On a Class of Control Systems with Takagi-Sugeno PI-Fuzzy Controllers

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Abstract: The paper proposes a new development method dedicated to a class of fuzzy control systems containing Takagi-Sugeno PI-fuzzy controllers to control SISO linear / linearized plants. The considered controllers are characterized by variable number of triangular input membership functions. The method is expressed in terms of relatively simple steps that use the prediction of the limit cycles on the basis of the gain-phase margin analysis and on the design results from the linear case. The method is applied to the development of a class of PI-fuzzy controllers for servo systems, and it is accompanied by the sensitivity analysis with respect to the parametric variations of the controlled plant and validated by a case study.

Keywords: Takagi-Sugeno PI-fuzzy controllers, development method, describing function, limit cycles, sensitivity models, servo systems.

1 Introduction

Fuzzy logic is widely used in modelling and control of complex systems. Depending on the structure of the inference rules, fuzzy systems can be characterized by three categories of models [1]: linguistic fuzzy models, fuzzy relational models, Takagi-Sugeno (TS) fuzzy models, and the first two categories are known as Mamdani fuzzy models.

The main feature of TS fuzzy models can be grouped under the form of the following steps that also point out their operating mode [2]:

- firstly, the input space is decomposed into subspaces;
- then, within each subspace (i.e., fuzzy regions in the input space), the system model can be approximated by simpler models, in particular linear ones;
- it is possible to use conventional controller development techniques to control these relatively simple local models;

- finally, the global fuzzy model in the state-space is derived by blending the subsystems' models in terms of the weighted average of rule contributions.

The presented favourable features determine the TS fuzzy models to be nowadays the most encountered in fuzzy control systems (FCSs).

However, the TS fuzzy models prove to have the following drawbacks:

- the behaviour of the global TS fuzzy model can significantly divert from the expected behaviour obtained by the merge of the local models [1];
- the stability analysis of TS fuzzy modelled systems is difficult because of the complex aggregation of the local models in the inference engine.

The mentioned drawbacks become difficult to handle when there are developed fuzzy controllers to control complex plants including servo systems or linear time-varying (LTV) plants [3].

In the general framework of the qualitative theory of nonlinear dynamical systems, several approaches have been widely used to the stability analysis of FCSs including [4]: the state-space approach, based on a linearized model of the nonlinear dynamical system [5], [6], Popov's hyperstability theory, Lyapunov's stability theory [8], [9], the circle criterion [4], [8], the describing function method referred to also as the harmonic balance method [8], etc.

Current approaches to the describing function method used in the stability analysis of FCSs are focussed on:

- its expression in terms of using the exponential-input describing function technique to FCSs using the representations of the fuzzy controllers (FCs) as multidimensional-multilevel relays [10], [11];
- its application to FCSs based on Mamdani FCs without dynamics [8] and to to multivariable TS FCSs [12];
- its formulation in the case of FCSs employing FCs with dynamics, by using the describing function of the saturation which occurs outside the universe of discourse region of the FCs [13].

All these approaches require the prediction of the limit cycles, specific to the describing function method [14] also called gain-phase margin analysis [15].

The main contribution of this paper is to propose a development method meant for a class of fuzzy control systems containing Takagi-Sugeno PI-fuzzy controllers (TS PI-FCs), with variable number of triangular input membership functions, controlling SISO linear / linearized plants. The method is based on:

- the prediction of the limit cycles in terms of the gain-phase margin analysis, and on

- the generally acknowledged approximate equivalence – under certain well-stated conditions – between FCs and the linear ones [16], [17] resulting in the acceptance of development methods for FCs by employing the merge between the knowledge on conventional linear PI controllers and the experience of experts in controlling the plant.

In this context, there is also proposed a sensitivity analysis method dedicated to the considered class of FCSs based on the proposing sensitivity models with respect to the parametric variations of the controlled plant (CP).

This paper is organized as follows. The next Section presents the considered class of TS PI-FCs in their version with output integration. In Section 3 the gain-phase margin analysis is applied, and the development method is expressed in terms of transparent development steps. Then, Section 4 is dedicated to the derivation of the sensitivity models which enable the sensitivity analysis of the FCSs when controlling a class of servo systems. Section 5 deals with an application accompanied by digital simulation results and the last Section outlines the conclusions.

