2}{2\gamma_{sj}}K>0\end{aligned}\tag{10}$$

Where γ_{mj} , γ_{sj} are positive scalar constants.

$$\lim_{t\to\infty}=\dot{q}_m^j=\lim_{t\to\infty}=\dot{q}_s=\lim_{t\to\infty}=\ddot{q}_m^j=\lim_{t\to\infty}=\ddot{q}_s=0,\forall j=1,2,...,p\tag{11}$$

Proof: Consider the following Lyapunov-Krasovskii functional:

$$\begin{aligned}&V\left(\dot{q}_m^j,\dot{q}_s,q_m^j-q_s,q_s,q_m^j\right)=\frac{1}{2}\sum_{j=1}^{p}\dot{q}_m^{jT}M_m^j\dot{q}_m^j+\frac{1}{2}\dot{q}_s^TM_s\dot{q}_s+\frac{1}{2}q_s^TK_eq_s\\&\frac{1}{2}\sum_{j=1}^{p}\left(1-\alpha_j\right)q_m^{jT}Kq_m^j+\sum_{j=1}^{p}\int_0^t-\dot{q}_m^{jT}\left(\xi\right)F_h^j\left(\xi\right)d\xi+\int_0^t-\dot{q}_m^{jT}\left(\xi\right)F_e\left(\xi\right)d\xi+\\&\sum_{j=1}^{p}\rho_m^j+\frac{1}{2}\sum_{j=1}^{p}\alpha_j\left(q_m^j-q_s\right)^TK\left(q_m^j-q_s\right)+\sum_{j=1}^{p}\alpha_j\int_{t-T_{mj}}^{t}\dot{q}_m^{jT}\left(\xi\right)K_1\dot{q}_m^j\left(\xi\right)d\xi\\&+\sum_{j=1}^{p}\alpha_j\int_{t-T_{sj}}^{t}\dot{q}_s^T\left(\xi\right)K_1\dot{q}_s\left(\xi\right)d\xi\end{aligned}\tag{12}$$

By taking the time-derivative of (12) along the system trajectories defined by (7) and (8), and using the passivity assumption 2 along with the property 2 of the robot dynamics, we have:

$$\begin{aligned}&\dot{V}=\sum_{j=1}^{p}\dot{q}_m^{jT}\left(K_{m1}^jq_m^j+K_{m2}^j\dot{q}_m^j+R_{m1}^jq_s\left(t-T_{sj}\right)+R_{m2}^j\dot{q}_s\left(t-T_{sj}\right)\right)+\\&\dot{q}_s^T\left(K_{s1}q_s+K_{s2}\dot{q}_s+\sum_{j=1}^{p}R_{s1}^jq_m^j\left(t-T_{mj}\right)+\sum_{j=1}^{p}R_{s2}^j\dot{q}_m^j\left(t-T_{mj}\right)-K_eq_s-B_e\dot{q}_s\right)\\&+\dot{q}_s^TK_eq_s+\sum_{j=1}^{p}\left(1-\alpha_j\right)\dot{q}_m^{jT}Kq_m^j+\sum_{j=1}^{p}\alpha_j\dot{q}_m^{jT}K\left(q_m^j-q_s\right)+\\&\sum_{j=1}^{p}\alpha_j\dot{q}_s^TK\left(q_s-q_m^j\right)+\sum_{j=1}^{p}\alpha_j\dot{q}_m^{jT}K_1\dot{q}_m^j-\sum_{j=1}^{p}\alpha_j\dot{q}_m^{jT}\left(t-T_{mj}\right)K_1\dot{q}_m^j\left(t-T_{mj}\right)\\&+\sum_{j=1}^{p}\alpha_j\dot{q}_s^TK_1\dot{q}_s-\sum_{j=1}^{p}\alpha_j\dot{q}_s^T\left(t-T_{sj}\right)K_1\dot{q}_s\left(t-T_{sj}\right)\end{aligned}\tag{13}$$

