# Extended Linear Regression and Interior Point Optimization for Identification of Model Parameters of Fixed Wing UAVs

**Huda Naji Al-sudany, Béla Lantos**

Budapest University of Technology and Economics, Hungary
Magyar tudósok krt. 2, H-1117 Budapest, Hungary
E-mail: alsudany@iit.bme.hu, lantos@iit.bme.hu

*Abstract: The paper deals with the identification of the system parameters in the nonlinear dynamic model of fixed wing UAVs. Fixed wings airplanes are popular in long distance applications and have to be modelled accurately to guarantee efficient control properties. Different methods are suggested to solve the parameter estimation beginning with the standard linear regression (LR) and continued with its extension (ELR), the optimization using interior point methods and finally using the FireFly technique, which is a metaheuristic algorithm. The methods illustrate the convergence and the speed of these approaches. The known default parrameter values of a Sekwa UAV were used to demonstrate that the elaborated identification methods can also reconstruct the numerical values of the dimensionless system parameters embedded into the nonlinear model using the physical weighting functions. After the extension of linear regression, MinMax optimization algorithm was used to get the best and optimal solution and reconstruct the parameters of the Sekwa aircraft. FireFly optimization gives also comparable results with minmax method. Flight data was needed for simulation and testing the different approaches. Matlab and toolboxes were used as simulation software. The results showed the estimated parameters are more accurate than linear regression estimation. Even though it is a small improvment this will reflect in all calculations.*

*Keywords: Aircraft model identification; linear regression;Min Max optimization; FireFly optimization*

# 1 Introduction

Analysis and parameter prediction are vital in achieving the control and stability properties of a somewhat complicated system like an aircraft. The parameter estimation method has been used with great success in the past to predict numerous parameters depending of real data of flight. Considering that the rigid body model is reliable, parameter estimate using flight data is currently employed often when applied to airplanes in the linear flight domain. As a result, the derivation of aircraft model is used to estimate procedure lacks elastic degrees of freedom. High levels of flexibility in an aircraft may make it more susceptible to the dynamics of a system with too many factors that must be evaluated [1] [2].

The identification of an accurate and verified mathematical model of aircraft apparatus is known as aircraft system identification. This is a crucial stage in the development of flight vehicles due to the generated model is vital for:

I. Comprehending the cause-and-effect relationship.
II. Examining the capabilities and characteristics of aircraft.
III. Verifying aerodynamic databases and upgrading flight control law designs are steps in step three.
IV. Supporting the expansion of the flying envelope.
V. Attempting to recreate the flight path, which incorporates incidence analysis and wind calculation.
VI. Running adaptive control and fault diagnosis.

This article is a natural extension of our previous research efforts. Building upon our prior work, titled "Comparison of Adaptive Fuzzy EKF and Adaptive Fuzzy UKF for State Estimation of UAVs Using Sensor Fusion," in [3] where we focused on state estimation techniques, and "Prediction of the Navigation Angles Using Random Forest Algorithm And Real Flight Data of UAVs," in [4] which explored machine learning-based prediction of navigation angles, this current article delves into a new dimension of UAV research. Specifically, it addresses the critical task of model parameter identification for Fixed Wing UAVs, utilizing extended linear regression and interior point optimization techniques.

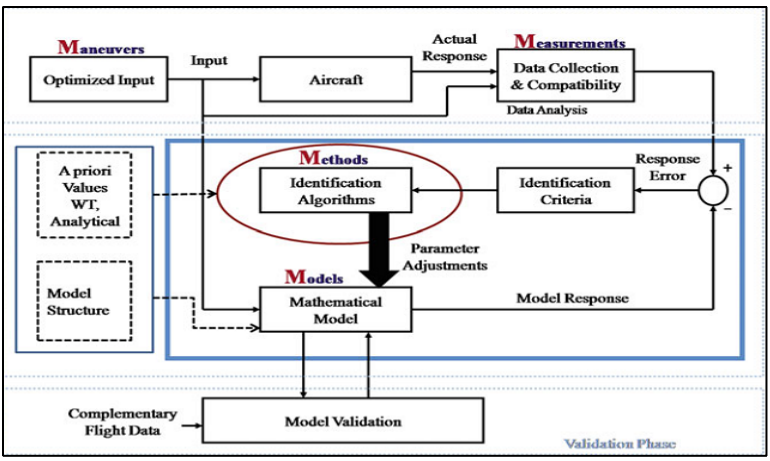



Figure 1
Block diagram of aircraft [5]

By citing our previous work, we establish the continuity and progression of our research, highlighting how our latest contribution complements and broadens the scope of our ongoing efforts in advancing UAV technology.

The method for identifying an aircraft system is described in Figure 1.

## 1.1 State of Art

The authors in [6] discussed how to use measurable input and output data to estimate parameters in aircraft flight dynamic models, like control and stability derivatives. In this method, the aircraft control effectors are moved using orthogonal phase-optimized multisines, frequency responses of MIMO systems are computed using Fourier analysis, and noise values of the parameters of model are determined using frequency response error (FRE), a maximum likelihood estimator. The T-2 generic transport system and the X-56A aeroelastic model are examples of airplanes whose flight test results are used to illustrate the technique. By using the maximum likelihood estimator, one may easily incorporate prior knowledge and combine data from many motions without additional correction while also giving precise statistical uncertainties for the expected values. However, the tactic still has several shortcomings. Although frequency responses provide physical insight into the dynamics, their usage restricts modeling to linear, time-invariant systems, necessitating flight test data with low disturbances compared to a reference state. While other approaches can benefit from fewer data records, a meaningful frequency response estimate requires steady-state data, which take longer to gather. If there is environmental disturbance, more loops of steady-state data may be needed. The strategy requires the capacity to extend the command path with computerized inputs. Uncertainties or uncertain environment can cause many problems that need to be solved [7].