2 A Class of Takagi-Sugeno PI-Fuzzy Controllers

The structure of the considered FCS is a conventional one, presented in Fig. 1 (a), where: r – the reference input, y – the controlled output, $e = r - y$ – the control error, u – the control signal, d1, d2, d3 – the disturbance inputs and the CP includes the actuator and the measuring device.

The TS PI-FC is a discrete-time FC with dynamics, introduced by the numerical differentiation of the control error ek expressed as the increment of control error, Δek, $\Delta e_k = e_k - e_{k-1}$, and by the numerical integration of the increment of control signal, Δuk, $u_k = u_{k-1} + \Delta u_k$, where k is the index of the current sampling interval (T_s – the sampling period). The TS PI-FC structure is shown in Fig. 1 (b), where B-FC represents the basic fuzzy controller, without dynamics.

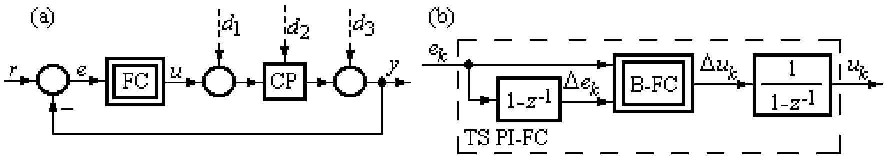

Figure 1
Structure of FCS (a) and of TS PI-FC with output integration (b)

The block B-FC is a nonlinear two inputs-single output (TISO) system, which includes among its nonlinearities the scaling of inputs and output as part of its fuzzification module.

Since the CP is a continuous-time system and the FC is a discrete-time one, the zero-order hold is necessary to be taken into account. By choosing the value of T_s in accordance with the requirements of quasi-continuous digital control (for example, [18]), the following relations can be written between the discrete-time TS PI-FC in Fig. 1 (b) and the continuous-time FC in Fig. 1 (a):

$$e_k \text{ corresponds to } e(t),\ \Delta e_k / T_s \text{ corresponds to } \dot{e}(t),$$
$$\Delta u_k / T_s \text{ corresponds to } \dot{u}(t). \tag{1}$$

Therefore, it is justified and convenient for the aspects presented in the sequel to consider that the TS PI-FC is a TISO system with the input variables e and $\dot{e}$, and the output variable $\dot{u}$ (the continuous time variable t is omitted). In these conditions, the inference rules belonging to the rule base of the continuous-time TS PI-FC can be expressed as:

$$R_{i,j} : \text{IF } \{e \text{ is } M_i \text{ AND } \dot{e} \text{ is } N_j\}$$
$$\text{THEN } \{\dot{u} = \dot{u}_{i,j} = k_{Ci,j} e + (k_{Ci,j} / T_{ii.j})\dot{e}\},\ i = -\overline{m, m},\ j = -\overline{n, n}, \tag{2}$$

where: Mi and Nj are the linguistic terms corresponding to the input linguistic variables e and $\dot{e}$, respectively (Fig. 2); kCi,j are the gains and Tii,j are the integral time constants of the linear PI controllers in the consequent part of Ri,j.

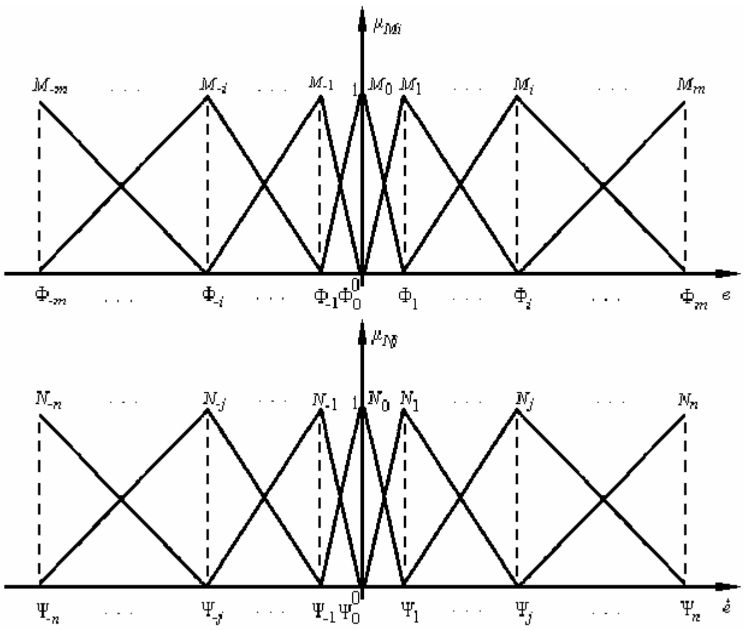

Figure 2
Shapes of input membership functions of TS PI-FC

The triangular input membership functions of the TS PI-FC μMi and μNj corresponding to the linguistic terms Mi and Nj are expressed in (3) and (4), respectively according to the approach in [14]:

