Stability of the Robotic System with Time Delay in Open Kinematic Chain Configuration

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Abstract: In this article, stability of the robotic manipulator with time delay in open kinematic chain configuration was analyzed. The dynamic equations of motions were derived for one five-degree-of-freedom (DOF) robotic system with system latency. The mathematical model includes the model of the actuators to define the parameters of the actuators that can stabilize such a system. Investigation of the system stability was performed using novel stability conditions. The system state responses and the system stability were analyzed for different time delays. The proposed control methodology was shown to be appropriate to maintain the stability of the robotic system during tracking tasks. To analyze the concept, we presented a numerical example together with an extensive system simulation. The stability analysis showed the full compliance of the system behavior with the desired system dynamics. The proposed method can be used for the stability analysis of any robotic system with state delays in the open kinematic chain configuration.

Keywords: stability; stability conditions; robotic systems; time delay

1 Introduction

Time delay plays an important role in the dynamics of robotic systems in some applications. For instance, accurate tracking might be challenging if time delay exists. The fact is especially pronounced in the medical, even in some industrial, applications where high accuracy and positioning are strictly demanded. Furthermore, in repeatable motions, time delay might influence the phase shifting, and consequently, increases the errors. In some cases, the system instability might appear as an unwanted consequence of neglecting the time delay of systems.

The influence of time delay on robotic systems was previously analyzed in literature [2, 3, 6, 10, 17-20, 22, 24, 25, and 35]. Different types of latencies have been analyzed in conjunction with system stability, such as mechanical latency, signal processing (transmission) latency, communication latency etc. Signal transmission latency was shown to be able to affect the robotic effective force-reflecting system, [24]. A large group of teleoperation robotic systems is affected by time delay due to communication drifts. The overview of telemanipulators with constant transmission time delay and control challenges was presented in [25]. The instability of the systems can often be caused by time delay. Many control strategies have been reported to solve this problem [4, 6, 17-18, 22, 24-25, 30, 35]. A control strategy for a robotic system where instability was caused by time delay was proposed to overcome instability, [2]. An adaptive tracking controller was introduced to solve the instability problem. A study [17] showed the advantages of the compliant control over the force feedback control for one six-DOF robotic system within the wide range of time delay. The stability analysis for multiple manipulators capable of sensing latency was analyzed in [22]. Some robotic manipulators use video feedback [18], and the delay appears in the image processing module. In these situations, the discrete time modeling [6], adaptive motion and force control [35] can be used to overcome the suboptimal results in operations. In some cases, the existing time delay can be neglected in the analysis, as in [30]. However, a broader approach, such as the robust control, was used for tracking control. Consequently, the latency problem does not need to be analyzed separately; it should rather be analyzed within a more general set of uncertainties which acts on the systems [4].

The initial approach presented in [2, 3, 6, 17-20, 22, 24, 25, and 35] took the system delay into account, which potentially could destabilize the system and degrade the performances. The group of stability criteria that take time delay into account for investigations is named delay-dependent conditions [40]. Different control methodologies were developed based on the delay-dependent criteria. The latest research results on this topic are presented in the sequel. In [8, 9], robust tracking tasks for robotic manipulators were performed using a gradient estimator and an adaptive compensator, respectively. The system trajectory control i.e. tracking task in [27, 28] was performed using time delayed control which was proven to be robust against nonlinearities in the robotic dynamic system. Tracking of industrial robotic systems with time delay was analyzed in [12-14] from different aspects. The control methodology included the time delay estimation to decrease nonlinearities, velocity feedback, and sliding mode control to converge time delay errors. Another sliding mode controller together with the impedance control was used in [33] for position tracking. Uncertain disturbances and time delay can pose a problem in the modeling of robotic systems [11]; the linearization procedure and application of the linear matrix inequalities were found suitable in this case. A teleoperated mobile robot with latency was presented in [29] where the usage of a sensor was recommended as a solution for the fulfillment of the desired tasks, similar to [26].

An overview of the stability problems, when the time delay is present in the systems, is analyzed in [41]. Another theoretic approach to the asymptotic stability for robotic systems with time delay was proposed in [1]. It was noticed that the stability of systems with time delay is often related to complicated numerical calculations that can make the stability criteria inapplicable. The numerical calculations of the system stability under the influence of latency were analyzed in [42]. In some articles, this approach was solved using delay-independent criteria [40]. The method avoided using complicated computations of the inverse system dynamics; a time delay estimation was used to obtain the adequate dynamics and local disturbances. The trajectory tracking problem for the analyzed class of robotic systems was solved using a neural network controller, as described in [31].

In this article, it is of interest to analyze trajectory tracking problems. The article [21] analyzed the control of a space robotic system with time delay to track the desired trajectory in the inertial space with several uncontrolled variables, such as the position of the base and vertical coordinates. The nonlinear feedback control law was applied. A discrete time control of a mobile robot with transport latency was suggested in [32], instead of the continuous time control strategy. A tracking control algorithm for an industrial six-DOF robot was proposed in [23]. The maximum value of time delay was estimated to maintain the desired tracking performances. Some of the latest classical and new theoretical results that include the control of robots, application, servos, and actuators are presented in [34], [36-39].

In this article, we analyzed the stability of a five-DOF robotic system with time delay. An extensive computer simulation was presented for the evaluation of the system behavior. In order to be able to perform a high precision contour tracking, we modeled the system with latency. Moreover, it was requested that the system end-effector should be in the repeatable desired positions in the equidistant time interval. Consequently, latency in the mechanical part of the system or in signal processing can significantly influence the fulfillment of the desired tasks.

Due to the specified requirements, the innovative modeling procedure that includes the mathematical modeling of both the robotic systems and the actuators was derived. The time delay was incorporated in the generalized coordinates. The novel stability conditions were presented to investigate the stability of the robotic system. Furthermore, the calculation of the control gains was proposed in the article. This method can be used for the stabilization of this class of systems, irrespective of the number of joints within the manipulators, as long as they are in the open kinematic chain configuration. To evaluate the efficacy of the novel controller, we compared it to a classical proportional-integral-derivative (PID) controller and investigated the stability with respect to the time delay.

2 Mathematical Framework

The second section describes the mathematical modeling procedure for a robotic system with time delay, which is used for the simulation. The detailed modeling procedure for the system without latency and time delay stability conditions can be found in [5, 7].

