Design of a Single-Master/Multi-Slave Nonlinear Teleoperation System through State Convergence with Time Varying Delays

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Abstract: This paper presents the design of a nonlinear teleoperation system which is comprised of a single master and multiple slave (SM/MS) units. The interaction between these units follows the extended state convergence architecture which allows multiple linear master units to influence multiple linear slave units. However, in this study, the nonlinear dynamics of the master and slave units is considered and the resulting nonlinear teleoperation system is analyzed in the presence of time delays. To be specific, the following objectives are defined: (i) the nonlinear teleoperation remains stable in the presence of time varying delays, (ii) the slave units follow the position commands of the master unit and (iii) the operator receives a force feedback proportional to the interaction forces of the slaves with their environments. Towards this end, Lyapunov-Krasovskii theory is utilized which provides guidelines to select the control gains of the extended state convergence architecture such that the aforementioned objectives are achieved. The efficacy of the proposed scheme is finally verified through simulations in MATLAB/Simulink environment by considering a two degrees-of-freedom (DoF) single-master/tri-slave nonlinear teleoperation system.

Keywords: Teleoperation; nonlinear dynamics; state convergence; MATLAB

1 Introduction

Teleoperation refers to the control of a distant process and has found diverse applications ranging from miniaturized medical procedures to large-scale industrial processes. It is usually accomplished through the use of master and slave robotic devices which are connected through a communication channel. Based on the number of these robotic devices, teleoperation systems can be classified as either bilateral or multilateral systems. In a typical bilateral teleoperation system, human operator drives the master manipulator and the resulting motion commands are transmitted across the communication channel towards the slave manipulator which performs the desired task at the remote site. A force feedback is also provided by the slave manipulator to improve human's perception of the remote environment. By deploying more than one slave manipulator, the task can be carried out more efficiently. The teleoperation system in such a setting is known as single-master/multi-slave system and is one of the topologies in a broader class of multilateral systems. Other arrangements in this category include dual user systems for training tasks, and multi-master/single-slave and multi-master/multi-slave systems for collaborative missions [1]-[3].

All these forms of teleoperation need an effective control system to achieve the required task. An ideal control algorithm should be able to ensure that the teleoperation system remains stable against the time delays of the communication channel while providing a superior position and force tracking performance under systems' uncertainties. This is a challenging task since stability and transparency (the position and force tracking requirement is collectively referred as transparency) are two conflicting objectives and the presence of uncertainties complicates the problem further. Many research efforts have been directed to address these performance issues in teleoperation systems. Passivity schemes are popular in research community as they transform the delay-vulnerable system into a delay-resilient one [3]-[11]. However, transparency of the teleoperation system is sacrificed during this transformation process especially when large time delays exist, for which some modifications to the standard passivity algorithms have also been proposed [12]. To ensure that the teleoperation system performs well under uncertainties, non-passive algorithms based on H-∞ [13], [14], sliding mode [15]-[18] and adaptive control [19]-[21] theories are also proposed. However, time delay appears to be a limiting factor in the complete success of these algorithms. The use of intelligent control techniques such as fuzzy logic [22]-[26] and neural networks [27], [28] has also been investigated. Encouraging results are reported based on the combination of neural networks and passivity algorithms [29], [30].

State convergence is another novel scheme which provides an elegant design procedure to determine control gains for bilateral teleoperation systems modeled on state space [31]. It was originally proposed for linear systems with small time delay in the communication channel. Later, the applicability of the scheme to nonlinear teleoperation systems suffering from time-varying delays was demonstrated through the use of Lyapunov-Krasovskii functional [32]. In our earlier work, we have employed the state convergence scheme to control a nonlinear teleoperation system which can be approximated by a class of Takagi-Sugeno fuzzy models. We have also extended this scheme to design controllers for multiple linear one DoF teleoperation systems [33].

This paper builds on our earlier framework of [33] and discusses the design of a multi-DoF SM/MS nonlinear teleoperation system in the presence of time varying delays. To the best of authors' knowledge, state convergence based design of SM/MS nonlinear teleoperation system has not been discussed in the literature. Further, the earlier methodology on the control of nonlinear bilateral teleoperation system through state convergence [32] has become a special case of the proposed multilateral controller. To proceed, we first define the control objectives to be the synchronization of master and slave position signals along with the mixed force reflection to the operator from the slave environments. Then, to achieve these objectives, Lyapunov-Krasovskii theory is utilized to design the control gains of the extended state convergence architecture following the lines of [32]. The proposed methodology is finally verified through MATLAB simulations on a 2-DoF single-master/tri-slave nonlinear teleoperation system in the presence of time delays.

This paper is structured as follows: We start by presenting the modeling of SM/MS teleoperation system in Section 2. Preliminaries are included in Section 3 while control objectives are listed in Section 4. Stability analysis and control design is discussed in Section 5. Simulation results are presented in Section 6. Finally, conclusions are drawn in Section 7.

2 Modeling of the SM/MS Teleoperation System

We consider a nonlinear teleoperation system which is comprised of *n*-DoF single master and *l*-slave manipulators/units with the following dynamics:

$$M_m\left(q_m\right)\ddot{q}_m + C_m\left(q_m,\dot{q}_m\right)\dot{q}_m + g_m\left(q_m\right) = \tau_m + F_h \tag{1}$$

$$M_s^i\left(q_s^i\right)\ddot{q}_s^i + C_s^i\left(q_s^i,\dot{q}_s^i\right)\dot{q}_s^i + g_s^i\left(\dot{q}_s^i\right) = \tau_s^i - F_e^i, \forall i = 1,2,...,l \tag{2}$$

Where $\left(M_m, M_s^i\right)\in \mathbb{R}^{n\times n}$, $\left(C_m, C_s^i\right)\in \mathbb{R}^{n\times n}$, $\left(g_m, g_s^i\right)\in \mathbb{R}^{n\times 1}$, $\left(q_m, q_s^i\right)\in \mathbb{R}^{n\times 1}$, $\left(\dot{q}_m, \dot{q}_s^i\right)\in \mathbb{R}^{n\times 1}$, $\left(\ddot{q}_m, \ddot{q}_s^i\right)\in \mathbb{R}^{n\times 1}$, $\left(\tau_m, \tau_s^i\right)\in \mathbb{R}^{n\times 1}$ denote inertia matrices, coriolis/centrifugal matrices, gravity vectors, joint positions, joint velocities, joint accelerations, and input torques of master and slave units respectively. Operator's forces are assumed to be constant while environments are assumed to be passive in this study. It is also assumed that the environments can be modeled as spring-damper systems, i.e. $F_e^i = K_e^i q_s^i + B_e^i \dot{q}_s^i$ where $K_e^i, B_e^i \in \mathbb{R}^{n\times n}$ are positive definite diagonal matrices.

Now, the communication between the master and slave units is established through the use of extended state convergence architecture. This communication framework is shown in Figure 1 and is comprised of the following parameters:

$K_m = \begin{bmatrix} K_{m1} & K_{m2} \end{bmatrix} \in \mathbb{R}^{n\times 2n}$: This parameter is the stabilizing feedback gain matrix for the master unit. Each of its constituent members $\left(K_{m1}, K_{m2} \in \mathbb{R}^{n\times n}\right)$ is an unknown but negative definite diagonal matrix and will be found through Lyapunov-Krasovskii based design procedure.

$K_s^i = \begin{bmatrix} K_{s1}^i & K_{s2}^i \end{bmatrix} \in \mathbb{R}^{n\times 2n}$: This parameter is the stabilizing feedback gain matrix for the i^{th} slave unit. Each of its constituent members $\left(K_{s1}^i, K_{s2}^i \in \mathbb{R}^{n\times n}\right)$ is an unknown but negative definite diagonal matrix and will be found through Lyapunov-Krasovskii based design procedure.

$R_s^i = \begin{bmatrix} R_{s1}^i & R_{s2}^i \end{bmatrix} \in \mathbb{R}^{n\times 2n}$: This parameter models the influence of the master unit's motion onto the i^{th} slave unit's motion. Each of its constituent members $\left(R_{s1}^i, R_{s2}^i \in \mathbb{R}^{n\times n}\right)$ is an unknown but positive definite diagonal matrix and will be found through Lyapunov-Krasovskii based design procedure.