$$\mu_{Mi}(e) = \begin{cases} (e-\Phi_{i-1})/(\Phi_i-\Phi_{i-1}), & \text{if } \Phi_{i-1} \le e < \Phi_i, \\ (e-\Phi_{i+1})/(\Phi_i-\Phi_{i+1}), & \text{if } \Phi_i \le e < \Phi_{i+1},\ i=\overline{-m,m}, \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

$$\mu_{Nj}(\dot{e}) = \begin{cases} (\dot{e}-\Psi_{j-1})/(\Psi_j-\Psi_{j-1}), & \text{if } \Psi_{j-1} \le \dot{e} < \Psi_j, \\ (\dot{e}-\Psi_{j+1})/(\Psi_j-\Psi_{j+1}), & \text{if } \Psi_j \le \dot{e} < \Psi_{j+1},\ j=\overline{-n,n}, \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

and the parameters of the TS PI-FC, which are the parameters of the input membership functions, fulfil the condition (5):

$$\Phi_{-i} = -\Phi_i,\ \Psi_{-j} = -\Psi_j,\ i=\overline{-m,m},\ j=\overline{-n,n}. \tag{5}$$

These parameters will be determined by the development method of the FC to be presented in the following Section.

The condition (5) together with the symmetric shapes of the input membership functions (Fig. 2) ensures that the nonlinearity of the fuzzy controller is an odd function, required by the application of the describing function method [8]. The second condition stating that the FCS should have only one time-invariant and memoryless nonlinear component, the block B-FC, is also fulfilled. The third condition requires that the CP should have the characteristics of a low-pass filter, and this condition restricts the area of application of the considered class of Takagi-Sugeno PI-fuzzy controllers.

By using the singleton fuzzifier, the product inference method and the weighted average method for defuzzification, the block B-FC of the TS PI-FC FC can be expressed by (6) corresponding to a TS fuzzy dynamic model [1], [3], [14]:

$$\dot{u} = f(e,\dot{e}) = \sum_{i=-m}^{m} \sum_{j=-n}^{n} \frac{\mu_{Mi}(e)\mu_{NJ}(\dot{e})}{\sum_{ii} \sum_{jj} \mu_{Mii}(e)\mu_{NJi}(\dot{e})} \dot{u}_{i,j} = \sum_{i=-m}^{m} \sum_{j=-n}^{n} \Omega_{i,j}(e,\dot{e})\dot{u}_{i,j} , \quad (6)$$

where f is the nonlinear input-output map of the block B-FC.

To apply the gain-phase margin analysis there will be presented as follows without proof a definition, a lemma and a theorem [14]:

Definition 1 It is supposed that there are fed the harmonic signals $e(t) = A\sin(\omega t)$ and, therefore, $\dot{e}(t) = \omega A\cos(\omega t)$, to the fuzzy dynamic model of B-FC, where $\Phi_{nn} \le A < \Phi_{nn+1}$ and $\Psi_{mm} \le \omega A < \Psi_{mm+1}$. Then there are defined the following sets of parameters:

$$\begin{aligned}
&\alpha_i = \arcsin(\Phi_i / A) \in [0, \pi/2),\ i = \overline{0, nn},\\
&\alpha_i = \pi - \alpha_{2nn+1-i} \in [\pi/2, \pi),\ i = \overline{nn+1, 2nn},\\
&\beta_0 = 0,\ \beta_j = \arccos(\Psi_{mm-j+1} / \omega / A) \in (0, \pi),\ j = \overline{1, 2mm+1},\\
&\beta_{2mm+2} = \pi.
\end{aligned} \quad (7)$$

Furthermore, the set of parameters γ_k, $k = \overline{0, hh}$, is defined as the sorted values of αi and βj in their ascending order.