In [8], Online system identification was covered by the authors; as technology advanced, it became a crucial step in the construction of methods for estimating aerodynamic parameters. In order to estimate the aerodynamic parameters of fixed-wing aircraft in unsteady conditions like stalls, this research proposes two online system identification (SID) algorithms that are based on Kalman filters. The suggested approaches, in contrast to previous SID ones, incorporate aerodynamic features associated with the upset state directly into the aircraft dynamics, such as modification of aerodynamic derivatives or flow separation point. To get the best estimations of the relevant aerodynamic characteristics, the standard or unscented Kalman filter is then applied in real-time. To illustrate the usefulness of the proposed approaches and their superiority to a previously proposed method, real flight data sets from a variety of aircraft are used in testing.

In [9], authors deal on the issue of fixed-wing UAV modeling and control. For control purposes dynamic models for aircraft were given in body, stabilization-axes, and wind-axes coordinate systems. It was possible to define and resolve a typical integral backstepping control issue that ensures stability in closed loops. Integral parts of the control can aid in reducing parameter changing and disruption impacts. The approach ensures that the combined 3D attitude system will remain stable in a closed loop even with changing reference signals. For motion portions that can be joined and smoothed, primitives for path design were developed. The amount of time required for each section can be specified. They worked on Sekwa aircraft.

The techniques for identification are used specifically for accurate representation of system of aircraft. Many studies might use nominal values but this will reflect in their works. Some studied attitude estimation using Kalman filter and artificial intelligence as in [9] [10] [11] and the results are great but still some errors because of not estimation the aircraft parameters. Optimization also has roles in medicine and communications [12] [13] [14].

In this approach, an extension of Linear regression is used to identify Sekwa aircraft parameters accurately with new mathematical representations. The goals of this search is to implement Linear regression with an extension for aircraft parameters estimation for a specific aircraft, the linear regression is implemented with interior point optimization algorithms , but our aim is to extend this to get more accurate results. The structure of this paper is as follows. Section 2 deals with the developed algorithms including the nonlinear UAV models, the basic regression model the interior point optimization, the extended linear regression, and FireFly optimization. Section 3 presents and analyses the identification results based on standard linear (LR) and extended linear regression (ELR), and FireFlytechnique. Since the default parameter values of the Sekwa UAV are available hence the identified parameter values are also compared in the parameter domain. The paper is finished with the Conclusions, and References.

# 2 Developed Algorithm

## 2.1 Nonlinear UAV Model

Fixed wing propeller driven aircrafts are nowadays popular for both long distance military and civilian applications. The paper assumes that the reader is familiar with the fundamentals of the nonlinear dynamic model and control of fixed wing aircraft. There are excellent classical books on this field, specialities regarding UAVs can be found in [9] [10]. The paper concentrates on the parameter identification of the models, see here only some details based on [16]. It is assumed that registered flight data are available obtained by data logging either within teleoperation or online control. The identification can be performed using batch-processing or online. Bach-processing makes it easy to use special sotwares, state estimation and filtering and differentiation of the estimated signals before starting the identification process. The notations used are the well spread ones in vehicles and robotics literature.

The use of coordinate systems (frames) are preferred, namely Kn, Kb, Ks and Kw are the Flat-Earth, body, stability axis and wind axis frames, respectively. The parametrisation of the model can often be performed in the stability axies frame or in the wind axis frame. Denote $v_b$ and $\omega_b$ the body linear velocity and angular velocity, the relative air speed is $v_r = v_b - R_n^b v_{wind}^n$ where the wind velocity is constant and $R_n^b$ transforms vectors from Kn to Kb, $m$ is the mass and $J = I_c$ is the inertia matrix, and $F_B$ and $T_b$ are the resulting external force and torque satisfying the Newton-Euler equations:

$$\dot{v}_b = -\omega_b \times v_b + F_B / m \quad \text{and} \quad J\dot{\omega}_b = -\omega_b \times (J\omega_b) + T_B \tag{1}$$

where the force and moment effects are

$$F_B = (F_x, F_y, F_z)^T = F_{BA} + F_{BT} + F_{BG} \quad \text{and} \quad T_B = (\overline{L}, M, N)^T = T_{BA} + T_{BT} \tag{2}$$

Here the second letters A,T and G in the indexes denote the aeodynamic, trust and gravity effects, respectively. The (nongravity) forces and torques depend on the wing reference area $S_{wa}$, the free-stream dynamic preassure $\overline{q} = 1/2\rho v_T^2$, different dimensionless coefficients $C_D, C_L, C_Y, C_l, C_m, C_n$ and, in the case of the torques, on the wing span $b$ and the wing mean geometric chord $\overline{c}$. The dimensionless coefficients depend in first line on the angle of attack $\alpha$ and the sideslip angle $\beta$, the control surfaces and the Mach-number:

$$\begin{aligned}
D_{stab} &= \overline{q} S_{wa} C_D && \text{drag} \\
L_{stab} &= \overline{q} S_{wa} C_L && \text{lift} \\
Y &= \overline{q} S_{wa} C_Y && \text{sideforce} \\
\overline{L} &= \overline{q} S_{wa} b C_l && \text{rolling moment} \\
M &= \overline{q} S_{wa} \overline{c} C_m && \text{pitching moment} \\
N &= \overline{q} S_{wa} b C_n && \text{yawing moment}
\end{aligned} \tag{3}$$

