

# Universal Fluctuations in Very Short ECG Episodes

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*Abstract: We propose a new algorithm for the detection of ventricular fibrillation (VF) in very short surface electrocardiogram (ECG) episodes. Ventricular fibrillation is the most commonly identified arrhythmia in cardiac arrest patients and can lead to syncope, within seconds. The fast detection of ventricular fibrillation is necessary for prompt defibrillation either with an implantable cardioverter/defibrillator or an automated external defibrillator. Ventricular fibrillation generates stochastic waveforms and recently it has been shown that it exhibits characteristics similar to a non-chaotic signal and contains deterministic Probability Density Function (PDF), for the different physical fluctuations was described previously. Accordingly, we describe scaling properties of very short shockable, VF and non-shockable ECG episodes and show that a universal PDF exists for the fluctuations of shockable ECG episodes. We compared the proposed algorithm with nine standard VF detection algorithms. The comparison indicated that our algorithm consistently produced more accurate detection results, then with standard algorithm. We conclude that the proposed method, based on fluctuation analysis, provides new information on the dynamics underlying VF, and allows a better detection compared to other algorithms.*

*Keywords: electrocardiogram; universal fluctuations; ventricular fibrillation; defibrillation; probability density function*

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# 1 Introduction

The ventricular myocardium is a dynamic system which can appear synchronized in sinus rhythm or irregular, anomalous in ventricular fibrillation. [1]. The ventricular fibrillation, (VF) is the most common cause of sudden cardiac death [2]. In the non-appearance of defibrillation, VF leads to death within a few minutes, therefore, accurate and timely detection of this arrhythmia is of great clinical importance.

An implantable cardioverter/defibrillator (ICD) or an automated external defibrillator (AED) should reliably decide between *shockable* (e.g. VF)-, and *non-shockable* ECG episodes. Shockable episodes demand the urgent response of electrical defibrillation. The decision is based on different VF detection algorithms, which in the embedded, real-time software environment should reveal sufficient performance. Widely proposed methods for this level are: complexity measure, peak analysis, threshold cutting and narrow band filter algorithms [3].

Historically, it has been accepted that VF generate stochastic ECG waveforms [4]. Using power spectral density, Kaplan et al. [5] have demonstrated, that the VF waveform exhibits characteristic similar to a non-chaotic signal. Most recent studies show that shockable ECG episodes own 80-90% determinism [6].

In this contribution the properties of the very short ECG episodes (4 sec duration, 1000 Hz sampling rate, 12-bit amplitude resolution) are examined to the general attributes of a critical system presented by Bramwell et al. [7]. The authors demonstrated a PDF (Bramwell-Holdsworth-Pinton, BHP PDF) exists for the fluctuations in different physical systems (magnetism, Danube water level [7]). The BHP PDF is always in systems which lack a characteristic time scale. The underlying common thread in all those applications is rooted in the attempt to quantify the fluctuations of different temporal patterns.

## 2 Method

### 2.1 Probability Density Function (PDF)

The universal PDF for the very short episodes of ECG recordings  $y$  can be determined by using the distribution of  $h = \frac{y - \bar{y}}{\sigma_y}$  in the thermodynamic limit (in the infinite system size), where  $\bar{y}$  is the mean and  $\sigma_y$  the standard deviation.

In statistics, the characteristic function  $\varphi(t)$  of any variable completely defines its probability distribution. If a variable admits a PDF, then  $\varphi(t)$  is the inverse Fourier transform of the PDF [7].

The intensity of the correlations within an ECG signal depends on the degrees of irregularity of the miscellaneous topological rhythm statements, and on the duration of the episodes. In a very short ECG signal the central limit theorem breaks down, consequently the limit function can be non-Gaussian.

Let the sum of the ECG recording  $Y = \sum_n y_n$  and each measured ECG value  $y_n$  be a gamma variable, with known PDF:

$$PDF(y_n) = \frac{n}{\Gamma(\gamma)} e^{-ny_n} y_n^{\gamma-1} \quad (1)$$

where  $\Gamma()$  is the gamma-function,  $\gamma = \frac{1}{2}$ , and  $n$  is the size of the episodes.

The logarithm of the characteristic function  $\varphi_n(t)$  of the  $y_n$  could be expanded as a series of statistical moments  $k_r(y_n)$ .

$$\log \varphi_n(t) = \sum_1^{\infty} \frac{(it)^r}{r!} k_r(y_n) \quad (2)$$

Where the  $r^{\text{th}}$  statistical moment  $k_r(y_n) = \gamma(r-1)!n^{-r}$ .