Lemma 1 For the inputs fed to the FC varying within the domains where $\Phi_{kk} \le e < \Phi_{kk+1}$ and $\Psi_{ll} \le \dot{e} < \Psi_{ll+1}$, the TS fuzzy dynamic model of the block B-FC (6) is re-expressed as the bilinear expression (8):

$$\begin{aligned}
\dot{u} &= f(e,\dot{e}) = \sum_{i=-pp}^{pp} \sum_{j=-qq}^{qq} \Omega_{i,j}(e,\dot{e})\dot{u}_{i,j} = \\
&= a_{kk,ll}(e/\Delta\Phi_{kk})(\dot{e}/\Delta\Psi_{ll}) + b_{kk,ll}(e/\Delta\Phi_{kk}) + c_{kk,ll}(\dot{e}/\Delta\Psi_{ll}) + d_{kk,ll} ,
\end{aligned} \quad (8)$$

where the parameters are defined as:

$$\Delta\Phi_{kk} = \Phi_{kk} - \Phi_{kk+1},\ \Delta\Psi_{ll} = \Delta\Psi_{ll} - \Delta\Psi_{ll},$$
$$a_{kk,ll} = \dot{u}_{kk,ll} - \dot{u}_{kk,ll+1} - \dot{u}_{kk+1,ll} + \dot{u}_{kk+1,ll+1},$$
$$b_{kk,ll} = [\Psi_{ll+1}(\dot{u}_{kk+1,ll} - \dot{u}_{kk,ll}) + \Psi_{ll}(\dot{u}_{kk,ll+1} - \dot{u}_{kk+1,ll+1})] / \Delta\Psi_{ll},$$
$$c_{kk,ll} = [\Phi_{kk+1}(\dot{u}_{kk,ll+1} - \dot{u}_{kk,ll}) + \Phi_{kk}(\dot{u}_{kk+1,ll} - \dot{u}_{kk+1,ll+1})] / \Delta\Phi_{kk},$$
$$d_{kk,ll} = (\Phi_{kk+1}\Psi_{ll+1}\dot{u}_{kk,ll} - \Phi_{kk+1}\Psi_{ll}\dot{u}_{kk,ll+1} - \Phi_{kk}\Psi_{ll+1}\dot{u}_{kk+1,ll} +$$
$$+ \Phi_{kk}\Psi_{ll}\dot{u}_{kk+1,ll+1}) / \Delta\Phi_{kk} / \Delta\Psi_{ll}. \tag{9}$$

Theorem 1 The describing function N(A, ω) of the TS fuzzy dynamic model of the block B-FC is expressed in terms of Equation (10):

$$N(A,\omega) = (b_1 + j \cdot a_1), \quad \text{where:} \tag{10}$$

$$a_1 = \frac{2}{\pi}\sum_{i=0}^{hh-1}\{-a_{ki,li}\frac{A^2\omega}{3\Delta\Phi_{ki}\Delta\Psi_{li}}[\cos^3(\gamma_{i+1}) - \cos^3(\gamma_i)] -$$
$$- b_{ki,li}\frac{A}{4\Delta\Phi_{ki}}[\cos(2\gamma_{i+1}) - \cos(2\gamma_i)] + c_{ki,li}\frac{A\omega}{2\Delta\Psi_{li}}[(\gamma_{i+1} -$$
$$- \gamma_i)(\sin(\gamma_{i+1})\cos(\gamma_{i+1}) - \sin(\gamma_i)\cos(\gamma_i)] +$$
$$+ d_{ki,li}[\sin(\gamma_{i+1}) - \sin(\gamma_i)]\},$$
$$b_1 = \frac{2}{\pi}\sum_{i=0}^{hh-1}\{a_{ki,li}\frac{A^2\omega}{3\Delta\Phi_{ki}\Delta\Psi_{li}}[\sin^3(\gamma_{i+1}) - \sin^3(\gamma_i)] +$$
$$+ b_{ki,li}\frac{A}{2\Delta\Phi_{ki}}[(\gamma_{i+1} - \gamma_i) - \sin(\gamma_{i+1})\cos(\gamma_{i+1}) +$$
$$+ \sin(\gamma_i)\cos(\gamma_i)] - c_{ki,li}\frac{A\omega}{4\Delta\Psi_{li}}[\cos(2\gamma_{i+1}) - \cos(2\gamma_i)] -$$
$$- d_{ki,li}[\cos(\gamma_{i+1}) - \cos(\gamma_i)]\}, \tag{11}$$

with γi and hh defined in Definition 1, and ki, li defined to fulfil the conditions (12):

$$\Phi_{ki} \le A\sin\gamma < \Phi_{ki+1},\ \Psi_{li} \le \omega A\cos\gamma < \Psi_{li+1},\ \text{for}\ \gamma_i \le \gamma < \gamma_{i+1}, \tag{12}$$

and the rest of parameters are calculated in Lemma 1.