2.1 Preliminaries

A general representation of the nonlinear control systems with time delay can be written as:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}\left(t, \mathbf{x}(t), \mathbf{x}(t-\tau), \mathbf{u}(t)\right), \quad t \geq 0 \\ \mathbf{x}(t) &= \boldsymbol{\varphi}(t), \ -\tau \leq t \leq 0 \end{aligned} \tag{1}$$

where $\mathbf{x}(t) \in \Re^n$ is a state-space vector, $\mathbf{u}(t) \in \Re^m$ is a control law vector, $\boldsymbol{\varphi} \in \aleph([-\tau, 0], \Re^n)$ is an admissible functional of the initial states, $\aleph = \aleph([-\tau, 0], \Re^n)$ is the continuous state-space function which maps interval $[-\tau, 0]$ to $\Re^n$, where $\Re$ is a real vector space. Vector function $\mathbf{f}$ satisfies the following condition:

$$\mathbf{f} : \Im \times \Re^n \times \Re^n \times \Re^m \to \Re^n . \tag{2}$$

Function $\mathbf{f}$ is assumed to be smooth to guarantee the existence and uniqueness of the solutions on time interval defines as

$$\Im = \left[t_0, \left(t_0 + \tau\right) \right[\in \Re_+ . \tag{3}$$

Quantity τ can be a positive real number or $+\infty$. The initial state of the function $\mathbf{f} = (t, \mathbf{0}, \mathbf{0})$ does not need to be equal to 0, which means that the origin does not need to be identical as an equilibrium state.

2.2 Mathematical Modeling

Fig. 1 represents a kinematic structure of the 5 DOF robotic system analyzed in this article. As shown in Fig. 1, generalized coordinates (q_1, q_2, q_3, q_4, q_5) were adopted to characterize the motion of the individual joints. A stationary coordinate frame was denoted as O_o, and the coordinate frames of the joints were marked as O_i, i = 1,…, 5. D_i, i = 1,…, 5, denote distances between the origins O_i. The coordinate systems were marked as $x_i y_i z_i$, i = 0, 1,…,5.

With the use of the energy-based Lagrange-Euler approach, the dynamic equation of the motion can be written as

$$\sum_{\alpha=1}^{n} a_{\alpha\gamma}(q)\ddot{q}^{\alpha} + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Gamma_{\alpha\beta,\gamma}(q)\dot{q}^{\alpha}\dot{q}^{\beta} = Q_{\gamma}^{g} + Q_{\gamma}^{u}, \tag{4}$$

where γ =1,…,5. $a_{\alpha\gamma}(q)$ represents the tensor coefficients, $\Gamma_{\alpha\beta,\gamma}(q)$ denotes the matrix coefficients, Q^g and Q^u are the major components of the generalized torque. Q^g represents the gravitation forces, and Q^u corresponds to the generalized torque, produced by the actuators.

A mathematical description of the actuators, Fig. 2, is given as in equation (5).

$$N_V N_m J_M \ddot{\theta} + F\dot{\theta} + M = C_n N_m I_R, \tag{5}$$

where θ is the rotation angle, M is the output torque of the actuator, equal to the sum of Q^g and Q^u. I_R is the current of the rotor, L_R is the inductance of the actuator, and U is the voltage of the actuator.

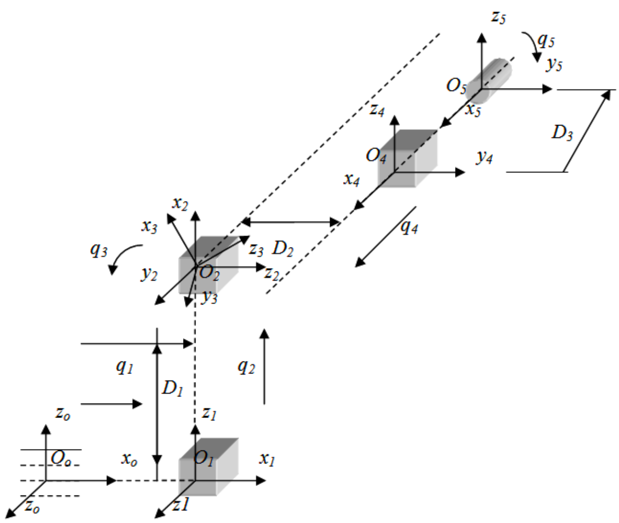

Figure 1

Model of the robotic system containing three translational and two rotational joints

The coefficient in equation (5) is denoted as follows: N_V is the reduction coefficient (ratio of the output velocity and input rotational velocity); N_m is the torque reduction coefficient (ratio of the input and output torques); J_M is the torque coefficient; F is the motor friction coefficient; C_n is the mechanical constant of the motor; R_R is the rotor circuit resistance; and C_E is the counter-electromotive force coefficient.

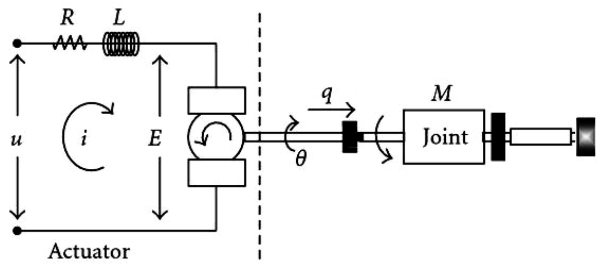

Figure 2

Schema of an actuator. Robotic joints are governed by the presented actuator.

Without the loss of any precession, it can be assumed that the inductance is $L_R \approx 0$. If the state-space vector for the motors is adopted as $x=(\theta,\ \dot{\theta},\ I_R)^T$, it can be concluded that the order of the mathematical model of the actuators is equal to two. Consequently, the state-space equation of each actuator is as follows

$$\dot{x}_i = A_i x_i + B_i u_i + d_i M_i \tag{6}$$

where A_i, b_i, and d_i are the matrices defined as:

$$A_i = \begin{bmatrix} 0 & 1 \\ 1 & -\frac{1}{J_M}\left(\frac{F_i}{N_V N_M} + \frac{C_M C_E}{R_R}\right) \end{bmatrix}, b_i = \begin{bmatrix} 0 \\ \frac{C_M}{N_V J_M R_R} \end{bmatrix}, d_i = \begin{bmatrix} 0 \\ -\frac{1}{N_V J_M N_m} \end{bmatrix} \tag{7}$$