Notice that the lift force $C_L$, the drug force $C_D$, etc. are usually defined in the stability frame, not in the wind-axes frame. The relative air-speed can be transformed using elementary transformations in the wind-axis frame by $v_r = R_w^b v_w = Rot(y, -a) Rot(z, \beta) v_w$ where $v_w = (1,0,0)^T v_T$. For the stability frame $\beta = 0$. The resulting external force in the body frame is

$$F_B = \overline{q} S_{wa} (C_X, C_Y, C_Z)^T + (1,0,0)^T F_T + m g_B \tag{4}$$

where $F_T$ is the thrust force and $g_B$ is the gravity acceleration in the body frame. The drug and lift forces in the stability frame satisfy

$$\begin{aligned}(-C_D, C_Y, -C_L)^T &= Rot(y,-\alpha)^T (C_X, C_Y, C_Z)^T \\ (-C_{DW}, C_Y, -C_{LW})^T &= Rot(z,\beta)^T (-C_D, C_Y, -C_L)^T\end{aligned} \tag{5}$$

In order to obtain forces and moments in standard SI dimensions (N and Nm) the dimensionless components should be multiplied by appropriate weighting functions. For example, the angular velocity in the stability frame can appear in the model as weighting function where $\omega_S = Rot(y,-\alpha)^T \omega_b$. Hence, identifying the parameters appearing in the model, the linear parameter estimation is embedded in the nonlinear models through the weighting functions. First, the user chooses the structure of the model, i.e. the number of parameters, the form of the high-level functions and the weighting signals in them:

$$C_i = C_i(p_{i1}, p_{i2}, \ldots, s_{i1}, s_{i2}, \ldots), \quad i \in \{D, L, Y, C_X, \ldots, C_n\} \tag{6}$$

Where $p$ denotes parameters and $s$ denotes weighting signals of the function. For constant weight $s=1$ is allowed.

On the other hand, the structure of the nonlinear model can contain nonlinear relations too, for example the drug may depend on the square of the lift, making the parameter estimation nonlinear or constrained linear. Such a situation is typical for many aircraft in steady state:

$$\begin{aligned}C_L &= C_{L0} + C_{L\alpha}\alpha \\ C_D &= C_{D0} + \frac{C_L^2}{\pi A_{sr} e_{Osw}} = \overline{C}_{D0} + \overline{C}_{D1}\alpha + \overline{C}_{D2}\alpha^2\end{aligned} \tag{7}$$

Where $A_{sr}$ and $e_{Osw}$ denote the wings aspect ratio and the Oswald efficiency factor, respectively. Taking the square, the introduced new parameters $\overline{C}_{D0}, \overline{C}_{D1}, \overline{C}_{D2}$ make the problem formally similar to linear parameter estimation in these parameters but it is evident that they are in relation with $C_{D0}$, $C_{L0}$ and $C_{L\alpha}$ generating constraints amongst them. Such a situation appears for the Sekwa fixed wing UAV causing problem in the identification. Neglecting the constraints for the parmeters $\overline{C}_{D0}, \overline{C}_{D1}, \overline{C}_{D2}, C_{D0}, C_{L0}, C_{L\alpha}$ the usual least square parameter esttimation is not necessarily convergent in $C_{D0}, C_{L0}, C_{L\alpha}$ to the correct values. Notice that this property is in force also for other methods if in them dominate the linear character together with some noise extension. Fortunately, the constraints are simple and sparse, hence using nonlinear optimization with constraints (see for example fmincon with options, starting from LS generated initial values) give an extra chance for parameter improvement. Returning back to the linear parameter estimation problem, one can assume that from the flight data and state estimation the states in the differential equations are available and the differentiation of the signals has already been performed. The LS problem can be considered in the form

$y(t) = \varphi^T(t)\vartheta + e(t)$ where $y(t)$ is a vector coming from the differential equations and $\vartheta$ is the (full) parameter vector. In case of Sekwa UAV the set of weighting functions may be

$$(1, \alpha, \alpha^2, \beta, \hat{p}, \hat{q}, \hat{r}, \delta_e, d_a, \delta r, \delta_{th}) \tag{8}$$

and in case of $C_D = (\overline{C}_{D0}, \overline{C}_{D1}, \overline{C}_{D2})$ this component can be represented by

$$(1, \alpha, \alpha^2, \beta, \hat{p}, \hat{q}, \hat{r}, \delta_e, d_a, \delta r, \delta_{th}) \times (1,1,1,0,\ldots,0)^T \tag{9}$$

Similar relations can be found for the other components in the differential equations. In this way the derivatives of the state equations subtracted by signals not containing identifiable parameters in the state equation playes the role of $y(t)$ and the right side is as above for $C_D$. Then, the components can be collected for every sampling time multiple $t$ and the LS problem can be built up and solved. A similar technique can be used for the examined more complex problems in the paper.

## 2.2 Problem Statement

After defining parameters and basic coefficients, the main problem of this research is how to estimate and predict these coefficients depending on real data. In regard, three main method are illustrated which will be explained in the next sections: Linear Regression (LR), Extended Linear Regression (ELR) and Optimized ELR by Firefly optimization algorithms. We deal with the solution of the parameter identification problem at two levels.

1) Normally, the results can be tested only to demonstrate that the output signals in the flight data can be well matched if the simulated model, inside with the identified parameters, is driven with the input (actuator) signals of the flight data. This can be checked in open loop or in closed loop under control. Notice that integral components in closed loop can reduce the effect of errors while matching during open loop testing is more difficult. Here, dominates signal comparison.
2) In rare situations the internal parametrs of the aircraft may also be known and we can test whether the default parameters can also be reconstructed using identification. Here, dominate parameter comparision, which is more general.