$R_m^i = \begin{bmatrix} R_{m1}^i & R_{m2}^i \end{bmatrix} \in \mathbb{R}^{n\times 2n}$: This parameter models the influence of the i^{th} slave unit's motion onto the master unit's motion. Each of its constituent members $\left(R_{m1}^i, R_{m2}^i \in \mathbb{R}^{n\times n}\right)$ is an unknown but positive diagonal matrix and will be found through Lyapunov-Krasovskii based design procedure.

$G_2^i \in \mathbb{R}^+$: This parameter models the influence of the operator's applied force onto the i^{th} slave unit.

$T_{mi}(t) \in \mathbb{R}^+$: This represents the time delay on the link which connects the i^{th} slave unit to the master unit. In this study, only the upper bounds on these time delays $\left(T_{mi}^+\right)$ are known.

$T_{si}(t) \in \mathbb{R}^+$: This represents the time delay on the link which connects the master unit to the i^{th} slave unit. In this study, only the upper bounds on these time delays $\left(T_{si}^+\right)$ are known.

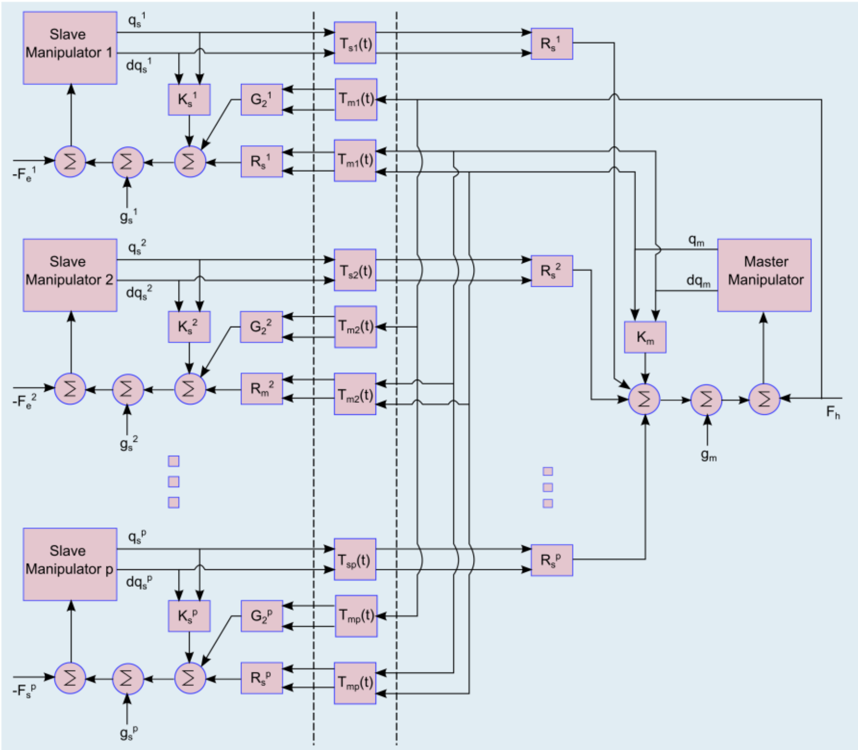


Figure 1
Single-master/multi-slave teleoperation system through state convergence

3 Premilinaries

3.1 Properties of Master/Slave Units

The master and slave units as modeled by (1),(2) possess the following properties:

(P1) The inertia matrices are symmetric, positive definite and bounded, i.e. there exist positive constants β_l and β_u such that $0 < \beta_l I < M(q) < \beta_u I \leq \infty$.

(P2) A skew-symmetric relation exists between the inertia and coriolis/centrifugal matrices such that $x^T\left(\dot{M}(q) - 2C\left(q, \dot{q}\right)\right)x = 0, \forall x \in \mathbb{R}^n$.

(P3) The coriolis/centrifugal force vectors are bounded i.e., there exists positive constant β_f such that $\left\| C\left(q, \dot{q}\right)\dot{q} \right\| \leq \beta_f \left\| \dot{q} \right\|$.

(P4) If the joint variables $\dot{q}$ and $\ddot{q}$ are bounded, then the time derivative of coriolis/centrifugal matrices is also bounded.

3.2 Lemmas

For any vectors $x, y \in \mathbb{R}^n$, positive definite diagonal matrix $F \in \mathbb{R}^{n\times n}$, scalar $\gamma > 0$ and variable time delay $T_i(t)$ having upper bound T_i^+, the following inequalities hold:

(L1) $-2\int_0^{t_f} \dot{x}^T F \int_0^{T_i(t)} \dot{y}(t-\sigma)d\sigma dt \leq \gamma \int_0^{t_f} \dot{x}^T Fxdt + \frac{T_i^{+2}}{\gamma}\int_0^{t_f} \dot{y}^T Fydt$

(L2) $x\left(t - T_i(t)\right) - x(t) = \int_0^{T_i(t)} \dot{x}(t-\sigma)d\sigma \leq T_i^{+\frac{1}{2}} \left\|\dot{x}\right\|_2$

4 Control Objectives

Besides establishing the stability, we intend to achieve the following objectives in SM/MS nonlinear teleoperation system:

Control Objective # 1: During the free motion, the joint positions of all the slave units should converge to the corresponding joint positions of the master unit in steady state i.e. $\lim_{t\to\infty}\left\|q_s^i(t) - q_m(t)\right\| = 0, \forall i = 1, 2, ..., l$

Control Objective # 2: During the contact motion, operator should feel a force proportional to the aggregated environmental forces, i.e. $F_h \propto \sum_{i=1}^{l} F_e^i$

5 Stability Analysis and Control Design

Consider the SM/MS teleoperation system of Fig. 1 with time varying delays in the communication channel. The control inputs for the master and slave units in this time-delayed teleoperation system are:

$$\tau_m = g_m\left(q_m\right) + K_{m1}q_m + K_{m2}\dot{q}_m + \sum_{i=1}^{l} R_{m1}^i q_s^i\left(t - T_{si}\left(t\right)\right) + \sum_{i=1}^{l} R_{m2}^i \dot{q}_s^i\left(t - T_{si}\left(t\right)\right) \tag{3}$$

$$\begin{aligned}\tau_s^i &= g_s^i\left(q_s^i\right) + K_{s1}^i q_s^i + K_{s2}^i \dot{q}_s^i + R_{s1}^i q_m\left(t - T_{mi}\left(t\right)\right) + R_{s2}^i \dot{q}_m\left(t - T_{mi}\left(t\right)\right) + \\ &\quad G_2^i F_h\left(t - T_{mi}\left(t\right)\right), \forall i = 1, 2, \ldots, l\end{aligned} \tag{4}$$

By plugging (3) in (1) and (4) in (2), we obtain the closed loop dynamics of the master and slave units as:

$$M_m \ddot{q}_m + C_m \dot{q}_m = K_{m1}q_m + K_{m2}\dot{q}_m + \sum_{i=1}^{l} R_{m1}^i q_s^i\left(t - T_{si}\left(t\right)\right) + \sum_{i=1}^{l} R_{m2}^i \dot{q}_s^i\left(t - T_{si}\left(t\right)\right) + F_h \tag{5}$$

$$\begin{aligned}M_s^i \ddot{q}_s^i + C_s^i \dot{q}_s^i &= K_{s1}^i q_s^i + K_{s2}^i \dot{q}_s^i + R_{s1}^i q_m\left(t - T_{mi}\left(t\right)\right) + R_{s2}^i \dot{q}_m\left(t - T_{mi}\left(t\right)\right) + \\ &\quad G_2^i F_h\left(t - T_{mi}\left(t\right)\right) - F_e^i, \forall i = 1, 2, \ldots, l\end{aligned} \tag{6}$$

In equilibrium points for master and slave units, we have:

$$\begin{aligned}&q_m = q_m\left(t - T_{mi}\left(t\right)\right) = \overline{q_m}, \dot{q}_m = \ddot{q}_m = 0 \\ &q_s^i = q_s^i\left(t - T_{si}\left(t\right)\right) = \overline{q_s^i}, \dot{q}_s^i = \ddot{q}_s^i = 0\end{aligned} \tag{7}$$