The correlation between the robotic system and the actuator is established via generalized coordinates and torques in the following way: $\theta_i = t_i q_i$, where t_i is a transfer coefficient vector for the individual joint i. The generalized torques on the actuators are defined as $M_i = \tau_{geni}$. A matrix representation of the coefficient is $T=diag(T_i)$, $T_i=(0\ t_i)$. Equation (4) can now be written as follows,

$$H(q)\ddot{q} + h(q,\dot{q}) = \tau_{gen}\,. \tag{8}$$

In equation (8), H represents an inertia matrix; h is a matrix that represents both the centrifugal and Coriolis effects, as well as the gravity. The relation between the state space vector and the generalized coordinates is adopted as $\mathbf{x} = T^{-1}\dot{q}_d$. The time delay joint variables are defined as:

$$\begin{aligned} q_d(t) &= q(t-\tau) \\ \dot{q}_d(t) &= \dot{q}(t-\tau) \end{aligned}, \tag{9}$$

where τ is the system latency. When (4), (6) and (9) are combined with (8), the nonlinear dynamic equations of the robotic systems governed by the actuators can be written as:

$$\dot{\mathbf{x}} = A_n(\mathbf{x}) + B_n(\mathbf{x})\mathbf{u}, \tag{10}$$

where **x** and **u** are the state-space and control vectors, respectively. $\mathbf{x} = (q_1, q'_1 \ldots, q_5, q'_5)$, and $\mathbf{u} = (U_1, \ldots, U_5)$, where U_i is the voltage on each actuator. Nonlinear matrices A_n and B_n are calculated as:

$$\begin{aligned} A_n(\mathbf{x}) &= [A + F(I_n - H(\mathbf{x})TF)^{-1} H(\mathbf{x})TA]\mathbf{x} + \\ &\quad F(I_n - H(x)TF)^{-1} h(\mathbf{x}) \\ B_n(\mathbf{x}) &= B + F(I_n - H(\mathbf{x})TF)^{-1} H(\mathbf{x})TB \end{aligned}, \tag{11}$$

where $A = diag\ (A_i)$, $B = diag\ (B_i)$, $F = diag\ (d_i)$. Equation (12) can be obtained through the derivation of equation (10) in the second order Taylor series around the nominal point. For derivation purposes, the deviation of the generalized coordinates due to time delay was expressed as $\Delta q_d(t) = q(t) - q(t-\tau)$.

$$\dot{\mathbf{x}}(t) = A_{L0}\mathbf{x}(t) + A_{L1}\mathbf{x}(t-\tau) + B_L\mathbf{u}(t) \tag{12}$$

where matrices $A_L = (A_{L0}\ A_{L1})^T$ and B_L have the following form:

$$\begin{aligned} A_L &= A + F(I_n - HTF)^{-1}\frac{\partial H}{\partial \mathbf{x}} TF(I_n - HTF)^{-1}[HTA\mathbf{x}(\mathbf{x}_0) + HTB\mathbf{u}(0) + h] + \\ &\quad F(I_n - HTF)^{-1}[\frac{\partial H}{\partial \mathbf{x}}(TA\mathbf{x}_0 + TB\mathbf{u}_0) + HTA + \frac{\partial h}{\partial \mathbf{x}}] \\ B_L &= B_n(\mathbf{x}_0, 0) \end{aligned} \tag{13}$$

3 Control Synthesis

In this part, the ability of the robotic system to guarantee the desired trajectory tracking within the strictly predefined time interval was investigated. Consequently, it was necessary to find the control law which will supply the actuator with appropriate control signals to perform the motion of the links according to the predefined trajectories within a specified time frame.

The analyzed latency includes latency in mechanical parts, signal processing latency and latency due to the unmeasured disturbances. The overall system latency affects the generalized coordinates and consequently, the system states.

The proposed control method deals with the latency of any source that can cause delay in the system states.

The objective of the control was to minimize the error Δq between the real generalized coordinates and the generalized coordinates (positions and velocities) under the influence of latency. The errors due to time delay can be presented as:

$$\begin{aligned} \Delta q_d(t) &= q(t) - q(t-\tau) \\ \Delta \dot{q}_d(t) &= \dot{q}(t) - \dot{q}(t-\tau) \, . \\ \Delta \ddot{q}_d(t) &= \ddot{q}(t) - \ddot{q}(t-\tau) \end{aligned} \tag{14}$$

Equation (8) is rewritten as:

$$H(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) = \Im \, . \tag{15}$$

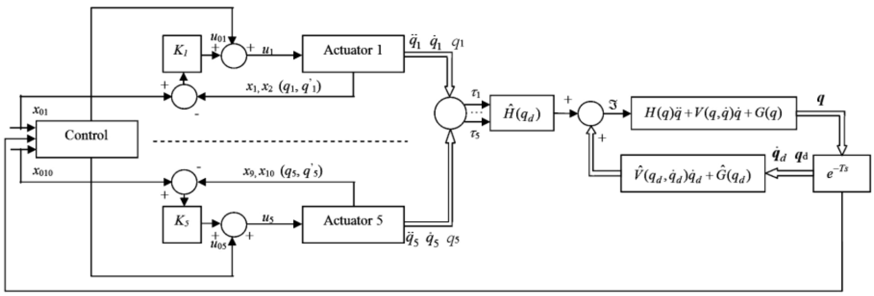

Figure 3

Control schema of the manipulator with time delay

Through the introduction of the estimated values of the system parameters, such as the estimated inertia matrix $\hat{H}(q)$, the estimated Coriolis and centrifugal matrix $\hat{V}(q,\dot{q})$, and the estimated gravitational vector $\hat{G}(q)$, the generalized form of equation (15) can be written as

$$\hat{H}(q)[\ddot{q} + K_v \Delta\dot{q} + K_p \Delta q] + \hat{V}(q,\dot{q})\dot{q} + \hat{G}(q) = \Im \, , \tag{16}$$

where K_v and K_p are the derivative and proportional gain matrices. Including (14), the controller equation for the system with time delay can be written as

$$\hat{H}(q_d)[\ddot{q} + K_v \Delta\dot{q}_d + K_p \Delta q_d] + \hat{V}(q_d,\dot{q}_d)\dot{q}_d + \hat{G}(q_d) = \Im \tag{17}$$

The proposed control methodology guarantees the asymptotic reduction of errors introduced by time delay. A block diagram of the proposed approach was presented in Fig. 3.