The studied aircraft was the Sekwa UAV with available known default model parameters. The details can be found in [15] [16] [17]. Hence. it was also possible that beside the convergence of the parameter identification also the parameter errors could have been tested. The comparison can be found later in table. Next section will explain the general regression model and Firefly optimization algorithm.

## 2.3 Basic Regression Model

The main idea of our work is re-modelling the mathematical equation of aircraft model from a new view, suppose the next model:

$$f_j = \sum_{i=1}^{M} H_j(x_i) + b_j + c_j + d_j, \quad j \in [1, n] \tag{10}$$

Where: $n$: is the number of all samples. $M$ number of variables to be estimated. $a, b, c$ are known values, $H_j$ is nonlinear function in general (it might be linear). $f$ is the dependent variable that can be measured or observed. $x_i$ are cofficients to be estimated (it is vector). As an example, $f$ might be the position and $x_i$ might be the velocity and $a, b, c$ are parameters that are initial values or noise parameters. In linear case, they reduce to next (* is the normal multiplication):

$$f_j = \sum_{i=1}^{M} H_{ji} * x_i + b_j + c_j + d_j, \quad j \in [1, n] \tag{11}$$

Whereas $H_{ji}$ are independent known variables. (H could be represented by vector)

$$\sum_{i=1}^{M} H_{ji} * x_i = \left[ H_{j1}, H_{j2}, \ldots, H_{jM} \right] * \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_M \end{bmatrix}$$

To solve this issue

$$f_j - \left(b_j + c_j + d_j\right) = \sum_{i=1}^{M} H_{ji} * x_i, \quad j \in [1, n] \tag{12}$$

And it became as a normal regression model; note that $H_i$ might contain old values of $f_i$, as an example $H_5 = h_1 x_1 + f_2 x_3 + \cdots$. To solve this type of regression model, we need to build equation as follows

$$\begin{gathered} Y = f_j - \left(b_j + c_j + d_j\right), \quad A \cdot X = \sum_{i=1}^{M} H_{ji} * x_i \\ Y = A \cdot X \\ X = \left(A\right)^{-1} \cdot Y \end{gathered} \tag{13}$$

Where, $\boldsymbol{A}$ is a matrix with $n$ rows (number of samples) and $M$ columns (number of parameters to be estimated. The matrix $\boldsymbol{A}$ is not square. The next equation show dimensions

$$\underset{n*1}{Y} = \underset{n*M}{A} \cdot \underset{M*1}{X} \tag{14}$$

Pseudo inverse matrix is needed here to calculate the vector $X$. The Sekwa model equations will be used to find the aircraft coefficients (Force equations, Torque equations and Navigation equations).

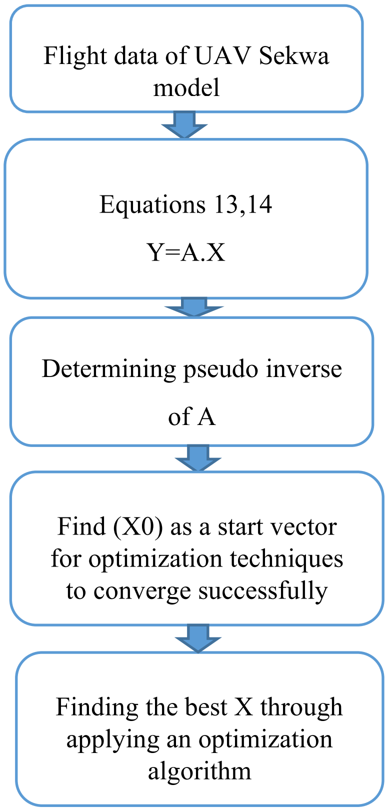



Figure 2
Flowchart of Sekwa model parameters estimation

Linear regression depends on Interior point optimizsation, which will be explained next.

### 2.3.1 Interior Point Optimization Function

Interior point algorithms are a certain class of algorithms that solve linear and nonlinear convex optimization problems. It enabled solutions of linear programming problems that were beyond the capabilities of the simplex method. Contrary to the simplex method, it reaches a best solution by traversing the interior of the feasible region. The method can be generalized to convex programming based on a self-concordant barrier function used to encode the convex set.

After we get the equation:

$$\underset{n*1}{Y} = \underset{n*M}{A} \cdot \underset{M*1}{X} \tag{15}$$

We run an optimization function to find best value of $X$ , we need this step since we got number of equations larger than number of coefficients. The optimization algorithm in [15] is to find minimum of constrained nonlinear multivariable function, it is available in Matlab [18].

## 2.4 Extended Linear Regression

The above representation of equation is accurate for ideal situations and ideal flight environment, so new term is added to each equation to get close to accurate calculations, but this added term is not for noise. Hence, if we want to simulate noise state too, we have to add $n$ term for $n$ sample. For aircraft, each coefficient might be dependent from all other ones; so if the number of coefficients is M, another M terms will be added to extend the regression as follows:

$$f_j = \sum_{i=1}^{M} H_j(x_i) + b_j + c_j + d_j + \sum_{i=1}^{M} G_j\left(eps_i\right), j \in \left[1, n\right] \tag{16}$$

The aim is to estimate $x_1, x_2, \dots, x_M$ so we add $eps_1, eps_2, \dots, eps_M$ term. Whereas $G_j$ are independent known variables . To solve this issue in linear case:

$$G_j(eps_i) = G_{ji}.f_j.esp_i$$

$$f_j - \left(b_j + c_j + d_j\right) = \sum_{i=1}^{M} H_j \cdot x_i + \sum_{i=1}^{M} G_{ji}.f_j.eps_i\,, j \in [1, n] \tag{17}$$