After simplifying and grouping the terms in (13), we get:

$$\begin{aligned}
\dot{V} &= \sum_{j=1}^{p} \dot{q}_m^{jT}\left(K_{m1}^j + K\right)\dot{q}_m^j + \sum_{j=1}^{p} \dot{q}_m^{jT}\left(R_{m1}^j q_s\left(t-T_{sj}\right) - \alpha_j K q_s\right) + \\
&\dot{q}_s^T\left(K_{s1} + \sum_{j=1}^{p}\alpha_j K\right) q_s + \sum_{j=1}^{p}\dot{q}_s^T\left(R_{s1}^j q_m^j\left(t-T_{mj}\right) - \alpha_j K q_m^j\right) + \\
&\sum_{j=1}^{p}\dot{q}_m^{jT}\left(K_{m2}^j + \alpha_j K_1\right)\dot{q}_m^j + \sum_{j=1}^{p}\dot{q}_s^T\left(K_{s2} + \sum_{j=1}^{p}\alpha_j K_1\right)\dot{q}_s - \dot{q}_s^T B_e \dot{q}_s \\
&+\sum_{j=1}^{p}\dot{q}_m^{jT} R_{m2}^j \dot{q}_s\left(t-T_{sj}\right) + \sum_{j=1}^{p}\dot{q}_s^T R_{s2}^j \dot{q}_m^j\left(t-T_{mj}\right) - \\
&\sum_{j=1}^{p}\alpha_j \dot{q}_m^{jT}\left(t-T_{mj}\right)K_1 \dot{q}_m^j\left(t-T_{mj}\right) - \sum_{j=1}^{p}\alpha_j \dot{q}_s^T\left(t-T_{sj}\right)K_1\dot{q}_s\left(t-T_{sj}\right)
\end{aligned} \tag{14}$$

By plugging the control gains of (9) in (14), and by adding and subtracting the terms $\sum_{j=1}^{p}\alpha_j \dot{q}_m^{jT} K_1 \dot{q}_m^j$, $\sum_{j=1}^{p}\alpha_j \dot{q}_s^T K_1 \dot{q}_s$ from (14), we have:

$$\begin{aligned}
\dot{V} &= \sum_{j=1}^{p}\alpha_j \dot{q}_m^{jT} K\left(q_s\left(t-T_{sj}\right) - q_s\right) + \sum_{j=1}^{p}\alpha_j \dot{q}_s^T K\left(q_m^j\left(t-T_{mj}\right) - q_m^j\right) \\
&-\sum_{j=1}^{p}\dot{q}_m^{jT}\left(3-2\alpha_j\right)K_1\dot{q}_m^j - \dot{q}_s^T K_1 \dot{q}_s - \dot{q}_s^T B_e \dot{q}_s \\
&-\sum_{j=1}^{p}\alpha_j\left(\dot{q}_m^{jT} K_1 \dot{q}_m^j + \dot{q}_m^{jT}\left(t-T_{mj}\right)K_1\dot{q}_m^j\left(t-T_{mj}\right) - 2\dot{q}_s^T K_1 \dot{q}_m^j\left(t-T_{mj}\right)\right) \\
&-\sum_{j=1}^{p}\alpha_j\left(\dot{q}_s^T K_1\dot{q}_s + \dot{q}_s^T\left(t-T_{sj}\right)K_1\dot{q}_s\left(t-T_{sj}\right) - 2\dot{q}_m^{jT} K_1 \dot{q}_s\left(t-T_{sj}\right)\right)
\end{aligned} \tag{15}$$

Now, we define the following error signals:

$$\begin{aligned}
e_{q_s^j} &= q_s - q_m^j\left(t-T_{mj}\right) \\
e_{q_m^j} &= q_m^j - q_s\left(t-T_{sj}\right)
\end{aligned} \tag{16}$$