The control law used in the described case can be expressed as

$$\mathbf{u} = -KC\mathbf{x} \, , \tag{18}$$

where C is the output system matrix and K is the gain matrix, K=diag(K_p K_v). The values of the proportional and derivative gains were calculated for each link according to the following formula:

$$\begin{aligned} K_p &< ((0.5\omega_0)^2 (H_{ii} + N_v N_m J_M) R_R) / N_m C_M \\ K_v &= 2(K_p (H_{ii} + N_v N_m J_m) R_R - FR_R)^{1/2} / N_m C_M - N_v C_E \end{aligned} . \quad (19)$$

By recalculating the control law for trajectory tracking with respect to the actuators and using equation (6), one can obtain:

$$\mathbf{u}(t) = (\dot{\mathbf{x}}_v(t) - A_i \mathbf{x}_v(t) - d_i \tau_{gen}(t)) B_i^{-1}, \quad (20)$$

where $\mathbf{x}_v$ denotes the velocity components of the state values, with matrices defined as in (6).

4 Stability Analysis

In this part, a brief stability analysis for such systems is presented. To evaluate the stability of the system described here, we performed an evaluation using a novel approach. System (12) in the free working regime was analyzed

$$\dot{\mathbf{x}}(t) = A_{L0}\mathbf{x}(t) + A_{L1}\mathbf{x}(t-\tau), \quad (21)$$

with an initial vector function as

$$\mathbf{x}(t) = \boldsymbol{\varphi}_{\mathbf{x}}(t), \quad -\tau \le t \le 0. \quad (22)$$

While the analyzed class of the systems is kept in mind, the following definitions are presented. The theorem presented here was used to evaluate the stability of system (21).

Definition 1: System (21) is stable with respect to $\{\alpha, \beta, -\tau, T, \|\mathbf{x}\|\}$, $\alpha \le \beta$ if for any trajectory $\mathbf{x}(t)$ condition $\|\mathbf{x}_0\| < \alpha$ implies $\|\mathbf{x}(t)\| < \beta$, $\forall t \in [-\Delta, T]$, $\Delta = \tau_{\max}$, [7].

Definition 2: Autonomous system (21) is contractively stable with respect to $\{\alpha, \beta, \gamma, T, \|\mathbf{x}\|\}$, $\gamma < \alpha < \beta$, if for any trajectory $\mathbf{x}(t)$ with condition $\|\mathbf{x}_0\| < \alpha$, implies:

(*i*) stability with respect to $\{\alpha, \beta, -\tau, T, \|\mathbf{x}\|\}$,

(*ii*) there exists $t^* \in]0, T[$ such that $\|\mathbf{x}(t)\| < \gamma$, for all $\forall t \in]t^*, T[$, [7].

Definition 3: System (21) satisfying initial condition (22) is finite time stable with respect to $\{\zeta(t), \beta, \Im\}$ if and only if $\boldsymbol{\varphi}_x(t) < \zeta(t)$, implies $\|\mathbf{x}(t)\| < \beta$, $t \in \Re$, $\zeta(t)$ is a scalar function with the property $0 < \zeta(t) \le \alpha$, $-\tau \le t \le 0$, where α is a real positive number and $\beta \in \Re_+$ and $\beta > \alpha$, [7].

Theorem 1: Suppose that the matrix defined as $\Pi = (A_{L0}^T + A_{L0} + A_{L1}^T A_{L1} + I)$ is positive definite. Then the autonomous system (21) with initial function (22) is finite time stable with respect to $\{\alpha, \beta, \tau, \Im\}$, if $\alpha < \beta$, such that the following condition holds

$$(1+\tau)e^{\lambda_{\max}(\Pi)\cdot t} < \beta / \alpha, \ \forall t \in \Im, \tag{23}$$

where $\lambda_{\max}$ is the maximum eigenvalue of the specific matrix, and $\Im$ is a finite time interval.

Proof: Let us consider the following Lyapunov-like, aggregation function:

$$V(\mathbf{x}(t)) = \mathbf{x}^T(t)\mathbf{x}(t) + \int_{t-\tau}^{t} \mathbf{x}^T(\vartheta)\mathbf{x}(\vartheta)d\vartheta, \tag{24}$$

Denote by $\dot{V}(\mathbf{x}(t))$ time derivative of $V(\mathbf{x}(t))$ along the trajectory of system (21), so one can obtain:

$$\begin{aligned} \dot{V}(\mathbf{x}(t)) &= \dot{\mathbf{x}}^T(t)\mathbf{x}(t) + \mathbf{x}^T(t)\dot{\mathbf{x}}(t) + \frac{d}{dt}\int_{t-\tau}^{t} \mathbf{x}^T(\vartheta)\mathbf{x}(\vartheta)d\vartheta \\ &= \mathbf{x}^T(t)(A_0^T + A_0)\mathbf{x}(t) + 2\mathbf{x}^T(t)A_1\mathbf{x}(t-\tau) \\ &\quad + \mathbf{x}^T(t)\mathbf{x}(t) - \mathbf{x}^T(t-\tau)\mathbf{x}(t-\tau) \end{aligned} \tag{25}$$

Based on the known inequality[1], and with the particular choice:

$$\mathbf{x}^T(t)\Gamma\mathbf{x}(t) = \mathbf{x}^T(t)I\,\mathbf{x}(t) > 0, \ \forall \mathbf{x}(t) \in \mathcal{S}_\beta, \tag{26}$$

so that:

$$\begin{aligned} \dot{V}(\mathbf{x}(t)) &\le \mathbf{x}^T(t)(A_0^T + A_0 + A_1A_1^T + I)\mathbf{x}(t) \\ &\le \mathbf{x}^T(t)\Pi\,\mathbf{x}(t) \\ &\le \lambda_{\max}(\Pi)\mathbf{x}^T\mathbf{x}(t) \end{aligned}, \tag{27}$$

under the assumption given in *Theorem* 1. Moreover, it can be calculated:

$$\begin{aligned} \dot{V}(\mathbf{x}(t)) &< \lambda_{\max}(\Pi)\mathbf{x}^T\mathbf{x}(t) + \lambda_{\max}(\Pi)\int_{t-\tau}^{t} \mathbf{x}^T(\vartheta)\mathbf{x}(\vartheta)d\vartheta \\ &< \lambda_{\max}(\Pi)\left(\mathbf{x}^T\mathbf{x}(t) + \int_{t-\tau}^{t} \mathbf{x}^T(\vartheta)\mathbf{x}(\vartheta)d\vartheta\right) < \lambda_{\max}(\Pi)V(\mathbf{x}(t)) \end{aligned}, \tag{28}$$

since $\int_{t-\tau}^{t} \mathbf{x}^T(\vartheta)\mathbf{x}(\vartheta)d\vartheta > 0$ and $\lambda_{\max}(\Pi) > 0$.