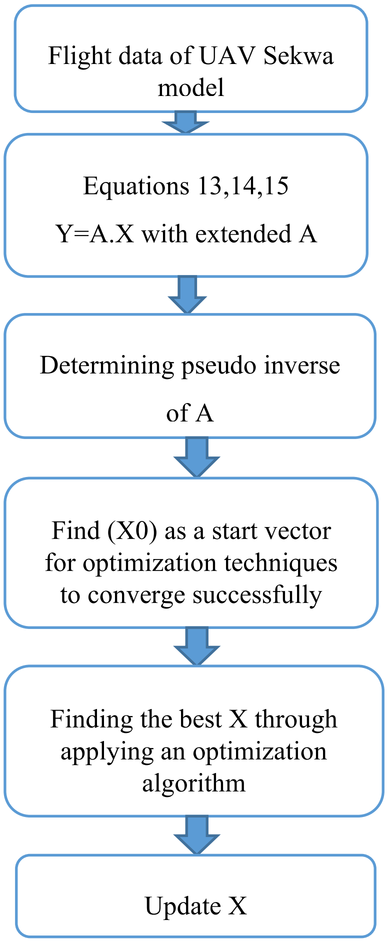
Suppose the term $\tilde{X} = [x_1, x_2, \dots x_M, eps_1, eps_2, \dots eps_M]$ whereas

$$\underset{n*1}{Y} = \underset{n*2M}{A} \cdot \underset{2M*1}{X} \tag{18}$$

This will extend the matrix and will end up with more accurate and reality representation of the aircraft model. The $G_{ji}$ are zeroes or ones; this according to the existence if $x_i$. This means if coefficient with index depends on coefficients with index $i$ then $G_{ji} = 1$ and then $eps_i$ should be estimated

$$G_{ji} = \begin{cases} 1 & if\ x_i\ is\ exist\ \ in\ \ \ equation \\ 0 & \ \ if\ x_i\ is\ \ not\ \ in\ \ equation \end{cases} \tag{19}$$

The final desired $X$ might be $X = \left[x_1, x_2, \dots x_M\right]$; this means that the calculated $eps_i$ compensates noise. Figure below shows these stages:




Flowchart of Sekwa model parameters estimation with extended regression

For ELR, MinMax optimization function is used, this will be explained next. It is a ready function in matlab toolbox.

### 2.4.1 MinMax Optimization Function

The optimization function here is "Solve minimax constraint problem". fminimax function searches for the best solution that minimizes the maximum of a set of objective functions. The problem includes any type of constraint. The optimization function needs an objective function to be minimized or in some cases to maximized (minimize loss or error, maximize accuracy) [19] [20] [21]. All optimization algorithms need a loss or an objective function to be minimzed during optimization phase (searching for optimal solution, this will be explained next).

### 2.4.2 Objective Function

For linear problems the tak is to find the solution of linear regression [22] [23] for equation (18):

$$Y = A \cdot X$$

To solve this problem, new objective function can be defined for minimizing:

$$\begin{aligned} fun = \min_X \|Y - A \cdot X\|^2 &= (Y - A \cdot X)^T (Y - A \cdot X) \\ &= X^T A^T A X - 2 X^T A^T Y + Y^T Y \end{aligned} \tag{20}$$

$X$ can be simply determined minimizing $fun$. In our case, it is an optimization problem and need to search for a solution to minimize the objective function. Best solution would be when *fun*=0. Matmatically, $(Y - A \cdot X)^T (Y - A \cdot X) = X^T A^T A X - 2X^T A^T Y + Y^T Y$ can also be solved, especially in case of constraints, by using the preferable wayof numerical optimum seeking.

Constraints may be the lower and upper bound for $X$ to defined. Then the defined values have to be satisfied during the algorithm.

Next, FireFly mehtod will be discussed to help ELR in searching for the best solution.

### 2.4.3 FireFly Optimization Algorithm

One of the newest metaheuristic algorithms for optimization issues is the firefly method. The program takes its cues from firefly flashing behavior. The flashlight is utilized as a warning system to keep the fireflies from potential predators [24]. The program will treat randomly produced solutions as firefly, and brightness will be allocated based on how well they perform on the objective function. They can divide into smaller groups due to their attractiveness, and each group converges around the local models [25]. The following three rules are the fundemantals of firefly described as follows [26]:

1. Fireflies come in both genders.
2. Attractiveness is proportional directly to their brightness.
3. The landscape of the objective function controls a firefly's brightness.

When compared to other algorithms, firefly offers two key advantages: automated subdivision and the capacity to handle multimodality.

The key parameters of the Firefly Algorithm are as follows:

**Population Size (n)**: The number of fireflies. We used n=150.

**Light Intensity (I)**: This represents the objective function value or fitness of a solution. Fireflies are attracted to brighter fireflies, meaning that solutions with higher light intensity values are considered better.

**Absorption Coefficient (γ)**: This parameter represents the light absorption during the propagation of light. It's used to reduce the attractiveness of a firefly based on the distance between them. If the distance is high, this means that this solution is not close, so will decrease attractive parameter. Where **Absorption Coefficient (γ)=0.99.**

**Maximum Generations (MaxGen)**: The number of iterations or generations the algorithm will run before terminating. It controls the stopping criterion for the optimization process. Where **Maximum Generations=500.**

**Objective Function**: The mathematical function that represents the problem to be optimized. This represents the function in equation (20).

**Search Space and Bounds**: This represents the constraints. It should be mentioned the initial values were determined after many experiments.