The gamma variables are statistically independent [7], therefore the characteristic function  $\Phi(t)$  of  $Y$  is the product of the  $\varphi(t)$ :

$$\Phi(t) = \prod_n \varphi_n(t) \quad (3)$$

Normalizing  $Y$  by its standard deviation  $\sigma_y$ , the  $r^{\text{th}}$  statistical moment

$$k_r\left(\frac{Y}{\sigma_y}\right) = \frac{\frac{1}{2}(r-1)! \sum_n \left(\frac{1}{n}\right)^r}{\left(\frac{1}{2} \sum_n \left(\frac{1}{n}\right)^2\right)^{\frac{r}{2}}} \quad (4)$$

Finally, the PDF of the very short episodes of ECG is the inverse Fourier transformation of the series [4].

### 3 Results and Discussion

#### 3.1 Probability Density Function

Figure 1 shows the PDF of the fluctuations,  $h = \frac{y - \bar{y}}{\sigma_y}$  as found from the inverse

Fourier transform of (4) and the histograms of the fluctuations applying 2400 short episodes of ECG signals with shockable arrhythmias and 2400 short episodes with non-shockable arrhythmias.

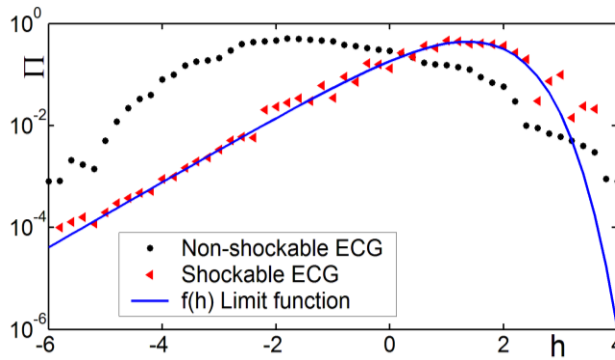


Figure 1

Comparison, without fitting parameters, between the histogram  $\sigma P(h)$  of the short episodes of shockable fluctuations (circle symbols), non-shockable fluctuations (triangle symbols) and the universal PDF (line). The data is plotted as  $\Pi = \sigma P(h)$  against  $h = \frac{y - \bar{y}}{\sigma_y}$ .

In the case of the shockable episodes, an excellent agreement can be seen with the universal PDF  $f(h)$ , however Figure 1 contains no fitting parameters.  $f(h)$  is asymmetric, with the position for fluctuations above the mean exponentially variations, yielding a probability for a positive fluctuation that might be larger than by a normal distribution.

For the non-shockable arrhythmias, the fluctuations curve has a Gaussian characteristic; hence, the normal sinus rhythm system cannot be approximated as a non-linear system.

The specific features of shockable signals are a consequence of strong correlations and self-similarity and do not share the same universal class as the non-shockable ones. These results imply that the degree of organization in short episodes of VF is low, which may explain the lack of convergence of fractal dimension of the VF reported in the literature [4].

To examine the linearity of the episodes, the ECG signals were analyzed with power spectral density (PSD). The PSD is the square of the coefficients in a Fourier series representation and measures the average variation of the arrhythmia at different frequencies. If the adjacent ECG points are uncorrelated, then the power spectrum will be constant, as a function of the normalized frequency. If the adjacent ECG points are correlated relative to points far apart the power spectrum will be large at small frequencies and small at large frequencies.

The PSD response of a non-shockable signal can be seen in Figure 2. The slope of the least squares fit shows clear signs of a weakly correlated origin.

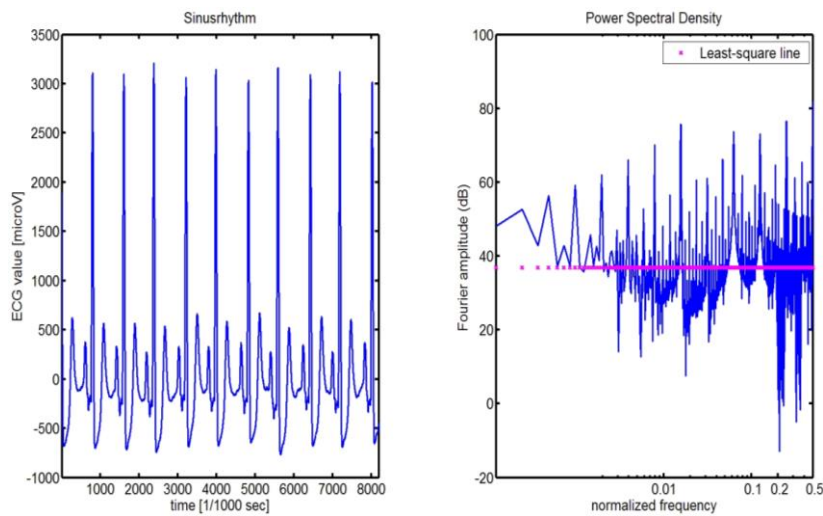


Figure 2

Left Panel: Typical sinus rhythm ECG. Right Panel: Power spectral density with least-squares fit. The fractal dimension of the 2400 signals  $\in [1.03-1.24]$ .