By re-writing (15) in terms of the time-derivatives of the error signals of (16) and using the relation $q\left(t-T\right) - q\left(t\right) = -\int_0^T \dot{q}\left(t-\sigma\right)d\sigma$, we have:

$$\dot{V} = -\sum_{j=1}^{p}\alpha_j \dot{q}_m^{jT} K \int_0^{T_{sj}} \dot{q}_s(t-\sigma)d\sigma - \sum_{j=1}^{p}\alpha_j \dot{q}_s^T K \int_0^{T_{mj}} \dot{q}_m^j(t-\sigma)d\sigma - \\ \sum_{j=1}^{p}\dot{q}_m^{jT}\left(3-2\alpha_j\right)K_1 \dot{q}_m^j - \dot{q}_s^T K_1 \dot{q}_s - \dot{q}_s^T B_e \dot{q}_s - \sum_{j=1}^{p}\alpha_j \dot{e}_{q_s^j}^T K_1 \dot{e}_{q_s^j} \\ -\sum_{j=1}^{p}\alpha_j \dot{e}_{q_m^j}^T K_1 \dot{e}_{q_m^j} \tag{17}$$

By integrating (17) over the time interval [0, t_f] and using lemma 1, we get:

$$\int_0^{t_f}\dot{V}\,ds \le \sum_{j=1}^{p}\alpha_j\left(\frac{\gamma_{sj}}{2}\int_0^{t_f}\dot{q}_m^{jT} K \dot{q}_m^j\,ds + \frac{T_{sj}^{\ 2}}{2\gamma_{sj}}\int_0^{t_f}\dot{q}_s^T K \dot{q}_s\,ds\right) + \\ \sum_{j=1}^{p}\alpha_j\left(\frac{\gamma_{mj}}{2}\int_0^{t_f}\dot{q}_s^T K \dot{q}_s\,ds + \frac{T_{mj}^{\ 2}}{2\gamma_{mj}}\int_0^{t_f}\dot{q}_m^{jT} K \dot{q}_m^j\,ds\right) - \\ \sum_{j=1}^{p}\int_0^{t_f}\dot{q}_m^{jT}\left(3-2\alpha_j\right)K_1 \dot{q}_m^j\,ds - \int_0^{t_f}\dot{q}_s^T K_1 \dot{q}_s\,ds - \int_0^{t_f}\dot{q}_s^T B_e \dot{q}_s\,ds \\ -\sum_{j=1}^{p}\alpha_j\int_0^{t_f}\dot{e}_{q_s^j}^T K_1 \dot{e}_{q_s^j}\,ds - \sum_{j=1}^{p}\alpha_j\int_0^{t_f}\dot{e}_{q_m^j}^T K_1 \dot{e}_{q_m^j}\,ds \tag{18}$$

By grouping the terms in (18), we can simplify the bound on the time-derivative of the Lyapunov- Krasovskii functional as:

$$V\left(t_f\right) - V\left(0\right) \le -\sum_{j=1}^{p}\lambda_{\min}\left(\left(3-2\alpha_j\right)K_1 - \frac{\alpha_j\gamma_{sj}}{2}K - \frac{T_{mj}^{\ 2}}{2\gamma_{mj}}K\right)\left\|\dot{q}_m^j\right\|_2^2 \\ -\sum_{j=1}^{p}\lambda_{\min}\left(K_1 - \frac{\alpha_j\gamma_{mj}}{2}K - \frac{T_{sj}^{\ 2}}{2\gamma_{sj}}K\right)\left\|\dot{q}_s\right\|_2^2 - \sum_{j=1}^{p}\lambda_{\min}\left(\alpha_j K_1\right)\left\|\dot{e}_{q_s^j}\right\|_2^2 \\ -\sum_{j=1}^{p}\lambda_{\min}\left(\alpha_j K_1\right)\left\|\dot{e}_{q_m^j}\right\|_2^2 - \lambda_{\min}\left(B_e\right)\left\|\dot{q}_s\right\|_2^2 \tag{19}$$