The problem is searching for best X that minimizes the function in equation (20). Firefly will be adopted to search for this X by its fireflies. Simple flowchart is shown below:

**Begin**
1) Objective function $f(x)$ to be minimized. As in equation (20)
2) Generate an initial population of fireflies (fireflies number must be defined); Each firefly will search for the best value of X values in Equation (20), they will attract other fireflies when they found minimum values to repopulate the fireflies near the minimum zone and search again till they reach minimum value of function in Equation (20)
3) Formulate light intensity $I$ so that it is associated with $f(x)$
4) Define absorption coefficient $\gamma$
**while** (t < MaxGeneration)
**for** i = 1 : n (all n fireflies)
**for** j = 1 : i (n fireflies)
**if** ($I_j > I_i$),
Vary attractiveness with distance $r$ via $e^{-\gamma r}$ ;
move firefly i towards j;
Evaluate new solutions and update light intensity; (the new solution here is the vector X)
**end if**
**end for** j
**end for** i
Rank fireflies and find the current best loss function that in equation (20);
**end while**
**end**

This algorithm is applied after extended regression is done. After implementing FireFly, the initial input for this algorithm is

$$x_0 = \alpha * x_{elr} + (1-\alpha) * x_{erlm} \tag{21}$$

Where $x_{elrm}$ is the modified extended linear regression output by using the extended parameters, $x_{elr}$ is a vector with 52 parameters (first 26 parameters are the basic parameters and the other 26 parameters are the extended parameters).

$$x = \left[x_{elr}, x_{erlm}\right]$$

If we consider that the type of the $x_{erlm}$ is not additive; the equation will be the following:

$$x_0 = \alpha * x_{elr} + (1-\alpha) * x_{elr} * x_{erlm} \tag{22}$$

This is known as multiplicative error. Then, Y is recalculated according to the initial values:

$$\underset{n*1}{Y} = \underset{n*2M}{A} \cdot \underset{2M*1}{X} \tag{23}$$

The algorithm stops when it reaches max generation. The algorithm saves the best solution each iteration, the solution contains the values of X and the value of function in Equation (20). The next section will illustrate the results of the developed algorithm. The X in Equation (20) is known for the aircraft, the developed algorithm will result also new X values, then there is a comparison to evaluate the developed algorithm to be used with another aircraft.

# 3 Identification Results

## 3.1 Flight Data

The used flight data is sampled by 0.01 seconds, it contains a real data of trip with all controls, and it contains all sensors data: accelerometers, gyroscopes and angular velocities, position data, velocity data and quaternion data.

## 3.2 Parameters Reconstruction

The code will result in a vector of length $M$ ; it contains all target parameters that are explained in Table 1. These parameters used exist in [9, 10]. The flowchart in Figures 2 and 3 illustrated estimation of Sekwa model parameters.

## 3.3 Extended Linear Regression Results

To check the quality of the algorithm for later unknown airplanes, the X in Equation (20) was assumed here to be known (it has 26 parameters). The developed algorithm will result also new $\tilde{X}$ values that are solutions of the optimization algorithm. Comparing them, the quality of the method can be judged for later use if the correct parameters are unknown. The real values of Sekwa UAV parameters are from [15]. LR and ELR were implemented with interior point optimizations. The figures below show the results of LR and ELR regression. The y axis in next plots refers to true values of coffiecients that was explained in the introduction.

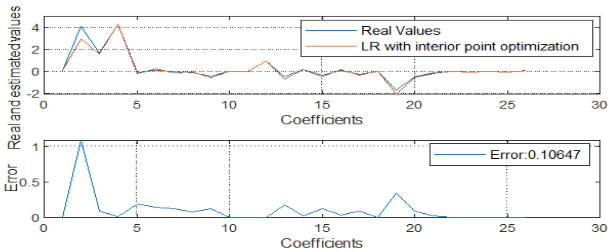



Figure 4

LR with interior point results

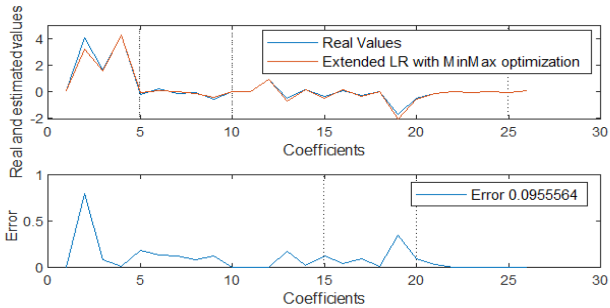



Figure 5

Extended LR with Min Max results

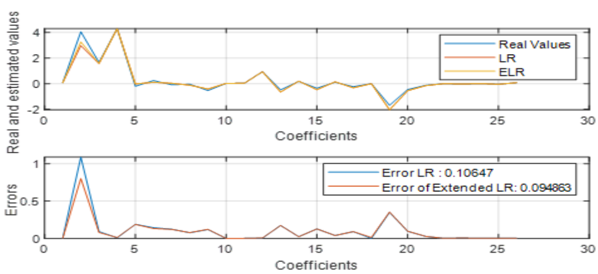



Figure 6

Compare results of LR and Extended LR

The formula for calculating the improvement ratio is used to minimize the relative error of two methods. It refers to the absolute value of the difference of the errors of (LR&Extended LR) divided by the error of LR.

$$improve\ ratio = \frac{abs(0.10647 - 0.094863)}{0.10647} = 10.9\%$$

## 3.4 FireFly Results

The estimation is shown below, where the figure contains our developed method Firefly ELR, linear regression and Firefly without the regression.