[1] $2\mathbf{u}^T(t)\mathbf{v}(t-\tau) \le \mathbf{u}^T(t)\Gamma^{-1}\mathbf{u}(t) + \mathbf{v}^T(t-\tau)\Gamma\mathbf{v}(t-\tau), \ \Gamma > 0$

Multiplying (28) with $e^{-\lambda_{\max}(\Pi)\cdot t}$, one can obtain:

$$\frac{d}{dt}\left(e^{-\lambda_{\max}(\Pi)\cdot t}V\left(\mathbf{x}(t)\right)\right)<0. \tag{29}$$

Integrating (29) from 0 to *t*, with $t\in\Im$, we have:

$$V\left(\mathbf{x}(t)\right)<e^{\lambda_{\max}(\Pi)\cdot t}\cdot V(0). \tag{30}$$

From (24), it can be seen:

$$\begin{aligned}V(0)=\mathbf{x}^T(0)\mathbf{x}(0)+\int_{-\tau}^{0}\boldsymbol{\varphi}^T(0)\boldsymbol{\varphi}(0)\,d\vartheta\le\mathbf{x}^T(0)\mathbf{x}(0)+\\ \boldsymbol{\varphi}^T(0)\boldsymbol{\varphi}(0)\int_{-\tau}^{0}d\vartheta\le\alpha+\alpha\cdot\tau=\alpha(1+\tau)\end{aligned}. \tag{31}$$

Combining (30) and (31) leads to:

$$V\left(\mathbf{x}(t)\right)<\alpha(1+\tau)\cdot e^{\lambda_{\max}(\Pi)\cdot t} \tag{32}$$

On the other hand:

$$\mathbf{x}^T(t)\mathbf{x}(t)<\mathbf{x}^T(t)\mathbf{x}(t)+\int_{t-\tau}^{t}\mathbf{x}^T(\vartheta)\mathbf{x}(\vartheta)\,d\vartheta=V\left(\mathbf{x}(t)\right)<\alpha(1+\tau)\cdot e^{\lambda_{\max}(\Pi)\cdot t}, \tag{33}$$

Condition (24) and the above inequality imply:

$$\mathbf{x}^T(t)\mathbf{x}(t)<\alpha(1+\tau)\cdot e^{\lambda_{\max}(\Pi)\cdot t}<\beta,\ \ \forall t\in\Im, \tag{34}$$

which was to be proven.

5 Numerical Example and Simulation

For the purpose of the simulations of such systems, the desired trajectory in the Cartesian space was defined as in Fig. 4.

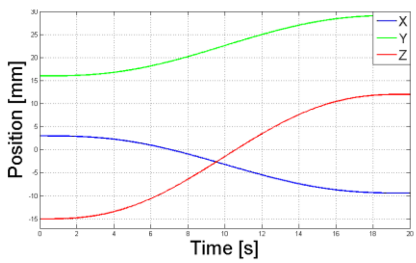

Figure 4

Tracking (desired) trajectories in the Cartesian space

It was requested that the coordinates of the absolute end-effector should follow the predefined trajectories within a time frame of 20 s and should maintain the stability in the interval.

Due to the described task, it is necessary to investigate the finite time stability of the time delay system.

Table 1

Geometric characteristics of the system and masses

Value/Joint	1	2	3	4	5
m (kg)	0.2	0.2	0.5	0.1	0.01
$\vec{\rho}_{ii}$ (m)	$(-0.2\ 0\ 0)^T$	$(0\ 0.18\ 0)^T$	$(0\ 0\ -0.17)^T$	$(0\ 0\ -0.165)^T$	$(0\ 0\ -0.2)^T$
$\vec{\rho}_i$ (m)	$(0.1\ 0\ 0)^T$	$(0\ -0.09\ 0)^T$	$(0\ 0\ 0.1)^T$	$(0\ 0\ 0.1)^T$	$(0\ 0\ 0.1)^T$
$\vec{e}_i$	$(1\ 0\ 0)^T$	$(0\ 1\ 0)^T$	$(1\ 0\ 0)^T$	$(0\ 0\ 1)^T$	$(0\ 0\ 1)^T$

Table 2

Numerical values of the parameters of the actuators

Value/Joint	1	2	3	4	5
R_r	8.40	8.40	84.30	2.10	16
J	6.50E-06	6.50E-06	5.40E-07	6.50E-06	9.00E-09
N_v	473	473	247	994	2100
C_m	2.02E+04	2.02E+04	74200	9800	2000
C_e	2.12E-03	2.12E-03	4.04E-04	4.04E-04	3.00E-03
N_m	115	115	111	111	870

In relation to Figure 1, the geometric characteristics of the system and the mass of the joints are presented in Table 1.

Table 3

Elements of the matrices

Actuator	a_{22}	b_2	d_2
1	-7.825*e*+05	7.822e+05	1.610e-02
2	-7.825*e*+05	7.822e+05	1.610e-02
3	-6.593*e*+04	6.588e+04	1.009e-01
4	-2.901*e*+05	7.371e+05	1.612e-02
5	-4.167*e*+07	6.614e+06	2.646e+01

Table 2 represents the numerical values of the parameters described in equation (5) and Figure 2.

The variables described in equation (6) which were used to determine control gains (19) are presented in Table 3. The coefficients a_{22}, b_2, and d_2 are the diagonal elements of the matrices (7).

K is the diagonal matrix and their elements are the position and velocity gains, $K = diag\{K_p\ K_v\}$. The gain values for each segment can be calculated using equation (19) and the values in Table 2.

Table 4
Gain elements

Joint	K_p	K_v
1	1.306E-02	1.164E+03
2	1.306E-02	1.164E+03
3	3.690E-01	7.461E+03
4	6.999E-03	2.143E+02
5	1.839E+00	2.488E+03

The detailed explanations for this procedure can be found in [18]. Using the control law (29), (4) and (30), it is possible to calculate the eigenvalues of system (5). The control gains are presented in Table 4.

Fig. 5 represents q_i, i=(1,2,…,5) trajectories in the joint space. The values on the y axis are in mm for joints 1, 2, and 4, and in rad/s for joints 3 and 5. The initial condition (22) transformed to the initial generalized coordinates in the joint space can be described as $\mathbf{q}_0$=[0 -17, 0, 1, 0]^T, as in Fig. 5. At this point, it is of interest to investigate the influence of time delay on the system stability.