Figure 3 depicts the typical PSD of the shockable ECG recordings. The relationship between the spectral power of the signal shows fractal power-law scaling similar to a fractal  $1/f^m$  framework, which is directly related to non-linearity of the input arrhythmia scale distribution and suggests a lower dynamic complexity for the ECG activity.

The signal shows fractal power - law scaling similar to a fractal structure  $1/f^m$ .

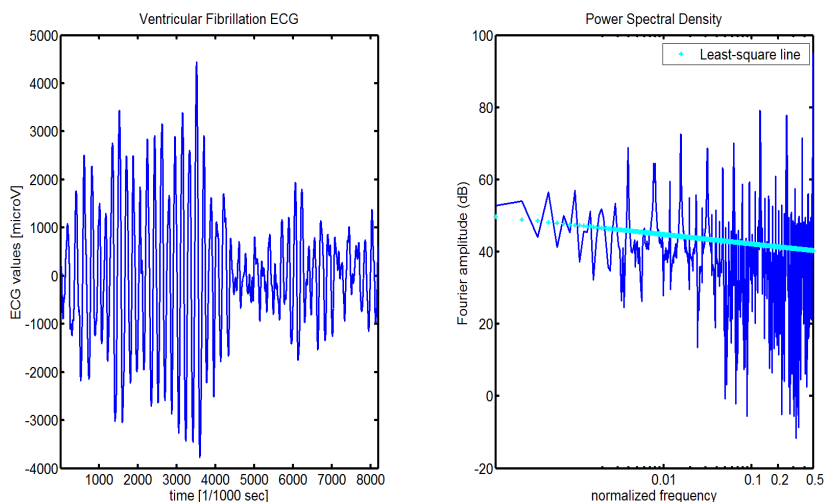


Figure 3

Left Panel: Ventricular fibrillation ECG. Right Panel: Power spectral density with least-squares fit. The fractal dimension of the 2400 signals  $\in [1.19-1.46]$ .

### 3.2 Scaling of Higher Moments

In the very short episodes of ECG it is expected that the standard deviation  $\sigma_y$  and the mean value of the order parameter  $\langle m \rangle$  scale with system size, so that a rescaled PDF is independent of the episode size.

The order parameter of ECG is a measure of the degree of order in a system; it ranges between zero for shockable arrhythmias and the saturation value for non-shockable ones. This can be expressed as:

$$\langle m \rangle \propto \sigma_y \propto t^\alpha \quad (5)$$

where  $\alpha$  is the scaling exponent [13, 14]

The scaling exponents can be used as a function of  $n$ . The 2400 short episodes of ECG signals with shockable arrhythmias approximately fulfil the Eq.(5) relationship, while the 2400 short episodes with non-shockable arrhythmias show divergences from multiscaling [15].

The multiscaling function is shown in Fig. 4.

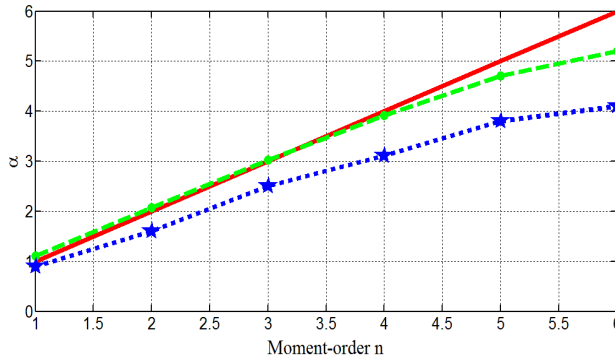


Figure 4

Multiscaling function Eq. 5. Simple scaling – [solid line]; the short episodes of ECG signals with shockable arrhythmias [o]  $\alpha(n) = 1.0899 - 0.04955n + 0.00271n^2$  and short episodes with non-shockable arrhythmias [\*]  $\alpha(n) = 0.91244 - 0.02618n$ .

We have investigated the skewness  $\frac{\langle(m - \langle m \rangle)^3\rangle}{\sigma^3}$  and kurtosis  $\frac{\langle(m - \langle m \rangle)^4\rangle}{\sigma^4} - 3$  of the

ECG episodes.