Considering the environmental models and evaluating (6), (7) at equilibrium, we have:

$$\begin{aligned}&0 = K_{m1}\overline{q_m} + \sum_{i=1}^{l} R_{m1}^i \overline{q_s^i} + F_h \\ &0 = K_{s1}^i \overline{q_s^i} + R_{s1}^i \overline{q_m} + G_2^i F_h\left(t - T_{mi}\left(t\right)\right) - K_e^i \overline{q_s^i}, \forall i = 1, 2, \ldots, l\end{aligned} \tag{8}$$

Let us now analyze the closed loop teleoperation system of (5), (6) in a new coordinate system formed by the variables q_m, q_s^i and their time delayed versions $q_m\left(t - T_{mi}\left(t\right)\right), q_s^i\left(t - T_{si}\left(t\right)\right)$ as defined below:

$$q_m = q_m - \overline{q_m} \tag{9}$$

$$q_m\left(t - T_{mi}\left(t\right)\right) = q_m\left(t - T_{mi}\left(t\right)\right) - \overline{q_m} \tag{10}$$

$$q_s^i = q_s^i - \overline{q_s^i} \tag{11}$$

$$q_s^i\left(t - T_{si}\left(t\right)\right) = q_s^i\left(t - T_{si}\left(t\right)\right) - \overline{q_s^i} \tag{12}$$

By using (9)-(12) with (5)-(8), we obtain the transformed teleoperation system as:

$$M_m \ddot{q}_m + C_m \dot{q}_m = K_{m1} q_m + K_{m2} \dot{q}_m + \sum_{i=1}^{l} R_{m1}^i q_s^i \left(t - T_{si}\left(t\right)\right) + \sum_{i=1}^{l} R_{m2}^i \dot{q}_s^i \left(t - T_{si}\left(t\right)\right) \quad (13)$$

$$\begin{aligned} M_s^i \ddot{q}_s^i + C_s^i \dot{q}_s^i = K_{s1}^i q_s^i + K_{s2}^i \dot{q}_s^i + R_{s1}^i q_m \left(t - T_{mi}\left(t\right)\right) + R_{s2}^i \dot{q}_m \left(t - T_{mi}\left(t\right)\right) - \\ K_e^i q_s^i - B_e^i \dot{q}_s^i, \forall i = 1, 2, ..., l \end{aligned} \quad (14)$$

Now we study the asymptotic stability and position coordination behavior of the time-delayed teleoperation system in Theorem 1 while the force reflection behavior is discussed in Theorem 2.

Theorem 1: The origin of the transformed time-delayed teleoperation system (13), (14) is asymptotically stable and the position coordination between the master and slave units is achieved in free motion when the control gains of (15), (16) are used and $l+1$ inequalities in (17), (18) are also satisfied.

$$\begin{aligned} &K_{m1} = -lK, K_{m2} = -\left(l+1\right) K_1 - \sum_{i=1}^{l} K_{md}^i \\ &K_{s1}^i = -K, K_{s2}^i = -2K_1 - K_{sd}^i, \forall i = 1, 2, ..., l \\ &R_{m1}^i = R_{s1}^i = K, R_{m2}^i = 2K_{md}^i, R_{s2}^i = 2K_{sd}^i, \forall i = 1, 2, ..., l \end{aligned} \quad (15)$$

$$K_{md}^i = \left(1 - \dot{T}_{sj}\left(t\right)\right) K_1, K_{sd}^i = \left(1 - \dot{T}_{mj}\left(t\right)\right) K_1 \quad (16)$$

$$K_1 - \frac{\gamma_{mj}}{2} K - \frac{T_{sj}^{+^2}}{2\gamma_{sj}} K > 0, \forall i = 1, 2, ..., l \quad (17)$$

$$K_1 - \sum_{i=1}^{l} \frac{\gamma_{sj}}{2} K - \sum_{i=1}^{l} \frac{T_{mj}^{+^2}}{2\gamma_{mj}} K > 0 \quad (18)$$

Where, γ_{sj}, γ_{mj} are positive constants, $K, K_1 \in \mathbb{R}^{n \times n}$ are positive definite diagonal matrices, $\dot{T}_{sj}, \dot{T}_{mj}$ are the time derivatives of communication delays which are assumed to be less than unity. Therefore, $K_{sd}^i, K_{md}^i \in \mathbb{R}^{n \times n}$ are also positive definite diagonal matrices.

Proof: Consider the following Lyapunov-Krasovskii functional candidate:

$$V\left(\dot{q}_m,\dot{q}_s^i,q_s^i-q_m,q_s^i\right)=\frac{1}{2}\dot{q}_m^{T}M_m\dot{q}_m+\frac{1}{2}\sum_{i=1}^{l}\dot{q}_s^{iT}M_s^i\dot{q}_s^i+$$
$$\frac{1}{2}\sum_{i=1}^{l}\left(q_s^i-q_m\right)^{T}K\left(q_s^i-q_m\right)+\sum_{i=1}^{l}\int_{t-T_{mj}(t)}^{t}\dot{q}_m^{T}(\eta)K_1\dot{q}_m(\eta)\,d\eta+ \tag{19}$$
$$\sum_{i=1}^{l}\int_{t-T_{sj}(t)}^{t}\dot{q}_s^{iT}(\eta)K_1\dot{q}_s^i(\eta)\,d\eta+\frac{1}{2}\sum_{i=1}^{l}q_s^{iT}K_e^iq_s^i$$

By taking the time derivative of (19) along the trajectories of the teleoperation system (13), (14) and using the property P2 of the master and slave units, we obtain:

$$\dot{V}=\dot{q}_m^{T}\left(K_{m1}q_m+\sum_{i=1}^{l}R_{m1}^iq_s^i\left(t-T_{si}(t)\right)+K_{m2}\dot{q}_m+\sum_{i=1}^{l}R_{m2}^i\dot{q}_s^i\left(t-T_{si}(t)\right)\right)+$$
$$\sum_{i=1}^{l}\dot{q}_s^{iT}\left(K_{s1}^iq_s^i+K_{s2}^i\dot{q}_s^i+R_{s1}^iq_m\left(t-T_{mi}(t)\right)+R_{s2}^i\dot{q}_m\left(t-T_{mi}(t)\right)-K_e^iq_s^i-B_e^i\dot{q}_s^i\right)+$$
$$\sum_{i=1}^{l}\left(\dot{q}_s^{iT}Kq_s^i-\dot{q}_s^{iT}Kq_m-\dot{q}_m^{T}Kq_s^i+\dot{q}_m^{T}Kq_m+\dot{q}_s^{iT}K_e^iq_s^i\right)+ \tag{20}$$
$$\sum_{i=1}^{l}\left(\dot{q}_m^{T}K_1\dot{q}_m-\dot{q}_m^{T}\left(t-T_{mi}(t)\right)\left(1-\dot{T}_{mi}(t)\right)K_1\dot{q}_m\left(t-T_{mi}(t)\right)\right)+$$
$$\sum_{i=1}^{l}\left(\dot{q}_s^{iT}K_1\dot{q}_s^i-\dot{q}_s^{iT}\left(t-T_{si}(t)\right)\left(1-\dot{T}_{si}(t)\right)K_1\dot{q}_s^i\left(t-T_{si}(t)\right)\right)$$

By grouping the terms in (20) and using the definition of the time varying matrices (16), we have:

$$\dot{V}=\dot{q}_m^{T}\left(K_{m1}+lK\right)q_m+\sum_{i=1}^{l}\dot{q}_m^{T}\left(R_{m1}^iq_s^i\left(t-T_{si}(t)\right)-Kq_s^i\right)+$$
$$\sum_{i=1}^{l}\dot{q}_s^{iT}\left(K_{s1}^i+K\right)q_s^i+\sum_{i=1}^{l}\dot{q}_s^{iT}\left(R_{s1}^iq_m\left(t-T_{mi}(t)\right)-Kq_m\right)+$$
$$\sum_{i=1}^{l}\left(\dot{q}_m^{T}\left(\frac{K_{m2}}{l}+K_1\right)\dot{q}_m+\dot{q}_m^{T}R_{m2}^i\dot{q}_s^i\left(t-T_{si}(t)\right)-\dot{q}_s^{iT}\left(t-T_{si}(t)\right)K_{md}^i\dot{q}_s^i\left(t-T_{si}(t)\right)\right)+$$
$$\sum_{i=1}^{l}\left(\dot{q}_s^{iT}\left(K_{s2}^i+K_1-B_e^i\right)\dot{q}_s^i+\dot{q}_s^{iT}R_{s2}^i\dot{q}_m\left(t-T_{mi}(t)\right)-\dot{q}_m^{T}\left(t-T_{mi}(t)\right)K_{sd}^i\dot{q}_m\left(t-T_{mi}(t)\right)\right) \tag{21}$$