For that purpose, the comparison between the two control strategies applied for system (12) was performed. The first one includes the classical approach using a PID controller. The second one includes the proposed methodology, as in (18) and Fig. 3. The comparison was presented in Fig. 6 and Fig. 7. The figures represent the step and sinusoidal responses of the system.

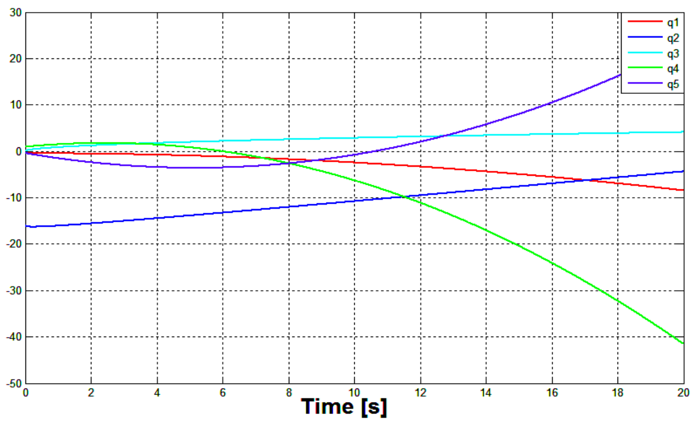

Figure 5
Generalized trajectories in the joint space

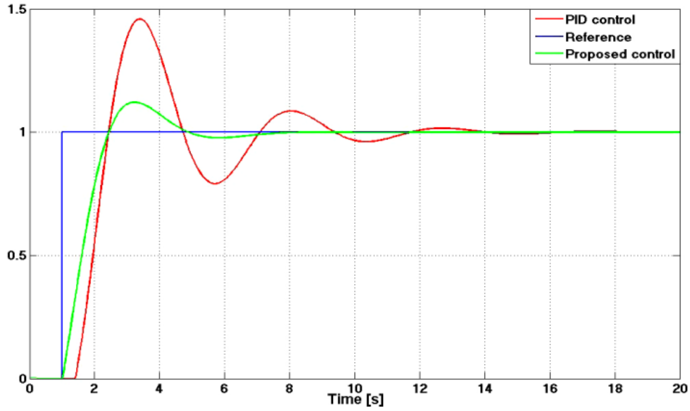

Figure 6

Step responses of the system for both PID and proposed control methodology vs. reference input signal

It was observed that the time delay had a significant influence on the dynamic behavior of system (12) when the PID controller was used. However, the proposed methodology in this article solved the latency problem of the system output, as shown in Figs. 6-7.

In the sequel, the stability of the robotic system represented by equation (21) with initial vector function (22) graphically presented in Fig. 4 was investigated. For the numerical stability analysis, *Theorem 1* was used.

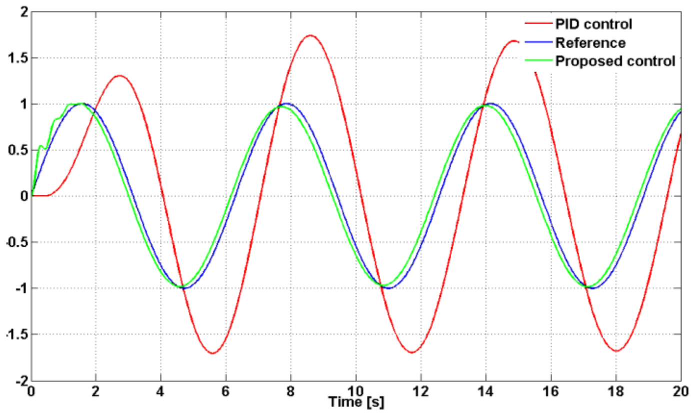

Figure 7

Sinusoidal responses of the system for both PID and proposed control methodology vs. reference input signal

The numerical values of matrices A_{L0} and A_{L1} are as follows, as in (35-36):

$$A_{L0} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3.9e6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.9e6 & 8.42 & -26.92 & -1.5e\text{-}4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.27e3 & 2.8e\text{-}2 & -4.6e4 & -0.24 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4.1e\text{-}2 & 0 & 4.4e\text{-}3 & -2.9e5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4.2e6 \end{bmatrix}, \tag{35}$$

$$A_{L1} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2.33e5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2.3e3 & 8.42 & -4.28 & -1.5e\text{-}3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3.3e4 & 0.18e\text{-}1 & -4.6e4 & -0.32 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.44e\text{-}1 & 0 & 2.4e\text{-}2 & -2.3e4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3.2e5 \end{bmatrix}. \tag{36}$$

System matrices A_{L0} and A_{L1} were calculated for the system with feedback, as in Fig. 3. For this example, the following was adopted: α = 2.5, β = 3, and τ = 200 ms. With the use of equation $\Pi = (A_{L0}^T + A_{L0} + A_{L1}^T A_{L1} + I)$, it was calculated that matrix Π is a positive definite matrix, i.e. $\Pi > 0$. The eigenvalues of the matrix were denoted as $\sigma(\Pi) = \{\lambda_1, \ldots, \lambda_{10}\}$.

The eigenvalues of the system were calculated using equation (37)

$$\begin{gathered} \det(A_L - KC - sE) = K \prod_{j=1}^{N} (s_j^0 - s) = \\ \det\left(\left(A_{0L} + A_{1L} e^{-s\tau}\right) - CK - sE\right) = \det\left(\left(A_{0L} + \bar{A}_{1L}\right) - CK - sE\right) \end{gathered}, \tag{37}$$

where $A_L = \left(A_{0L} + \bar{A}_{1L}\right)$ is a decomposition of matrix A^L. After calculation, it was obtained: $\sigma(\Pi)$ = {8.3, 2.8e5, 7.2e5, 1.2, 6.4e5, 1245, 12,4, 2.4e5, 4234, 4.1e6 }.

It can be seen that $\lambda_{\max}(\Pi) = 4.1e7$. Now it is possible to calculate condition (23) and to estimate T_{est} - time after the system is stable under the influence of control feedback. $(1+\tau)e^{\lambda_{\max}(\Pi)\cdot t} = (1+0.2)e^{\lambda\max(\Pi)t} < 1.2$. For this specific case, it was calculated that system (21) with control feedback (20) would obtain and maintain stability after $T_{\text{est}} = 3.9e\text{-}8$.