Now, if the inequalities in (10) are satisfied, the right hand side of (19) will remain negative. Since $V(0)$ and $V(t_f)$ are positive and right hand side of (19) is negative, it can be concluded that $V(t_f)$-$V(0)$ remains bounded ensuring that $V(t_f)$ will remain bounded. Taking the limit as $t_f \to \infty$ and using the robot properties, it can be said that the signals $\left\{\dot{q}_m^j, \dot{q}_s, q_m^j - q_s, q_s, q_m^j\right\} \in L_\infty$ and $\left\{\dot{q}_m^j, \dot{q}_s, \dot{e}_{q_s^j}, \dot{e}_{q_m^j}\right\} \in L_2$. To prove the system stability in the sense of (11), we have to show that the acceleration signals of master/slave systems and their time derivatives remain bounded. To this end, we rewrite (7) and (8) without external forces (since they are assumed to be passive and bounded) as:

$$\ddot{q}_m^j = \left(M_m^j\right)^{-1}\left[-C_m^j \dot{q}_m^j + K_{m1}^j q_m^j + K_{m2}^j \dot{q}_m^j + R_{m1}^j q_s\left(t-T_{sj}\right) + R_{m2}^j \dot{q}_s\left(t-T_{sj}\right)\right] \tag{20}$$

$$\ddot{q}_s = M_s^{-1}\left[-C_s \dot{q}_s + K_{s1} q_s + K_{s2} \dot{q}_s + \sum_{j=1}^{p} R_{s1}^j q_m^j\left(t-T_{mj}\right) + \sum_{j=1}^{p} R_{s2}^j \dot{q}_m^j\left(t-T_{mj}\right)\right] \tag{21}$$

By considering the control gains of (9) along with (20) and (21), it is now required to show that the signals $\left\{q_m^j - \alpha_j q_s\left(t-T_{sj}\right), q_s - \sum_{j=1}^{p}\alpha_j q_m^j\left(t-T_{mj}\right)\right\} \in L_\infty$. These signals can be written as:

$$\begin{aligned} q_m^j - \alpha_j q_s\left(t-T_{sj}\right) &= \left(q_m^j - \alpha_j q_s\right) + \alpha_j\left(q_s - q_s\left(t-T_{sj}\right)\right) \\ q_s - \sum_{j=1}^{p}\alpha_j q_m^j\left(t-T_{mj}\right) &= \left(q_s - \sum_{j=1}^{p}\alpha_j q_m^j\right) + \sum_{j=1}^{p}\alpha_j\left(q_m^j - \alpha_j q_m^j\left(t-T_{mj}\right)\right) \end{aligned} \tag{22}$$

The first set of parentheses on the right hand sides of (22) are bounded by virtue of $\left\{\dot{q}_m^j, \dot{q}_s, q_m^j - q_s\right\} \in L_\infty$ while the second set of parentheses are bounded by virtue of lemma 2 and $\left\{\dot{q}_m^j, \dot{q}_s\right\} \in L_\infty$. This implies that the left hand sides of (22) are also bounded. By using the properties 1 and 3 of the robot dynamics and the result $\left\{\dot{q}_m^j, \dot{q}_s, q_m^j - q_s, q_s, q_m^j, q_m^j - \alpha_j q_s\left(t-T_{sj}\right), q_s - \sum_{j=1}^{p}\alpha_j q_m^j\left(t-T_{mj}\right)\right\} \in L_\infty$, it can be concluded that the signals $\left\{\ddot{q}_m^j, \ddot{q}_s\right\}$ are bounded. Since the signals $\left\{\dot{q}_m^j, \dot{q}_s\right\}$ also belong to L_2, then by Barbalat's lemma: $\lim_{t\to\infty}\dot{q}_m^j = \lim_{t\to\infty}\dot{q}_s = \lim_{t\to\infty}\dot{e}_{q_m^j} = \lim_{t\to\infty}\dot{e}_{q_s^j} = 0$. Now, it is left to show the boundedness of the time derivatives of (20) and (21) to complete the proof. By taking their time derivatives, we have:

$$\begin{aligned} \frac{d\,\ddot{q}_m^j}{dt} &= \frac{d}{dt}\left(M_m^j\right)^{-1}\left[-C_m^j \dot{q}_m^j + K_{m1}^j q_m^j + K_{m2}^j \dot{q}_m^j + R_{m1}^j q_s\left(t-T_{sj}\right) + R_{m2}^j \dot{q}_s\left(t-T_{sj}\right)\right] \\ &\quad + \left(M_m^j\right)^{-1}\frac{d}{dt}\left[-C_m^j \dot{q}_m^j + K_{m1}^j q_m^j + K_{m2}^j \dot{q}_m^j + R_{m1}^j q_s\left(t-T_{sj}\right) + R_{m2}^j \dot{q}_s\left(t-T_{sj}\right)\right] \end{aligned} \tag{23}$$