Let us now define the following position error signals:

$$\begin{aligned} e_{q_m^i} &= q_m - q_s^i\left(t - T_{si}\left(t\right)\right) \\ e_{q_s^i} &= q_s^i - q_m\left(t - T_{mi}\left(t\right)\right) \end{aligned} \tag{22}$$

By substituting the control gains of (15) in (21) and using the time derivative of (22) in the resulting expression, we obtain:

$$\begin{aligned} \dot{V} = {} & \sum_{i=1}^{l} \dot{q}_m^{T} K\left(q_s^i\left(t - T_{si}\left(t\right)\right) - q_s^i\right) - \sum_{i=1}^{l} \dot{e}_{q_m^i}^{T} K_{md}^i \dot{e}_{q_m^i} + \sum_{i=1}^{l} \dot{q}_s^{i\,T} K\left(q_m\left(t - T_{mi}\left(t\right)\right) - q_m\right) - \\ & \sum_{i=1}^{l} \dot{e}_{q_s^i}^{T} K_{sd}^i \dot{e}_{q_s^i} - \dot{q}_m^{T} K_1 \dot{q}_m - \sum_{i=1}^{l} \dot{q}_s^{i\,T} K_1 \dot{q}_s^i - \sum_{i=1}^{l} \dot{q}_s^{i\,T} B_e^i \dot{q}_s^i \end{aligned} \tag{23}$$

By integrating (23) over the time interval $\left[0, t_f\right]$, rewriting first and third terms in integral form and finally using lemma L1, we have:

$$\begin{aligned} \int_0^{t_f} \dot{V}\, d\eta \le {} & \sum_{i=1}^{l} \left(\frac{\gamma_{si}}{2} \int_0^{t_f} \dot{q}_m^{T} K \dot{q}_m\, d\eta + \frac{T_{si}^{+2}}{2\gamma_{si}} \int_0^{t_f} \dot{q}_s^{i\,T} K \dot{q}_s^i\, d\eta \right) + \\ & \sum_{i=1}^{l} \left(\frac{\gamma_{mi}}{2} \int_0^{t_f} \dot{q}_s^{i\,T} K \dot{q}_s^i\, d\eta + \frac{T_{mi}^{+2}}{2\gamma_{mi}} \int_0^{t_f} \dot{q}_m^{T} K \dot{q}_m\, d\eta \right) - \\ & \sum_{i=1}^{l} \int_0^{t_f} \dot{e}_{q_m^i}^{T} K_{md}^i \dot{e}_{q_m^i}\, d\eta - \sum_{i=1}^{l} \int_0^{t_f} \dot{e}_{q_s^i}^{T} K_{sd}^i \dot{e}_{q_s^i}\, d\eta - \\ & \int_0^{t_f} \dot{q}_m^{T} K_1 \dot{q}_m\, d\eta - \sum_{i=1}^{l} \int_0^{t_f} \dot{q}_s^{i\,T} K_1 \dot{q}_s^i\, d\eta - \sum_{i=1}^{l} \int_0^{t_f} \dot{q}_s^{i\,T} B_e^i \dot{q}_s^i\, d\eta \end{aligned} \tag{24}$$

The simplification of (24) leads to:

$$\begin{aligned} \int_0^{t_f} \dot{V}\, d\eta \le {} & -\int_0^{t_f} \dot{q}_m^{T} \left(K_1 - \sum_{i=1}^{l} \frac{\gamma_{si}}{2} K - \sum_{i=1}^{l} \frac{T_{mi}^{+2}}{2\gamma_{mi}} K \right) \dot{q}_m\, d\eta - \\ & \sum_{i=1}^{l} \int_0^{t_f} \dot{q}_s^{i\,T} \left(K_1 - \frac{\gamma_{mi}}{2} K - \frac{T_{si}^{+2}}{2\gamma_{si}} K \right) \dot{q}_s^i\, d\eta - \\ & \sum_{i=1}^{l} \int_0^{t_f} \dot{e}_{q_m^i}^{T} K_{md}^i \dot{e}_{q_m^i}\, d\eta - \sum_{i=1}^{l} \int_0^{t_f} \dot{e}_{q_s^i}^{T} K_{sd}^i \dot{e}_{q_s^i}\, d\eta - \sum_{i=1}^{l} \int_0^{t_f} \dot{q}_s^{i\,T} B_e^i \dot{q}_s^i\, d\eta \end{aligned} \tag{25}$$

$$
\begin{aligned}
V\left(t_f\right)-V\left(0\right) \le & -\mu\left(K_1-\sum_{i=1}^{l}\frac{\gamma_{si}}{2}K-\sum_{i=1}^{l}\frac{T_{mi}^{+^2}}{2\gamma_{mi}}K\right)\left\|\dot{q}_m\right\|_2^2 - \\
& \sum_{i=1}^{l}\mu\left(K_1-\frac{\gamma_{mi}}{2}K-\frac{T_{si}^{+^2}}{2\gamma_{si}}K\right)\left\|\dot{q}_s^i\right\|_2^2 - \\
& \sum_{i=1}^{l}\mu\left(K_{md}^i\right)\left\|\dot{e}_{q_m^i}\right\|_2^2-\sum_{i=1}^{l}\mu\left(K_{sd}^i\right)\left\|\dot{e}_{q_s^i}\right\|_2^2-\sum_{i=1}^{l}\mu\left(B_e^i\right)\left\|\dot{q}_s^i\right\|_2^2
\end{aligned}
\tag{26}
$$

Where $\mu\left(X\right)$ denotes the minimal Eigen value of X and the notation $\left\|x\left(t\right)\right\|_2$ represents the L_2 norm of the signal $x\left(t\right)$ in the time interval $\left[0,t_f\right]$. Now, if the inequalities in (17), (18) are satisfied and the time derivative of the communication delays remains less than unity, then the right hand side of (26) remains negative. Taking the limit as $t_f \to \infty$, it can be concluded that the signals $\left\{\dot{q}_m,\dot{q}_s^i,q_s^i-q_m,q_s^i\right\}\in L_\infty$ and $\left\{\dot{q}_m,\dot{q}_s^i,\dot{e}_{q_m^i},\dot{e}_{q_s^i}\right\}\in L_2$. The boundedness of the signals $\left\{q_s^i-q_m,q_s^i\right\}$ implies that q_m is also bounded and therefore $q_m \in L_\infty$. Now, we study the boundedness of the signals $\left\{\ddot{q}_m,\ddot{q}_s^i\right\}$. Towards this end, we rewrite (13) and (14) as:

$$
\ddot{q}_m=-\left(M_m\right)^{-1}\left[C_m\dot{q}_m-K_{m1}q_m-\sum_{i=1}^{l}R_{m1}^i q_s^i\left(t-T_{si}\left(t\right)\right)-K_{m2}\dot{q}_m-\sum_{i=1}^{l}R_{m2}^i\dot{q}_s^i\left(t-T_{si}\left(t\right)\right)\right]
\tag{27}
$$

$$
\ddot{q}_s^i=-\left(M_s^i\right)^{-1}\left[\begin{array}{l} C_s^i\dot{q}_s^i-K_{s1}^i q_s^i-R_{s1}^i q_m\left(t-T_{mi}\left(t\right)\right)+K_e^i q_s^i-K_{s2}^i\dot{q}_s^i- \\ R_{s2}^i\dot{q}_m\left(t-T_{mi}\left(t\right)\right)+B_e^i\dot{q}_s^i \end{array}\right]
\tag{28}
$$