3 Gain-Phase Margin Analysis-Based Development

The considered class of servo systems representing the controlled plant (CP in Fig. 1 (a)) is characterized by the transfer function (t.f.) (13):

$$H_{CP}(s) = k_P / [(1 + sT_\Sigma)(1 + sT_1)] = B(s) / A(s), \tag{13}$$

where: T1 – large time constant, TΣ – small time constant or time constant corresponding to the sum of parasitic time constants (TΣ << T1), and kP – gain. In these conditions, from a theoretical point of view, HCP(s) has a quasi-integral behaviour, and the following benchmark can be used to approximate (14):

$$H_{CP}(s) = k_P / [s(1 + sT_\Sigma)] . \qquad (14)$$

For (14) the use of linear PI controllers having the t.f. (15) used as linear continuous-time PI controllers (the consequence part in (2)):

$$H_C(s) = k_c (1 + sT_i) / s,\ k_c = k_C / T_i , \qquad (15)$$

tuned in terms of the Extended Symmetrical Optimum (ESO) method [19] can ensure good control system (CS) performance:

$$k_c = 1/(\sqrt{\beta^3} T_\Sigma^2 k_P), T_i = \beta T_\Sigma , k_C = k_c T_i , \qquad (16)$$

where β represents a single design parameter.

The tuning Equations (16) were obtained by the optimization conditions (17):

$$\sqrt{\beta} a_0 a_2 = a_1^2,\ \sqrt{\beta} a_1 a_3 = a_2^2 , \qquad (17)$$

specific to the ESO method, to the closed-loop t.f. with respect to the reference input Hr(s) (the CS structure is that in Fig. 1 (a), with the continuous-time PI controller playing the role of FC):

$$H_r(s) = (b_0 + b_1 s) / (a_0 + a_1 s + a_2 s^2 + a_3 s^3),\ b_0 = a_0,\ b_1 = a_1 . \qquad (18)$$

The ESO method represents a generalization of the Symmetrical Optimum method [20], and it performs the optimization since it guarantees the maximum value of the phase margin φm for constant CP parameters and a minimum φm in the case of variable kP. The CS performance indices {σ1 – overshoot, $\hat{t}_s = t_s / T_\Sigma$ – settling time, $\hat{t}_r = t_r / T_\Sigma$ – rise time, φm} can be modified by the choice of the design parameter β in the recommended domain, 1 < β < 20. A compromise to these indices can be reached by using the diagrams in Fig. 3.

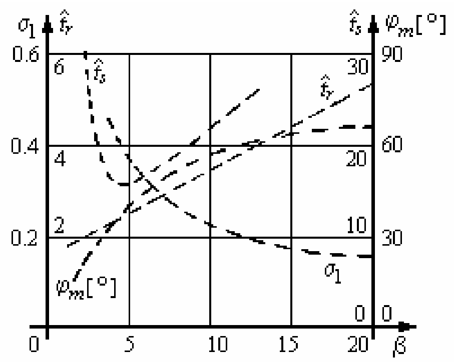

Figure 3
Control system performance indices versus β

The CS performance indices can be improved by adding feedforward filters [19]. This is the way the control structure obtains the features specific to control structures with 2 DOF controllers.

In the case of the controlled plant (13) and the PI controller with the t.f. (15), the coefficients of Hr(s) in (18) can be expressed as (19):

$$a_0 = k_c k_P,\, a_1 = k_c k_P T_1,\, a_2 = T_1 + T_\Sigma,\, a_3 = T_1 T_\Sigma \,. \tag{19}$$

Applying the optimization conditions (17) leads to the tuning Equations (18) corresponding to an extension of the ESO method [21]:

$$k_c = (1+m_P)^3 / (\sqrt{\beta^3} T_\Sigma k_P m_P),\; T_i = \beta T_{\Sigma m},$$
$$T_{\Sigma m} = T_\Sigma [m_P^2 + (2-\sqrt{2}) m_P + 1]/(1+m_P)^3,\; m_P = T_\Sigma / T_1 << 1. \tag{20}$$