$$\frac{d\ddot{q}_s}{dt} = \frac{d}{dt}\left(M_s^{-1}\right)\left[-C_s\dot{q}_s + K_{s1}q_s + K_{s2}\dot{q}_s + \sum_{j=1}^{p} R_{s1}^j q_m^j\left(t-T_{mj}\right) + \sum_{j=1}^{p} R_{s2}^j \dot{q}_m^j\left(t-T_{mj}\right)\right]$$
$$+ M_s^{-1}\frac{d}{dt}\left[-C_s\dot{q}_s + K_{s1}q_s + K_{s2}\dot{q}_s + \sum_{j=1}^{p} R_{s1}^j q_m^j\left(t-T_{mj}\right) + \sum_{j=1}^{p} R_{s2}^j \dot{q}_m^j\left(t-T_{mj}\right)\right] \tag{24}$$

By using the properties 3 and 4 of the robot dynamics and the earlier result $\left\{\dot{q}_m^j, \dot{q}_s, q_m^j - q_s, q_s, q_m^j, q_m^j - \alpha_j q_s\left(t-T_{sj}\right), q_s - \sum_{j=1}^{p}\alpha_j q_m^j\left(t-T_{mj}\right), \ddot{q}_m^j, \ddot{q}_s\right\} \in L_\infty$, it can be concluded that the second derivative terms in (23) and (24) are bounded. The boundedness of the first derivative terms in (23) and (24) follows from the properties 1 and 2 of the robot dynamics, the boundedness of the signals $\left\{\dot{q}_m^j, \ddot{q}_m^j, \dot{q}_s, \ddot{q}_s\right\}$ and considering $\dot{M}^{-1} = -M^{-1}\dot{M}M^{-1} = -M^{-1}\left(C + C^T\right)M^{-1}$. Thus the right hand sides of (23) and (24) remain bounded implying that the signals $\left\{\ddot{q}_m^j, \ddot{q}_s\right\} \in L_\infty$ are uniformly continuous. Therefore, we have: $\lim_{t\to\infty}\ddot{q}_m^j = \lim_{t\to\infty}\ddot{q}_s = 0$. This completes the proof.

Theorem 4.2: The desired position of the slave system, as mentioned in (3), is achieved under the control gains of (9) and the condition that the signals $\left\{\dot{q}_s, \dot{q}_m^j, \ddot{q}_s, \ddot{q}_m^j, \dot{e}_{q_s^j}, \dot{e}_{q_m^j}\right\}$ converge to zero as $t \to \infty$.

Proof: The convergence of the signals $\left\{\dot{q}_s, \dot{q}_m^j, \ddot{q}_s, \ddot{q}_m^j, \dot{e}_{q_s^j}, \dot{e}_{q_m^j}\right\}$ has been shown in Theorem 4.1. Thus, by using the results from Theorem 4.1 and by substituting the control gains of (9) in (8), we have:

$$\lim_{t\to\infty}\left\| q_s - \sum_{j=1}^{p}\alpha_j q_m^j\left(t-T_{mj}\right)\right\| = 0 \tag{25}$$

By using the relation $q_m^j\left(t-T_{mj}\right) = q_m^j - \int_{t-T_{mj}}^{t}\dot{q}_m^j\left(\xi\right)d\xi$, and the result $\lim_{t\to\infty}\dot{q}_m^j = 0$, we can write (25) as: $\lim_{t\to\infty}\left\| q_s - \sum_{j=1}^{p}\alpha_j q_m^j\right\| = 0$. Thus, slave position coordination is achieved in the absence of environmental force as time goes to infinity and control objective 1 is achieved. This completes the proof.