In (27) and (28), boundedness of the signals, $q_m-q_s^i\left(t-T_{si}\left(t\right)\right),q_s^i-q_m\left(t-T_{mi}\left(t\right)\right)$ needs to be established in order to draw conclusions on the boundedness of the perturbed acceleration signals. These position error signals can be written as:

$$
q_m-q_s^i\left(t-T_{si}\left(t\right)\right)=\overbrace{q_m-q_s^i}^{1}+\overbrace{q_s^i-q_s^i\left(t-T_{si}\left(t\right)\right)}^{2}
\tag{29}
$$

$$q_s^i - q_m\left(t - T_{mi}\left(t\right)\right) = \overbrace{q_s^i - q_m}^{1} + \overbrace{q_m - q_m\left(t - T_{mi}\left(t\right)\right)}^{2} \tag{30}$$

The first terms in (29), (30) have already been shown to be bounded. The second terms in these relations can be re-written using lemma L2 as:

$$\begin{aligned} q_s^i - q_s^i\left(t - T_{si}\left(t\right)\right) &= \int_0^{T_{si}(t)} \dot{q}_s^i\left(t-\sigma\right) d\sigma \le T_{si}^{+\frac{1}{2}} \left\|\dot{q}_s^i\right\|_2 \\ q_m - q_m\left(t - T_{mi}\left(t\right)\right) &= \int_0^{T_{mi}(t)} \dot{q}_m\left(t-\sigma\right) d\sigma \le T_{mi}^{+\frac{1}{2}} \left\|\dot{q}_m\right\|_2 \end{aligned} \tag{31}$$

It can now be concluded from (31) that the signals $\left\{q_m - q_s^i\left(t - T_{si}\left(t\right)\right), q_s^i - q_m\left(t - T_{mi}\left(t\right)\right)\right\} \in L_\infty$. Using the properties P1, P3 of the manipulators and the combined result, $\left\{\dot{q}_m, \dot{q}_s^i, q_s^i - q_m, q_s^i, q_m - q_s^i\left(t - T_{si}\left(t\right)\right), q_s^i - q_m\left(t - T_{mi}\left(t\right)\right)\right\} \in L_\infty$, it is established that the perturbed acceleration signals of master and slave units are bounded, i.e. $\left\{\ddot{q}_m, \ddot{q}_s^i\right\} \in L_\infty$. By Barbalat's lemma, this boundedness of the transformed acceleration signals in conjunction with the result $\left\{\dot{q}_m, \dot{q}_s^i\right\} \in L_2$ leads to the zero convergence of the perturbed velocity signals, i.e. $\lim_{t\to\infty} \dot{q}_m = \lim_{t\to\infty} \dot{q}_s^i = \lim_{t\to\infty} \dot{e}_{q_m^i} = \lim_{t\to\infty} \dot{e}_{q_s^i} = 0$. Next, we analyze the time derivative of (27) and (28):

$$\begin{aligned} \dddot{q}_m = &-\frac{d}{dt}\left(M_m\right)^{-1} \begin{bmatrix} C_m \dot{q}_m - K_{m1} q_m - \sum_{i=1}^{l} R_{m1}^i q_s^i\left(t - T_{si}\left(t\right)\right) \\ -K_{m2} \dot{q}_m - \sum_{i=1}^{l} R_{m2}^i \dot{q}_s^i\left(t - T_{si}\left(t\right)\right) \end{bmatrix} \\ &-\left(M_m\right)^{-1} \frac{d}{dt} \begin{bmatrix} C_m \dot{q}_m - K_{m1} q_m - \sum_{i=1}^{l} R_{m1}^i q_s^i\left(t - T_{si}\left(t\right)\right) \\ -K_{m2} \dot{q}_m - \sum_{i=1}^{l} R_{m2}^i \dot{q}_s^i\left(t - T_{si}\left(t\right)\right) \end{bmatrix} \end{aligned} \tag{32}$$

$$\begin{aligned}\dddot{q}_s^i &= -\frac{d}{dt}\left(M_s^i\right)^{-1}\begin{bmatrix} C_s^i\,\dot{q}_s^i - K_{s1}^i q_s^i - R_{s1}^i q_m\left(t-T_{mi}\left(t\right)\right) + K_e^i q_s^i \\ -K_{s2}^i\,\dot{q}_s^i - R_{s2}^i\,\dot{q}_m\left(t-T_{mi}\left(t\right)\right) + B_e^i\,\dot{q}_s^i \end{bmatrix} \\ &\quad -\left(M_s^i\right)^{-1}\frac{d}{dt}\begin{bmatrix} C_s^i\,\dot{q}_s^i - K_{s1}^i q_s^i - R_{s1}^i q_m\left(t-T_{mi}\left(t\right)\right) + K_e^i q_s^i \\ -K_{s2}^i\,\dot{q}_s^i - R_{s2}^i\,\dot{q}_m\left(t-T_{mi}\left(t\right)\right) + B_e^i\,\dot{q}_s^i \end{bmatrix}\end{aligned} \tag{33}$$

The derivative terms involving inertia matrices in (32), (33) are computed as:

$$\begin{aligned}\frac{d}{dt}\left(M_m\right)^{-1} &= -\left(M_m\right)^{-1}\left(C_m + C_m^{\,T}\right)\left(M_m\right) \\ \frac{d}{dt}\left(M_s^i\right)^{-1} &= -\left(M_s^i\right)^{-1}\left(C_s^i + C_s^{iT}\right)\left(M_s^i\right)\end{aligned} \tag{34}$$

The properties P1 and P3 of the master and slave units along with the earlier result $\left\{q_m, q_s^i, \dot{q}_m, \dot{q}_s^i\right\} \in L_\infty$ dictate the boundedness of the derivative terms in (34). The remaining derivative terms in (32), (33) also turn out to be bounded following the application of properties P1, P3, P4 and the earlier result:

$\left\{\begin{matrix} \dot{q}_m, \dot{q}_s^i, q_s^i - q_m, q_s^i, q_m - q_s^i\left(t-T_{si}\left(t\right)\right), \\ q_s^i - q_m\left(t-T_{mi}\left(t\right)\right), \ddot{q}_m, \ddot{q}_s^i \end{matrix}\right\} \in L_\infty$. Since all the terms on right hand sides of (32), (33) are bounded, we have $\left\{\dddot{q}_m, \dddot{q}_s^i\right\} \in L_\infty$. By using the results, $\lim_{t\to\infty}\dot{q}_m = \lim_{t\to\infty}\dot{q}_s^i = 0$ and $\left\{\dddot{q}_m, \dddot{q}_s^i\right\} \in L_\infty$, it can be concluded that $\lim_{t\to\infty}\ddot{q}_m = \lim_{t\to\infty}\ddot{q}_s^i = 0$.

With the zero convergence of perturbed velocity and acceleration signals, the closed loop teleoperation system of (13), (14) in combination with (15) becomes:

$$\lim_{t\to\infty}\sum_{i=1}^{l}\left\|q_m - q_s^i\left(t-T_{si}\left(t\right)\right)\right\| = 0 \tag{35}$$

$$\lim_{t\to\infty}\left\|q_s^i - q_m\left(t-T_{mi}\left(t\right)\right)\right\| = -K^{-1}K_e^i q_s^i \tag{36}$$

The time delay terms in (35) and (36) can be written as:

$$
\begin{aligned}
q_s^i\left(t-T_{si}\left(t\right)\right) &= q_s^i - \int_{t-T_{si}(t)}^{t} \dot{q}_s^i \, d\eta \\
q_m\left(t-T_{mi}\left(t\right)\right) &= q_m - \int_{t-T_{mi}(t)}^{t} \dot{q}_m \, d\eta
\end{aligned} \tag{37}
$$

Since $\lim_{t\to\infty} \dot{q}_m = \lim_{t\to\infty} \dot{q}_s^i = 0$, then the integral terms in (37) disappear. By using this result in (35), (36) and considering the free motion behavior of the teleoperation system, it can be concluded that the perturbations in joint position errors converge to zero, i.e. $\lim_{t\to\infty} q_m = \lim_{t\to\infty} q_s^i = 0$. Thus the origin of the transformed teleoperation system $\left\{\dot{q}_m, \dot{q}_s^i, q_m, q_s^i\right\}$ is asymptotically stable. This further implies that $\lim_{t\to\infty} q_m = \overline{q_m}, \lim_{t\to\infty} q_s^i = \overline{q_s^i}$. By using these results in the original time-delayed teleoperation system (5), (6), it is found that the position error between the master and slave units is vanished in the absence of operator and environmental forces and the control objective #1 is achieved□

Remark 1: In case of SM/MS teleoperation system with time varying delays in the communication channel, the control gains for the joint velocities of master and slave units depend on the derivative of time delays as can be seen from (16). These gains are unrealizable since no information about the trajectories of time delays is available except for their upper bounds. In order to overcome this limitation, we transmit extra ramp signals across the communication channel and their time derivatives are computed to realize the velocity control gains as:

$$
\begin{aligned}
K_{md}^i &= \dot{r}\left(t-T_{sj}\left(t\right)\right)K_1, \forall i = 1,2,...,l \\
K_{sd}^i &= \dot{r}\left(t-T_{mj}\left(t\right)\right)K_1, \forall i = 1,2,...,l
\end{aligned} \tag{38}
$$

Theorem 2: During the contact motion of the teleoperation system under the control gains of (15), static force is reflected to the operator which is proportional to the aggregated environmental force.