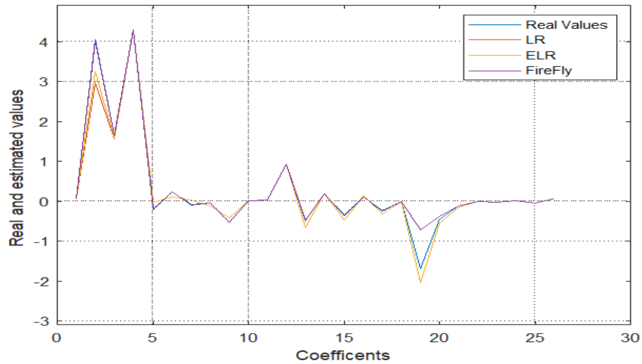



Figure 7
FireFly results

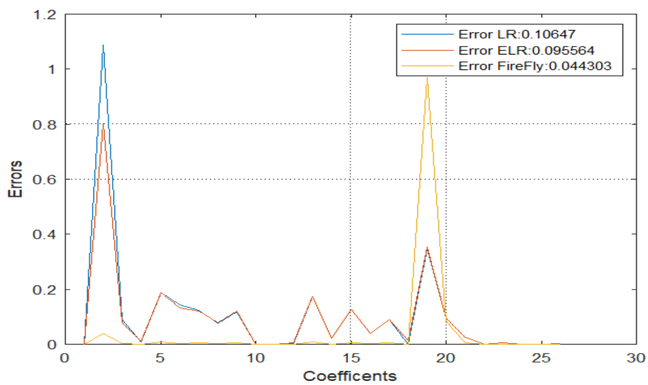



Figure 8
All methods results

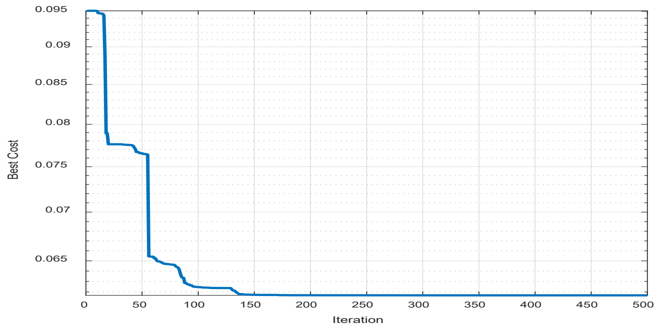



Figure 9
FireFly Cost function

## 3.5 Comparison of Parameter Estimation Results

The table below shows the comparison between our approach, Basic LR, only FireFly and FireFly ELR comparing to the true values.

Table 1
Results comparison

| True value | FireFly | LR (Interior point) | Firefly ELR (Min Max) | name |
|---|---|---|---|---|
| 0.0633 | 0.0633 | 0.0629 | 0.0625 | $C_{L0}$ |
| 4.0543 | 4.0324 | 2.9675 | 3.2533 | $C_{Lq}$ |
| 1.6524 | 1.6529 | 1.5611 | 1.5732 | $C_{L\delta e}$ |
| 4.3 | 4.3006 | 4.2918 | 4.2882 | $C_{L\alpha}$ |
| -0.2114 | -0.2021 | -0.0236 | -0.0244 | $C_{Yp}$ |
| 0.2409 | 0.2345 | 0.0977 | 0.1096 | $C_{Yr}$ |
| -0.094 | -0.0879 | 0.0284 | 0.0272 | $C_{Y\delta a}$ |
| -0.0478 | -0.0516 | -0.1231 | -0.1251 | $C_{Y\delta r}$ |
| -0.5401 | -0.5342 | -0.4216 | -0.419 | $C_{Y\beta}$ |
| -0.4848 | -0.4934 | -0.6578 | -0.6578 | $C_{lp}$ |
| 0.1704 | 0.1715 | 0.1933 | 0.193 | $C_{lr}$ |
| -0.352 | -0.3584 | -0.4787 | -0.4787 | $C_{l\delta a}$ |
| 0.1056 | 0.1075 | 0.1443 | 0.1443 | $C_{l\delta r}$ |
| -0.2381 | -0.2426 | -0.3282 | -0.3282 | $C_{l\beta}$ |
| 0 | 0 | 0 | 0 | $C_{m0}$ |
| -1.6945 | -0.7247 | -2.0426 | -2.0426 | $C_{mq}$ |
| -0.4583 | -0.3731 | -0.5528 | -0.5528 | $C_{m\delta e}$ |
| -0.1288 | -0.1238 | -0.1552 | -0.1552 | $C_{m\alpha}$ |
| -0.0021 | -0.0021 | -0.0029 | -0.0029 | $C_{np}$ |

| | | | | |
|---|---|---|---|---|
| -0.0354 | -0.0351 | -0.0296 | -0.0299 | $C_{nr}$ |
| 0.0018 | 0.0018 | 0.0016 | 0.0016 | $C_{n\delta a}$ |
| -0.0478 | -0.0478 | -0.0489 | -0.0489 | $C_{n\delta r}$ |
| 0.0658 | 0.0659 | 0.0679 | 0.0679 | $C_{n\beta}$ |
| 0.0001* [185,275,934.4] | 0.0001* [185,275, 934.2] | 0.0001* [185,273,930.9] | 0.0001* [185,271,929.3] | $C_D$ |

The results clearly show that our approach is accurate and the plots above present also the improvement ratio. The final results demonostrate that the order of the improvement in the parameter estimation results is 10.9% of the parameters and the error between real and estimated parameter values decreased around 6.3%.

## Conclusions

This work showed the importance of the mathematical representation of extended linear regression algorithm for aircraft system model. The idea is to represent the equation from another view to get the model to more accurately represent reality. The estimation of parameters is improved, this can be used later in aircarft with unkown parameters to use them in other topics of aircraft applications. The improvement ratio declared that the Extended Linear Regression with FireFly method is better than normal Linear Regression and this can be used in many problems of model analysis in different fields of applications.