4.2 Force Reflection Behavior

Let us now investigate the force experienced by the operators in steady state when the slave is in contact with the environment. Towards this end, the steady state behavior of the closed loop master systems' (when the velocity and acceleration signals converges to zero) is first found from (7) as:

$$K_{m1}^{j} q_{m}^{j} + R_{m1}^{j} q_{s}\left(t - T_{sj}\right) + F_{h}^{j} = 0, \forall j = 1, 2, ..., p \tag{26}$$

By using the relation $q_s\left(t - T_{sj}\right) = q_s - \int_{t-T_{sj}}^{t} \dot{q}_s\left(\xi\right) d\xi$ and the earlier result from stability analysis $\lim_{t\to\infty} \dot{q}_s = 0$, and the control gains of (9), operators' forces in (26) can be given as:

$$F_{h}^{j} = K\left(q_{m}^{j} - \alpha_{j} q_{s}\right), \forall j = 1, 2, ..., p \tag{27}$$

Similarly, the behavior of the slave system in steady state including the environmental force can be obtained from (8), (9) and two earlier theorems as:

$$F_{e} = -Kq_{s} + \sum_{j=1}^{p} \alpha_{j} K q_{m}^{j} \tag{28}$$

By adding and subtracting $\sum_{j=1}^{p} \alpha_j^2 K q_s$ from (28), we can write the environmental force in terms of operators' forces as:

$$F_{e} = \sum_{j=1}^{p} \alpha_{j} F_{h}^{j} - \left(1 - \sum_{j=1}^{p} \alpha_{j}^{2}\right) K q_{s} \tag{29}$$

From (29), it is evident that the environmental force is indeed proportional to the weighted effect of the operators' forces. Thus, the second control objective of (4) is also achieved.

5 Simulation Results

The proposed state convergence based scheme for multi-master-single-slave teleoperation system is verified in MATLAB/Simulink environment using a two degrees-of-freedom three masters and one slave manipulators with the dynamics of (1), (2). The corresponding inertia matrices, coriolis/centrifugal matrices and gravity vectors are given in (30)-(33):

$$M(q)=\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, C\left(q,\dot{q}\right)=\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}, g(q)=\begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \tag{30}$$

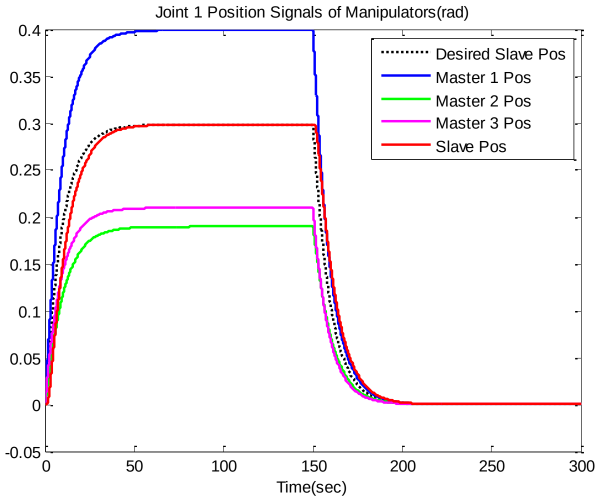
$$\begin{aligned} m_{11} &= m_2 l^2 + (m_1+m_2) l^2 + 2m_2 l^2 \cos(q_2) \\ m_{12} &= m_{21} = m_2 l^2 + m_2 l^2 \cos(q_2) \\ m_{22} &= m_2 l^2 \end{aligned} \tag{31}$$

$$\begin{aligned} c_{11} &= -\dot{q}_2\, m_2 l^2 \sin(q_2), c_{12} = -\left(\dot{q}_1+\dot{q}_2\right) m_2 l^2 \sin(q_2) \\ c_{21} &= \dot{q}_1\, m_2 l^2 \sin(q_2), c_{22} = 0 \end{aligned} \tag{32}$$