Proof: Consider the steady state behavior of the teleoperation system (1), (2) in the presence of operator and environmental forces. By plugging the control gains (15) in (5), (6), we have:

$$
F_h = K\sum_{i=1}^{l}\left(\overline{q_m} - \overline{q_s^i}\right) \tag{39}
$$

$$
F_e^i = K\left(\overline{q_m} - \overline{q_s^i}\right) + G_2^i F_h \tag{40}
$$

By taking the summation ($i=1$ to $i=l$) on both sides of (40) and using (39), we obtain:

$$F_h = \frac{\sum_{i=1}^{l} F_e^i}{1+\sum_{i=1}^{l} G_2^i} \tag{41}$$

It can be seen from (41) that the operator experiences a static force which is a scaled version of the aggregated environmental force. If all the slave units experience the same force while working in their environments and the force transmission coefficients $\left(G_2^i\right)$ are all unity, then $F_h = \frac{l}{l+1} F_e \approx F_e$ for large l . This completes the proof.

Remark 2: By setting l as unity in (41), the earlier state convergence based nonlinear bilateral controller [32] becomes a special case of the proposed multilateral controller.

6 Simulation Results

In order to validate the proposed scheme, a single-master/tri-slave nonlinear teleoperation system is setup in MATLAB/Simulink environment. The master and slave units forming this teleoperation system are all two link manipulators with two degrees-of-freedom motion. Their dynamical system representation is given by (1), (2) with the following matrix/vector entries:

$$M(q) = \begin{bmatrix} 3ml^2 + 2ml^2\cos(q_2) & ml^2 + ml^2\cos(q_2) \\ ml^2 + ml^2\cos(q_2) & ml^2 \end{bmatrix} \tag{42}$$

$$C\left(q,\dot{q}\right) = \begin{bmatrix} -\dot{q}_2\, ml^2\sin(q_2) & -\left(\dot{q}_1 + \dot{q}_2\right) ml^2\sin(q_2) \\ \dot{q}_1\, ml^2\sin(q_2) & 0 \end{bmatrix} \tag{43}$$

$$g(q) = \begin{bmatrix} a_g ml\sin(q_1+q_2) + 2a_g ml\sin(q_1) \\ a_g ml\sin(q_1+q_2) \end{bmatrix} \tag{44}$$

Where, $m_1 = m_2 = m$ denotes the mass of the links, $l_1 = l_2 = l$ denotes the link lengths and $a_g = 9.8ms^{-2}$ is the acceleration due to gravity. The numerical values of these parameters are chosen as $\{m_m = 2.0, l_m = 1.0\}$, $\{m_{s1} = 10.0, l_{s1} = 1.5\}$,

$\{m_{s2} = 5.0, l_{s2} = 2.0\}$ and $\{m_{s3} = 8.0, l_{s3} = 2.5\}$. The other parameters of the teleoperation system are the stiffness and damping of the remote environments which are selected as $\{K_e^1 = diag(100,100), B_e^1 = diag(20,20)\}$, $\{K_e^2 = diag(50,50), B_e^2 = diag(10,10)\}$ and $\{K_e^3 = diag(20,20), B_e^1 = diag(5,5)\}$.

Let us now consider that the time varying delays exist in the communication channel between the master and slave units as shown in Figure 2. We assume that the upper bound on these time delays is 0.8. We further assume that all the gamma constants are unity. The inequalities in (17), (18) are then solved which leads to the selection of decisive matrices as $K = diag(20,10)$ and $K_1 = diag(60,30)$. With the knowledge of these matrices, control gains of the teleoperation system are found to be:

$$\begin{aligned}
K_{m1} &= \begin{bmatrix} -60 & 0 \\ 0 & -30 \end{bmatrix}, K_{m2} = \begin{bmatrix} -80 & 0 \\ 0 & -40 \end{bmatrix} - \sum_{i=1}^{3} r\left(t - T_{si}(t)\right) \times \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix} \\
K_{s1}^i &= \begin{bmatrix} -20 & 0 \\ 0 & -10 \end{bmatrix}, K_{s2}^i = \begin{bmatrix} -40 & 0 \\ 0 & -20 \end{bmatrix} - r\left(t - T_{mi}(t)\right) \times \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix}, \forall i = 1,2,3 \\
R_{m1}^i &= \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix}, R_{m2}^i = 2r\left(t - T_{si}(t)\right) \times \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix}, \forall i = 1,2,3 \\
R_{s1}^i &= \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix}, R_{s2}^i = 2r\left(t - T_{mi}(t)\right) \times \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix}, \forall i = 1,2,3
\end{aligned} \tag{45}$$

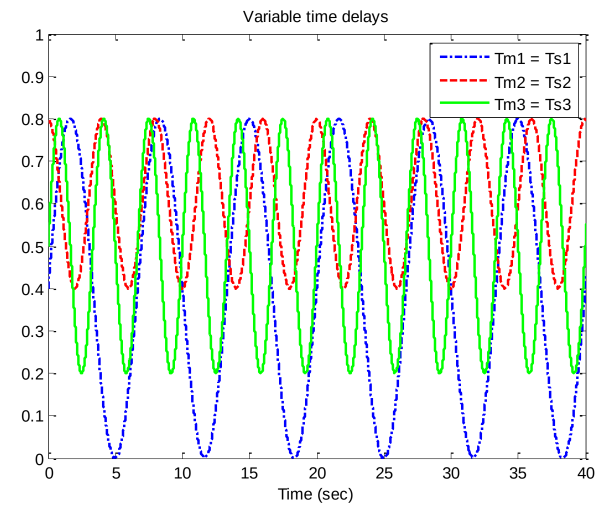


Figure 2
Time delays between master and slave units

We first analyze the time delayed teleoperation system in free motion when the operator applies a constant force as shown in Figure 3. The resultant joint positions of the master and slave units are shown in Figures 4-6. It can be seen that the slave units are following the master unit in the presence of time varying delays and the joint positions converge when the applied force becomes zero. This shows that the free motion behavior of the teleoperation system is stable.