Theoretically, the skewness and kurtosis might scale with the size of episode [7]. Nevertheless, both the short episodes of ECG signals with shockable and non-shockable arrhythmias, skewness and the kurtosis, as functions of the size, do not adapt to a power-law relationship (Fig. 5) nor is there a multiscale relationship:  $kurtosis \propto skewness^{2.08}$  for the shockable and  $kurtosis \propto skewness^{1.74}$  for the non-shockable episodes. The ECG episodes change to multiscaling and the skewness and kurtosis do not scale with the size of the episode.

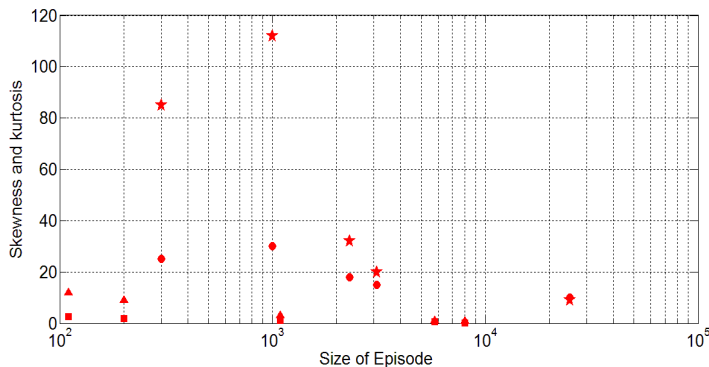


Figure 5

Skewness [o] and kurtosis [\*] of the short episodes of ECG signals with shockable arrhythmias, skewness [□] and kurtosis [Δ] of the short episodes of ECG signals with non-shockable arrhythmias

### 3.3 Comparison with Standard Algorithm

To gain insight into the quality of our algorithm it is essential to compare the results with other VF detection algorithms under equal conditions, with very short ECG episodes (4 sec duration, 1000 Hz sampling rate, 12-bit amplitude resolution). The following methods have been considered, for details refer the noted literature:

- Threshold crossing algorithm (TCIA) [3]
- VF filter algorithm (VFFA) [9]
- Spectral algorithm (SPECA) [10]
- Complexity measure algorithm (CPLXA) [11]
- Standard exponential algorithm (STEA) [3]
- Signal comparison algorithm (SCA) [3]
- Wavelet based algorithm (WBA) [3]
- Li algorithm (LIA) [12]
- Pan-Tompkins QRS based algorithm (QRSB) [15]

Table 1 shows the values for the sensitivity and the specificity of the algorithms using annotated databases: MIT-BIH malignant ventricular arrhythmia database (Massachusetts Institute of Technology (MIT), Beth Israel Hospital (BIH)), Creighton University ventricular tachyarrhythmia database (CU) and American Heart Association database (AHA).

Table 1  
Quality of VF detection algorithms. Specificity (SP), Sensitivity (SN) in %

<i>Algorithm</i>	<i>MIT BIH</i>		<i>CU</i>		<i>AHA</i>	
	SP	SE	SP	SE	SP	SE
TCIA	68.4	73.2	49.8	66.1	74.3	70.9
VFFA	86.2	25.9	82.8	19.1	80.6	12.7
SPECA	83.5	30.7	78.3	24.2	76.1	23.9
CPLXA	88.6	17.2	72.7	20.5	82.6	59.4
STEA	77.1	62.8	59.4	54.2	43.1	64.9
SCA	91.3	63.4	88.7	57.1	88.4	71.5
LIA	92.9	16.7	95.7	8.3	87.2	40.4
WBA	94.5	68.2	91.8	54.7	93.3	49.9
QRSB	33.2	50.8	51.5	63.1	48.3	85.2
BHPPDFA	97.3	98.6	96.2	99.4	93.9	97.3

MIT BIH: MIT-BIH malignant ventricular arrhythmia database; CU: Creighton University ventricular tachyarrhythmia database; AHA: American Heart Association database; SE: sensitivity; SP: specificity; VF: ventricular fibrillation; BHPPDFA: Bramwell-Holdsworth-Pinton probability density function.



For the very short episodes the PSD algorithm yields the best value both the sensitivity and specificity, followed by the algorithms SCA and VFFA. All other algorithms present heterogeneous simulations. We can interpret here, that algorithms for QRS detection or those developed in the spectral domain are not adequate for very short episodes.

### Conclusions

In this work fluctuation universality of very short ECG episodes was analyzed. The Universal Probability Density (PDF) function is the inverse Fourier transform of the statistical moments of the fluctuations within the thermodynamic limit.

For the non-shockable arrhythmias, e.g. sinus rhythm, the function has Gaussian characteristics, while in the case of shockable, the VF shows an excellent agreement with the BHP PDF, without fitting parameters. The BHP PDF is very well suited to classify the fluctuations of very short ECG episodes.

Our results show, that fluctuation analysis provides new information for the dynamics underlying VF, and allows for a better detection as compared to other algorithms.

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