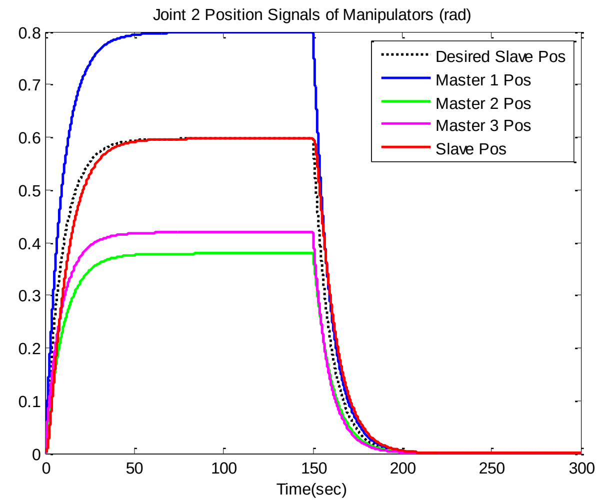
$$g_1 = a_g m_2 l \sin(q_1+q_2) + a_g (m_1+m_2) l \sin(q_1), g_2 = a_g m_2 l \sin(q_1+q_2) \tag{33}$$

Where m_1, m_2 are the masses of links 1 and 2 respectively; $l_1 = l_2 = l$ are the lengths of links and a_g is the acceleration due to gravity. By setting the parameters for the master systems as: $m_{1m} = m_{2m} = 2kg, l_m = 1m$ and the slave system as: $m_{1s} = m_{2s} = 10kg, l_s = 1m$ and by defining the alpha influencing factors for the master systems as: $\alpha_1 = 0.5, \alpha_2 = 0.3, \alpha_3 = 0.2$ and by setting the gamma values as unity and by assuming the time delays in the communication channel as: $T_{m1} = T_{s1} = 1s, T_{m2} = T_{s2} = 0.5s, T_{m3} = T_{s3} = 2s$, we solve the inequalities of (10) and subsequently select the decisive positive definite matrices as: $K = diag(20,10), K_1 = diag(40,20)$. The control gains for the multi-master-single-slave nonlinear teleoperation system can now be found using (9).

We first simulate the tri-master-single-slave nonlinear teleoperation system under the control of constant operators' forces and in the absence of environment forces. The human forces are $F_h^j(t) = F_{op}^j, t \le 150s, \left(F_{op}^1 = 5N, F_{op}^2 = 2N, F_{op}^3 = 3N\right)$ which vanish after 150 *sec*. The results for this simulation are shown in Figure 2. It can be seen that the multilateral system remains stable, owing to the boundedness of the signals and both the joint positions of the slave system converge to the desired references. We also consider the realistic case where the operators' forces vary linearly with time. The simulation results for this case are shown in Figure 3. It can again be seen that the slave position signals indeed follow the desired reference positions. Finally, we simulate the nonlinear teleoperation system in the presence of environment forces considering the constant operators' forces. The simulation results for this case are depicted in Figure 4 where the slave comes in contact with the environment at *t*=150 *sec*. After the contact is made, masters' positions are reduced and the slave is unable to follow the set references. This is in line with the theoretical results. The reduction in masters' positions is due to the force reflected onto the masters' systems by the slave system when it is in contact with the environment while the error in slave's desired trajectory is the result of its direct interaction with the environment as can be seen from (28).



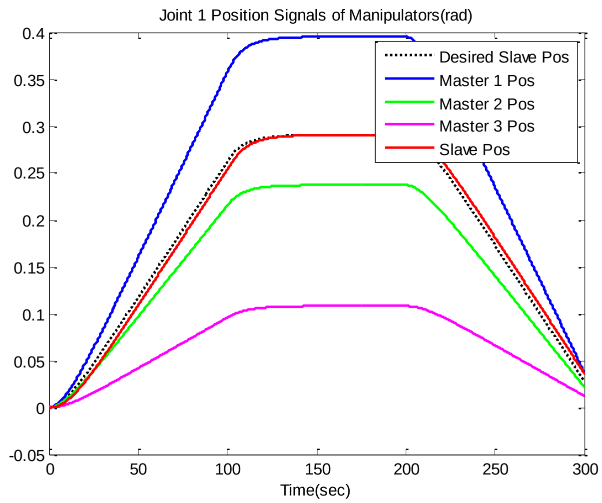
(a)