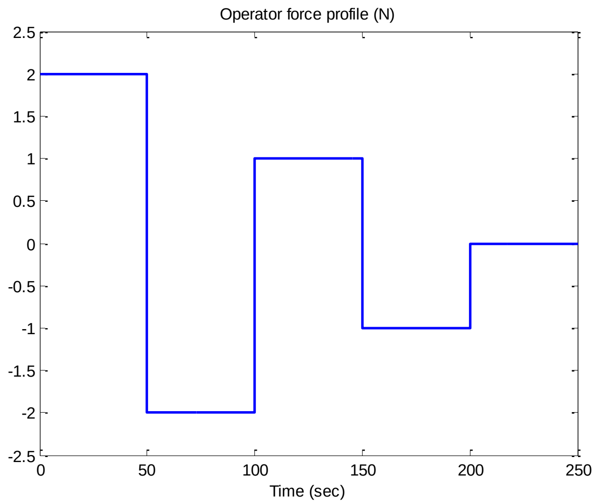
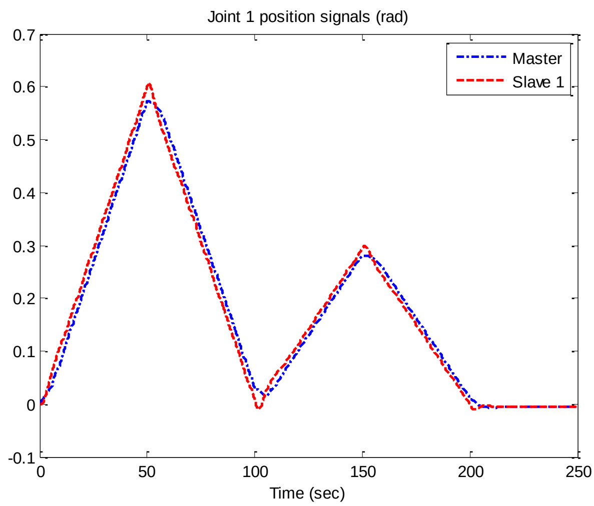
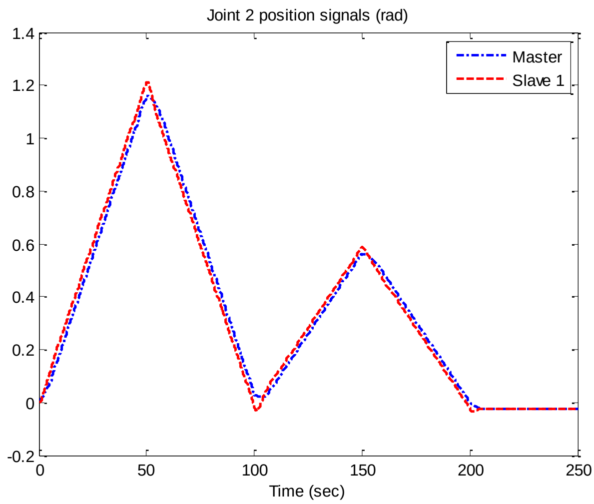


Figure 3
Operator's force profile

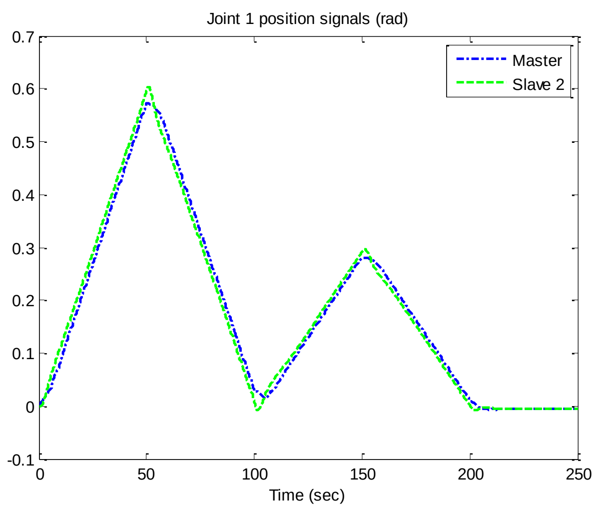


(a)

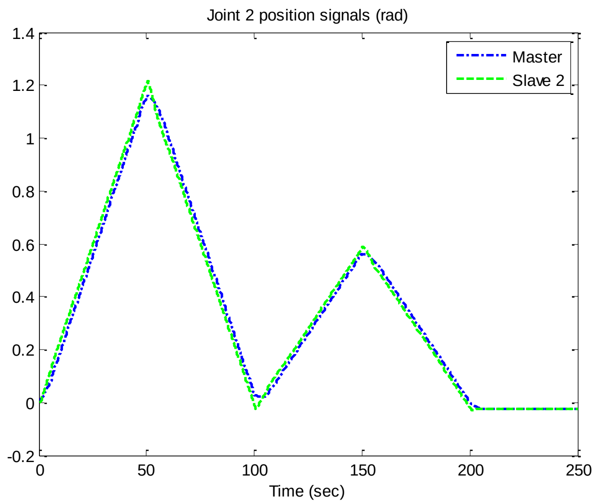


(b)

Figure 4
Free motion of Slave # 1 (a) Joint 1 position (b) Joint 2 position



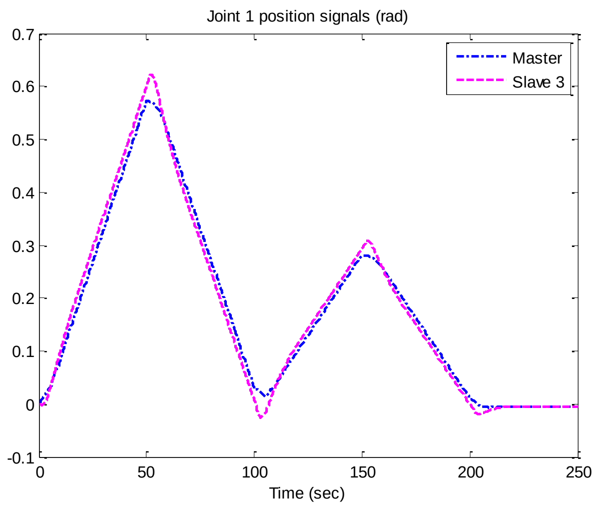
(a)



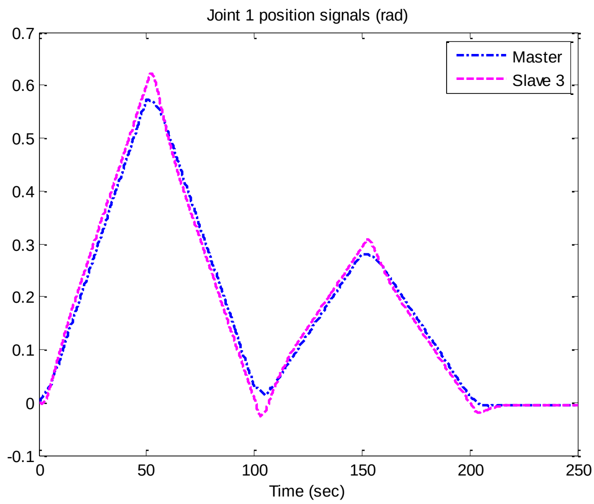
(b)

Figure 5

Free motion of Slave # 2 (a) Joint 1 position (b) Joint 2 position



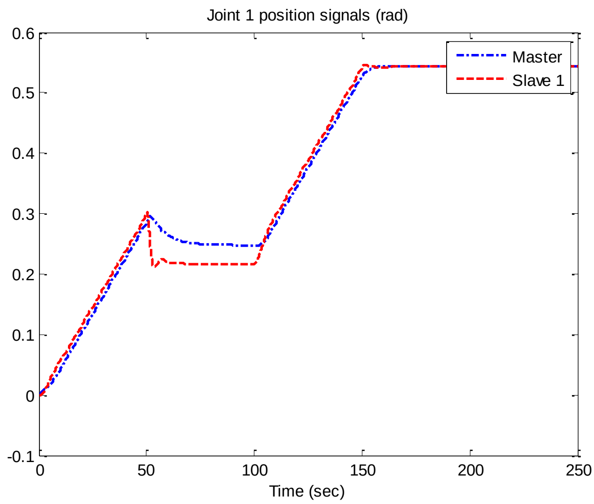
(a)



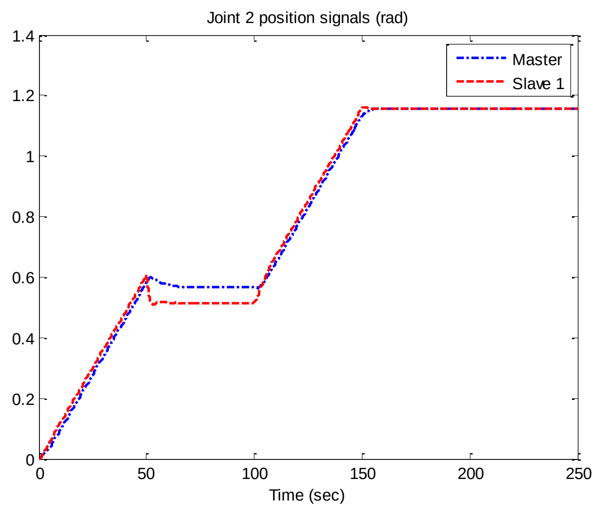
(b)

Figure 6
Free motion of Slave # 3 (a) Joint 1 position (b) Joint 2 position

We now consider the contact motion of the slaves when the operator exerts a constant force of 1 N that lasts for 150 s. The contact motion of all the slave units starts at $t = 50s$ and ends at $t = 100s$. The resultant position trajectories of the master and slave units are depicted in Figures 7-9. It is evident that the joint positions of the slave units are tracking the corresponding joint positions of the master unit and all the position signals remain bounded which implies that the teleoperation system is stable during both the free and contact motion cases. The force reflection ability of the time delayed teleoperation system is also analyzed. Theoretical result of (41) indicates that the static force reflection should be $0.25 \times \left(F_e^1 + F_e^2 + F_e^3\right)$, which is confirmed through simulation results on force reflection as shown in Figure 10.