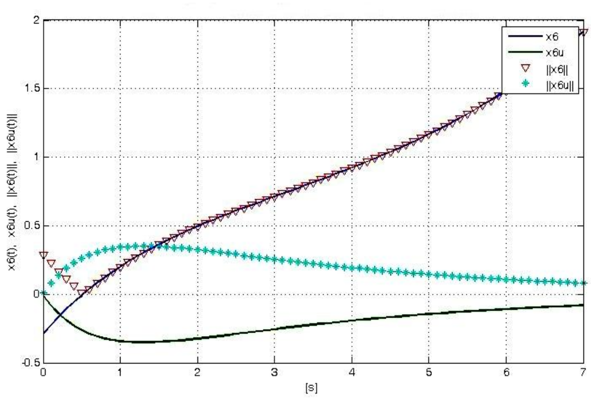

Figure 8

Trajectory and square norms of the representative state trajectory for the controlled and uncontrolled system

Fig. 8 represents the result graphically. The figure shows the trajectory and norm of the trajectory for controlled and uncontrolled systems. The norm of the representative state trajectory was presented to depict its convergence to the stable zero state during the time interval of interest.

Conclusions

In this article, a mathematical modeling procedure of the robotic system with time delay was presented. This procedure includes the mathematical model of the actuators, and it can be used for any robotic system in the open kinematic configuration. The time delay was included in the mathematical model. A time delay controller capable of system stabilization under the influence of the time delay was developed. The novel stability conditions were derived for the investigation of the stability of the system. These conditions were used to evaluate the proposed controller under the influence of system latency. The comparative results for both the PID and the time delay controller were presented. The proposed control methodology resulted in a stable dynamic behavior of the system. It was observed that the proposed controller could nullify the latency presented in each link. Consequently, the time delay did not influence the overall system performances. The performance investigation of the system using novel stability conditions showed the full compliance of the system behavior with the desired system dynamics. The future work of this study will include further rigorous dynamic analyses and the influence of the specific value of time delay on the system, and it will also define the stability boundaries for such a system.

References

[1] Ailon A.: Asymptotic Stability in Flexible-Joint Robots with Multiple Time Delays, Proc. of the IEEE Conference on Decision and Control (CDC), 2003, pp. 4375-4380

[2] Anderson R. J., Spong M. W.: Asymptotic Stability for Force Reflecting Teleoperators with Time Delay, International Journal of Robotics Research Vol. 11, No. 2, 1992, pp. 135-149

[3] Llama M. A., Santibanez V., Flores J.: A Passivity-based Stability Analysis for a Fuzzy PD+ Control for Robot Manipulators, Proc. of 18th IEEE International Conference of the North American, New York, NY, USA June, 1999, pp. 665-669

[4] Buzurovic I. M., Debeljkovic D. Lj.: Robust Control for Parallel Robotic Platforms, Proc. of 16th IEEE International Symposium on Intelligent Systems and Informatics (INES), Lisbon, Portugal, 2012, pp. 45-50

[5] Buzurovic I, Podder T. K., Yu Y.: Force Prediction and Tracking for Image-guided Robotic System using Neural Network Approach: IEEE Biomedical Circuits and Systems Conference (BioCAS), Baltimore, MA, USA, November, 2008, 41-44

[6] Debeljkovic D. Lj., Buzurovic I. M., Nestorovic T., Popov D.: A New Approach to the Stability of Discrete Descriptor Time Delay Systems in the Sense of Non-Lyapunov Delay Independent Conditions, Proc. of 24th IEEE Chinese Control and Decision Conference (CCDC), Taiyuan, China, May 23-25, 2012, pp. 1155-1160

[7] Debeljkovic D. Lj., Buzurovic I. M., Simeunovic G. V, Misic M.: Asymptotic Practical Stability of Time Delay Systems, Proc. of 10th IEEE International Symposium on Intelligent Systems and Informatics (SISY), Subotica, Serbia, September 20-22, 2012, pp. 379-384

[8] Han D. K., Chang P. H., Jin M.: Robust Trajectory Control of Robot Manipulators using Time Delay Control with Adaptive Compensator, IFAC Proc. Vol., 2008, pp. 2276-2281

[9] Han D. K., Chang P.: Robust Tracking of Robot Manipulator with Nonlinear Friction using Time Delay Control with Gradient Estimator, Journal of Mechanical Science and Technology, Vol. 24, No. 8, 2010, pp. 1743-1752

[10] Gu K., Chen J., Kharitonov V.: Stability of Time-Delay Systems, Springer-Verlag, Berlin, Heidelberg, New York, 2003

[11] Jiang W.: Robust $H\infty$ Controller Design for Wheeled Mobile Robot with Time-Delay, Proc. of International Conference on Intelligent Computation Technology and Automation (ICICTA), 2008, pp. 450-454

[12] Jin M., Jin Y., Chang P. H., Choi C.: High-Accuracy Trajectory Tracking of Industrial Robot Manipulators using Time Delay Estimation and Terminal Sliding Mode, IECON Proc. on Industrial Electronics Conference, 2009, pp. 3095-3099

[13] Jin M., Jin Y., Chang P. H., Choi C.: High-Accuracy Tracking Control of Robot Manipulators using Time Delay Estimation and Terminal Sliding Mode, International Journal of Advanced Robotic Systems, Vol. 8, No. 4, 2011, pp. 65-78

[14] Jin M., Kang S. H., Chang P. H.: A Robust Compliant Motion Control of Robot with Certain Hard Nonlinearities using Time Delay Estimation, IEEE International Symposium on Industrial Electronics, 2006, pp. 311-316

[15] Precup R. E., Doboli S., Preitl S.: Stability Analysis and Development of a Class of Fuzzy Control Systems, Engineering Applications of Artificial Intelligence, Vol. 13, No. 3, 2000, pp. 237-247

[16] Zhang W., Cai X. S., Han Z. Z.: Robust Stability Criteria for Systems with Interval Time-Varying Delay and Nonlinear Perturbations, Journal of Computational and Applied Mathematics, Vol. 234, Vol. 1, 2010, pp. 174-180

[17] Kim W. S., Hannaford B., Fejczy, A. K.: Force-Reflection and Shared Compliant Control in Operating Telemanipulators with Time Delay, IEEE Transactions on Robotics and Automation, Vol. 8, No. 2, 1992, 176-185