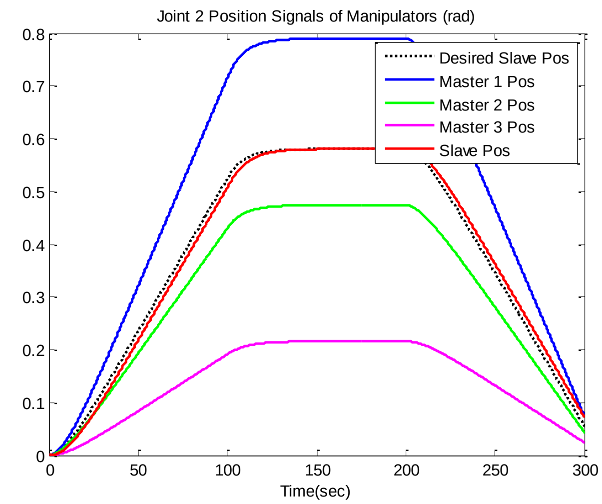
(b)

Figure 2

Tri-master-single-slave nonlinear teleoperation system with constant operator forces in free motion (a) Joint 1 position signals (b) Joint 2 position signals



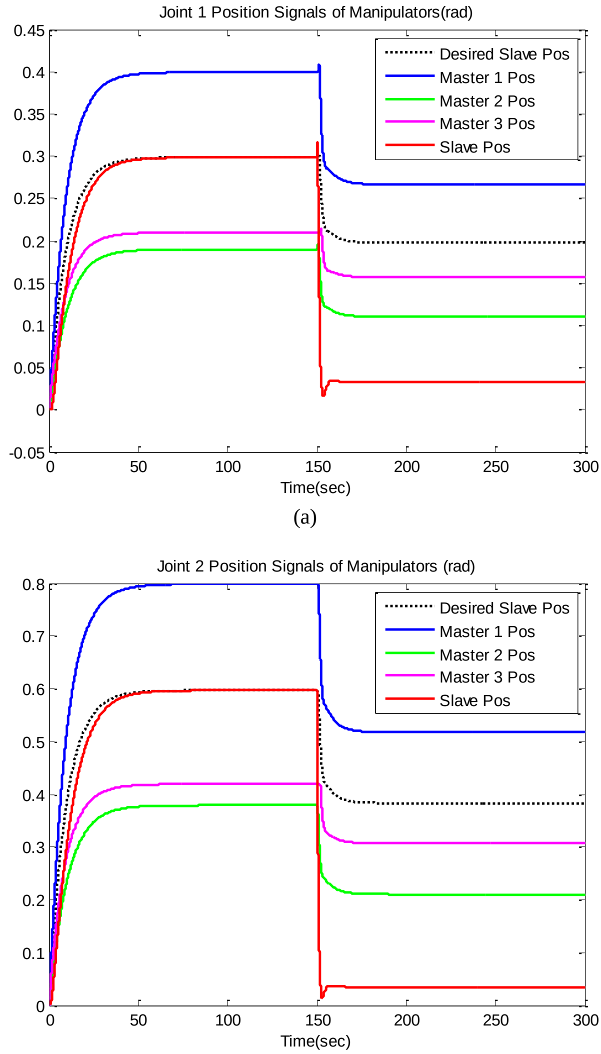
(a)



(b)

Figure 3

Tri-master-single-slave nonlinear teleoperation system with time varying operator forces in free motion (a) Joint 1 position signals (b) Joint 2 position signals



(b)

Figure 4

Tri-master-single-slave nonlinear teleoperation system in free plus contact motion (a) Joint 1 position signals (b) Joint 2 position signals

Conclusions

In this work, the design of a multi-master-single-slave nonlinear teleoperation system in the presence of asymmetric constant communication time delays is presented, based on the extended state convergence theory. A Lyapunov-based stability analysis is carried out to find the control gains for the modified extended state convergence architecture. The efficacy of the proposed scheme is finally verified through simulations in the MATLAB/Simulink environment by considering a two degrees-of-freedom, tri-master-single-slave robotic system. Future work involves the design of state convergence based multilateral nonlinear teleoperation system in the presence of time varying delays with experimental validation.

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