## References


[1] M. Mohamed: System identification of flexible aircraft in frequency domain, Aircraft Engineering and Aerospace Technology, Vol. 89, No. 6, pp. 826-834, 2017

[2] M. K. Samal, A. Singhal, and A. K. Ghosh: Estimation of equivalent aerodynamic parameters of an aeroelastic aircraft using neural network, Journal of the Institution of Engineers (India), Aerospace Engineering Division, Vol. 90, pp. 3-9, 2009

[3] H. N. Al-sudany, B. Lantos: Comparison of Adaptive Fuzzy EKF and Adaptive Fuzzy UKF for State Estimation of UAVs Using Sensor Fusion. Periodica Polytechnica Electrical Engineering and Computer Science, Vol. 66, No. 3, pp. 215-266, 2022

[4] H. N. Al-sudany, B. Lantos: Prediction of the Navigation Angles Using Random Forest Algorithm and Real Flight Data of UAVs, IEEE 20th Jubilee International Symposium on Intelligent Systems and Informatics (SISY), pp. 000097-000102, 2022

[5] M. Mohamed, V. Dongare: Aircraft Aerodynamic Parameter Estimation from Flight Data Using Neural Partial Differentiation, Springer Nature, 2021

[6] J. A. Grauer, M. J. Boucher: Aircraft system identification from multisine inputs and frequency responses. Journal of Guidance, Control, and Dynamics, Vol. 43, No. 12, pp. 2391-2398, 2020

[7] L. I. N. Chun-Yueh: Fuzzy AHP-based prioritization of the optimal alternative of external equity financing for start-ups of lending company in uncertain environment. Sci. Technol, Vol. 25, No. 2, pp.133-149, 2022

[8] G. G. Seo, Y. Kim, S. Saderla: Kalman-filter based online system identification of fixed-wing aircraft in upset condition. Aerospace Science and Technology, Vol. 89, pp. 307-317, 2019

[9] Z. Bodó, B. Lantos: Modeling and control of fixed wing UAVs. IEEE 13th International Symposium on Applied Computational Intelligence and Informatics (SACI), pp. 332-337, 2019

[10] R. F. Stengel: Some effects of parameter variations on the lateral-directional stability of aircraft. Journal of Guidance and Control, Vol. 3, No. 2, pp. 124-131, 1980

[11] A. Assad, W. Khalaf, I. Chouaib: Radial basis function Kalman filter for attitude estimation in GPS-denied environment. IET Radar, Sonar & Navigation, Vol. 14, No. 5, pp. 736-746, 2020

[12] G. Rigatos, P. Siano, D. Selisteanu, R. E. Precup: Nonlinear optimal control of oxygen and carbon dioxide levels in blood. Intelligent Industrial Systems, Vol. 3, pp. 61-75, 2017

[13] D. Singh, A. Shukla: Manifold optimization with MMSE hybrid precoder for Mm-Wave massive MIMO communication. Sci. Technol, Vol. 25, No. 1, pp. 36-46, 2022

[14] C. A. Bojan-Dragos, R. E. Precup, S. Preitl, R. C. Roman, E. L. Hedrea, A. I. Szedlak-Stinean: GWO-based optimal tuning of type-1 and type-2 fuzzy controllers for electromagnetic actuated clutch systems. IFAC-PapersOnLine, Vol. 54, No. 4, pp. 189-194, 2021

[15] Blaauw, Deon: Flight control system for a variable stability blended-wing-body unmanned aerial vehicle. Diss. Stellenbosch: University of Stellenbosch, 2009

[16] Broughton, B. A., and R. Heise. :Optimisation of the Sekwa blended-wing-Body research UAV,2008

[17] Z. Bodó: Modern control methods for unmanned aerial and ground vehicles, (Doctoral dissertation, Budapest, Hungary), 2021

[18] R. H. Byrd, J. C. Gilbert, J. Nocedal: A trust region method based on interior point techniques for nonlinear programming. Mathematical programming, Vol. 89, No. pp. 149-185, 2000

[19] Z. Drezner: On minimax optimization problems. Mathematical programming, Vol. 22, pp. 227-230, 1982

[20] M. Milovančević, D. Milčić, B. Andjelkovic, L. Vračar: Train Driving Parameters Optimization to Maximize Efficiency and Fuel Consumption. Acta Polytechnica Hungarica, Vol. 19, No. 3, pp. 143-154, 2022

[21] R. E. Precup, E. L. Hedrea, R. C. Roman, E. M. Petriu, A. I. Szedlak-Stinean, C. A. Bojan-Dragos: Experiment-based approach to teach optimization techniques. IEEE Transactions on Education, Vol. 64, No. 2, pp. 88-94, 2020

[22] Tian Z: Backtracking search optimization algorithm-based least square support vector machine and its applications. Engineering Applications of Artificial Intelligence. 2020 Sep 1;94:103801

[23] B. Lantos, L. Márton: Nonlinear control of vehicles and robots. Springer Science & Business Media, 2010

[24] Johari NF, Zain AM, Noorfa MH, Udin A: Firefly algorithm for optimization problem. Applied Mechanics and Materials. 2013 Dec 12; pp. 512-517

[25] S. K. Sarangi, R. Panda, S. Priyadarshini, A. Sarangi: A new modified firefly algorithm for function optimization. IEEE international conference on electrical, electronics, and optimization techniques (ICEEOT), pp. 2944-2949, 2016

[26] Ezzeldin R, Zelenakova M, Abd-Elhamid HF, Pietrucha-Urbanik K, Elabd S: Hybrid Optimization Algorithms of Firefly with GA and PSO for the Optimal Design of Water Distribution Networks. Water. 2023 May 17;15(10), 1906