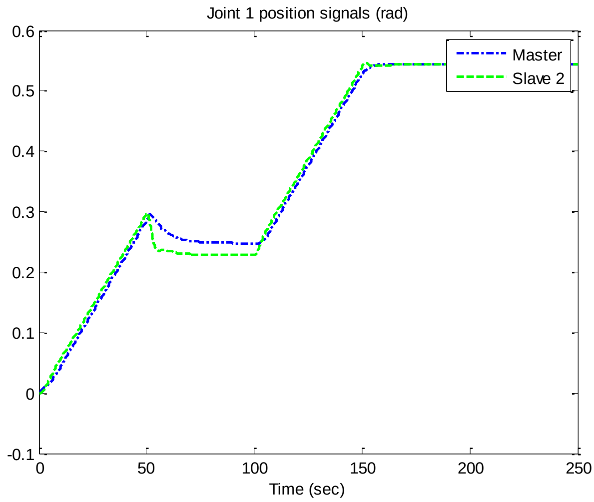
(a)



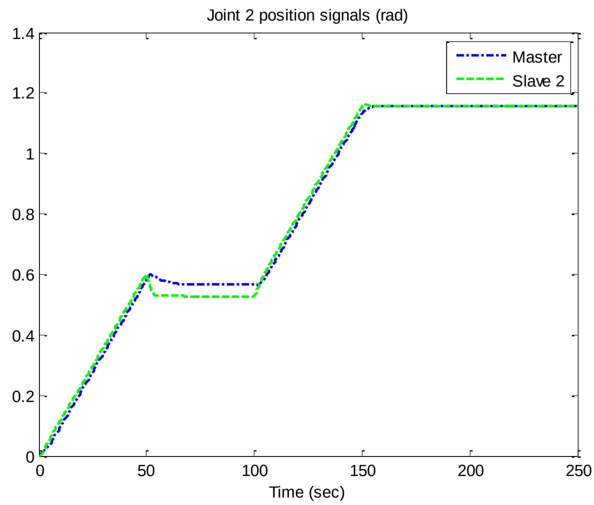
(b)

Figure 7

Free plus contact motion of Slave # 1 (a) Joint 1 position (b) Joint 2 position

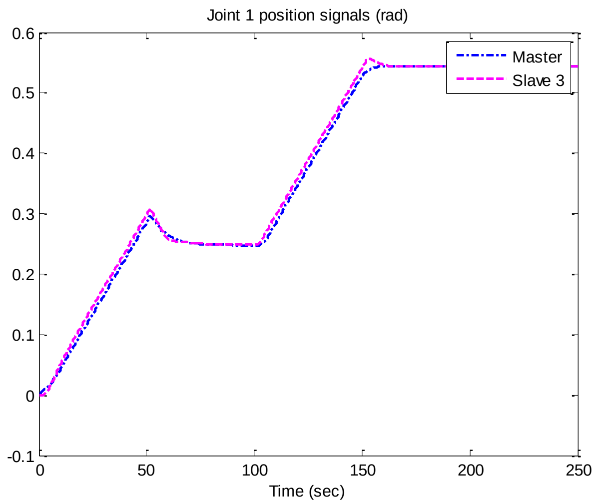


(a)

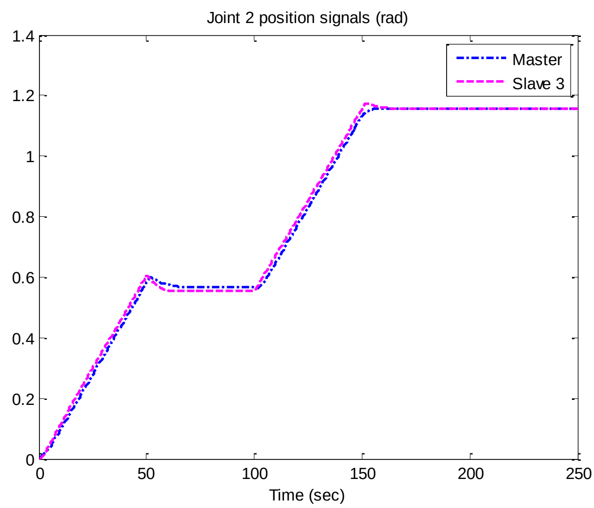


(b)

Figure 8
Free plus contact motion of Slave # 2 (a) Joint 1 position (b) Joint 2 position



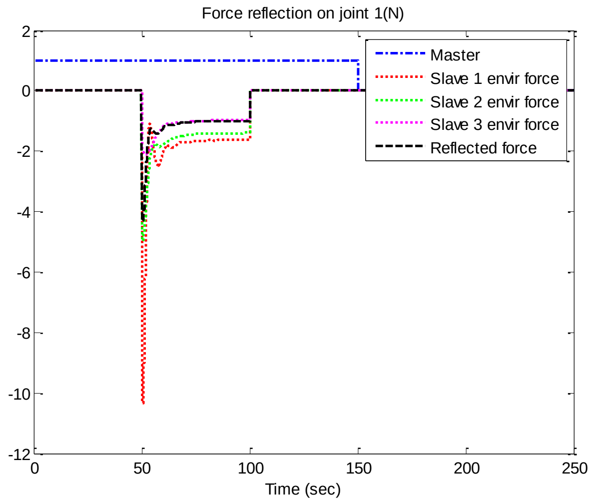
(a)



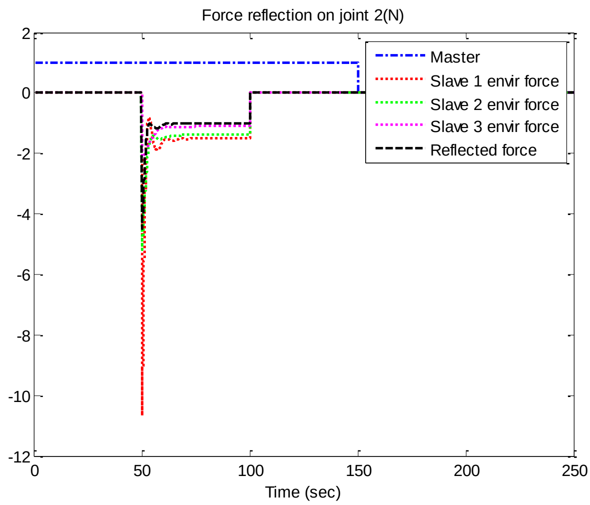
(b)

Figure 9

Free plus contact motion of Slave # 3 (a) Joint 1 position (b) Joint 2 position



(a)



(b)

Figure 10
Free plus contact motion of Slave # 3 (a) Joint 1 position (b) Joint 2 position

Conclusions

The design of a SM/MS nonlinear teleoperation system is presented in this paper. The proposed design builds upon our earlier work on the multilateral linear teleoperation systems, but considers the nonlinear dynamics of the master and slave units as well as the asymmetric time delays of the communication channel. With the help of Lyapunov-Krasovskii control theory, it is shown that the origin of the teleoperation system is asymptotically stable and the slave units track the position commands of the master unit. It is also shown that the proposed teleoperation system possess force reflection ability. To validate the theoretical findings, MATLAB simulations are performed on a single-master/tri-slave nonlinear teleoperation system where each master/slave unit has two DoF motion. It is found that all the control objectives including stability, position synchronization and force reflection are achieved. Future work involves the real time implementation of the proposed scheme.

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