[18] Koivo A. J., Houshangi N.: Real-Time Vision Feedback for Servoing Robotic Manipulator with Self-Tuning Controller, IEEE Transactions on Systems, Man and Cybernetics, Vol. 21, No. 1, 1991, pp. 134-142

[19] Galambos P., Baranyi P.: Representing the Model of Impedance Controlled Robot Interaction with Feedback Delay in Polytopic LPV Form: TP Model Transformation based Approach, Acta Polytechnica Hungarica, Vol. 10, No. 1, 2013, pp. 139-157

[20] Kuti J., Galambos P., Baranyi P.: Delay and Stiffness Dependent Polytopic LPV Modelling of Impedance Controlled Robot Interaction, in: Issues and Challenges of Intelligent Systems and Computational Intelligence, Springer International Publishing, 2014, pp. 163-174

[21] Liang J., Chen L.: Improved Nonlinear Feedback Control for Free-Floating Space-based Robot with Time-Delay Based on Predictive and Approximation of Taylor Series, Hangkong Xuebao/Acta Aeronautica et Astronautica Sinica, Vol. 33, No. 1, 2012, pp. 163-169

[22] Liu Y., Passino K. M., Polycarpou M. M.: Stability Analysis of M-Dimensional Asynchronous Swarms with a Fixed Communication Topology, IEEE Transactions on Automatic Control, Vol. 48, No. 1, 2003, pp. 76-95

[23] Liu H., Zhang T.: Tracking Control of Industrial Robot Based on Time Delay Estimation and Robust H ∞ Control, Huanan Ligong Daxue Xuebao/Journal of South China University of Technology (Natural Science) Vol. 40, No. 1, 2012, pp. 77-81

[24] Niemeyer G., Slotine J. E.: Stable Adaptive Teleoperation, IEEE Journal of Oceanic Engineering, Vol. 16, No. 1, 1991, pp. 152-162

[25] Niemeyer G., Slotine J. E.: Telemanipulation with Time Delays, International Journal of Robotics Research, Vol. 23, No. 9, 2004, pp. 873-890

[26] Ogaki F., Suzuki K.: Adaptive Teleoperation of a Mobile Robot under Communication Time Delay, Proc. of ROSE 2007 - International Workshop on Robotic and Sensor Environments, 2007, pp. 1-6

[27] Park J., Cho B., Lee J.: Trajectory Control of Underwater Robot Using Time Delay Control, Transactions of the Korean Society of Mechanical Engineers, A, Vol. 32, No. 8, 2008, pp. 685-692

[28] Park J., Cho B., Lee J.: Trajectory-Tracking Control of Underwater Inspection Robot for Nuclear Reactor Internals Using Time Delay Control, Nuclear Engineering and Design, Vol. 239, No. 11, 2009, pp. 2543-2550

[29] Sanders D.: Analysis of the Effects of Time Delays on the Teleoperation of a Mobile Robot in Varous Modes of Operation, Industrial Robot, Vol. 36, No. 6, 2009, pp. 570-584

[30] Slotine J. E.: Robust Control of Robot Manipulators, International Journal of Robotics Research, Vol. 4, No. 2, 1985, pp. 49-64

[31] Sree Krishna Chaitanya V., Srinivas Reddy M.: A Neural Network Controller for the Path Tracking Control of a Hopping Robot Involving Time Delays, International Journal of Neural Systems, Vol. 16, No. 1, 2006, pp. 47-62

[32] Velasco-Villa M., Alvarez-Aguirre A., Rivera-Zago G.: Discrete-Time Control of an Omnidireccional Mobile Robot Subject to Transport Delay, Proc. of the American Control Conference, 2007, pp. 2171-2176

[33] Zeng Q., Xu T., Xu J., Song A., Tian X.: Predictive Control for Force Telepresence Teleoperation Robot System with Time Delay, Dongnan Daxue Xuebao (Ziran Kexue Ban)/Journal of Southeast University (Natural Science Edition), Vol. 34, 2004, pp. 160-164

[34] Juang J. G., Yu C. L., Lin C. M., Yeh R. G., Rudas I. J.: Real-Time Image Recognition and Path Tracking of a Wheeled Mobile Robot for Taking an Elevator, Acta Polytechnica Hungarica, Vol. 10, No. 6, 2013, pp. 5-23

[35] Zhu W., Salcudean S. E.: Stability Guaranteed Teleoperation: an Adaptive Motion/Force Control Approach, IEEE Transactions on Automatic Control, Vol. 45, No. 11, 2000, pp. 1951-1969

[36] Baranyi P., Yam Y.: TP Model Transformation Based Observer Design to 2-D Aeroelastic System, Acta Polytechnica Hungarica, Vol. 1, No. 2, 2004, pp. 63-78

[37] Preitl S., Precup R. E., Fodor J., Bede B.: Iterative Feedback Tuning in Fuzzy Control Systems. Theory and Applications, Acta Polytechnica Hungarica, Vol. 3, No. 3, 2006, pp. 81-96

[38] Petra M. I., DeSilva L. C.: Implementation of Folding Architecture Neural Networks into an FPGA for an Optimized Inverse Kinematics Solution of a Six-Legged Robot, International Journal of Artificial Intelligence, Vol. 10, No. S13, 2013, pp. 123-138

[39] Triharminto H. H., Adji T. B., Setiawan N. A.: 3D Dynamic UAV Path Planning for Interception of Moving Target in Cluttered Environment, International Journal of Artificial Intelligence, Vol. 10, No. S13, 2013, pp. 154-163

[40] Debeljkovic D. Lj., Nestorovic T., Buzurovic I. M., Dimitrijevic N. J.: A New Approach to the Stability of Time-Delay Systems in the Sense of Non-Lyapunov Delay-Independent and Delay-Dependent Criteria, Proc. of 8th IEEE International Symposium on Intelligent Systems and Informatics (SISY) Subotica, Serbia, September, 2010, pp. 213-218

[41] Debeljkovic D. Lj., Buzurovic I. M.: Dynamics of Singular and Descriptive Time Delayed Control Systems: Stability, Robustness, Optimization, Stabilizability and Robustness Stabilizability, University of Belgrade, Serbia, ISBN: 978-86-7083-77, 2013

[42] Buzurovic I., Debeljkovic D. Lj., Jovanovic A. M.: An Efficient Method for Finite Time Stability Calculation of Continuous Time Delay Systems, Proc. of IEEE Asian Control Conference (ASCC), Istanbul, Turkey, June, 2013, pp. 1-5