Based on the expressions (10) and (13), the characteristic Equation of the closed-loop FCS is (19):

$$g(j\omega) = B(j\omega) + N(A,\omega) A(j\omega) = 0\,, \tag{21}$$

which can include in the forward channel of the open-loop transfer function a gain-phase margin tester ($k \cdot \exp(-j\theta)$) [15]. The gain margin conditions can be obtained as the values of A and k for θ = 0 (two conditions obtained from the real and imaginary part of g in (21)). The phase margin conditions can be obtained as the values of A and θ for k = 0 (also two conditions). These conditions correspond to the limit cycles and must be avoided.

The development method of the TS PI-FCs consists of the following steps:

- step 1: based on the knowledge and experience concerning the CP operation, determine the number of inference rules to control the plant and the partition of the input space in fuzzy regions, and assign the linguistic terms to the input linguistic variables e and $\dot{e}$;

- step 2: for each inference rule of type (2) design the linear PI controller with the parameters $k_{Ci,j}$ and $T_{ii,j}$ by the (extension of the) ESO method;
- step 3: set the parameters of the TS PI-FC (the parameters characterizing the input membership functions in Fig. 2) to ensure a stable FCS by avoiding the limit cycles by applying the modal equivalence principle;
- step 5: validate the TS PI-FC by digital simulation of the CS behaviour for simulation scenarios that take into consideration the significant operating points / trajectories of the controlled plant.

4 Sensitivity Analysis

The sensitivity models (SMs) enable the sensitivity analysis of the FCSs accepted, as mentioned in Section 1, to be approximately equivalent with the linear control systems. This justifies the approach to be presented in the sequel, that the sensitivity models of the FCSs are approximately equivalent to the SMs of the linear CSs. To derive the SMs of the linear CS with respect to the variations of CP parameters kP and TΣ it will be considered the CP structure in Fig. 1 (a) corresponding to a d3 type disturbance input. By considering the state variables x1 (the controlled output), x2 (the output of the integral element) of the CP (14) and x3 (the output of the integral component of the PI controller), tuning the PI controller in terms of (16) will result in the state model of the CS:

$$\begin{aligned}
&\dot{x}_1(t) = -(1/T_\Sigma)x_1(t) + (k_P/T_\Sigma)x_2(t) + (k_P/T_\Sigma)d_3(t),\\
&\dot{x}_2(t) = -[1/(\beta^{1/2}k_{P0}T_{\Sigma 0})]x_1(t) + [1/(\beta^{1/2}k_{P0}T_{\Sigma 0})]x_3(t) +\\
&\qquad + [1/(\beta^{1/2}k_{P0}T_{\Sigma 0})]r(t),\\
&\dot{x}_3(t) = -[1/(\beta T_{\Sigma 0})]x_1(t) + [1/(\beta T_{\Sigma 0})]r(t),\\
&y(t) = x_1(t).
\end{aligned} \tag{22}$$

For the system (22) there can be derived the sensitivity functions {λ1, λ2, λ3} and the output sensitivity function, σ [22]:

$$\lambda_j(t) = [\partial x_j(t)/\partial\alpha]_{\alpha 0},\quad \sigma(\mathrm{t}) = [\partial y(t)/\partial\alpha]_{\alpha 0},\quad \mathrm{j} = \overline{1,3}, \tag{23}$$

where the lower index 0 stands for the nominal values of the controlled plant parameters, $\alpha \in \{kP, T\Sigma\}$.

In this context, the sensitivity analysis can be applied for two parametric variations, of kP and TΣ, and for the dynamic regimes characterized by: the step modification of the reference input r for d3(t) = 0, or the step modification of the disturbance input d3 for r(t)=0. This leads to four SMs obtained by computing the partial derivatives with respect to kP and TΣ, but only two are presented here in (24) (the sensitivity model with respect to the variation of kP, the step modification of r, and d3(t) = 0) and (25) (the sensitivity model with respect to the variation of TΣ, the step modification of r, and d3(t) = 0):

$$\begin{aligned}
&\dot{\lambda}_1(t) = \lambda_2(t),\\
&\dot{\lambda}_2(t) = -[1/(\beta^{1/2}T_{\Sigma 0}^2)]\lambda_1(t) - (1/T_{\Sigma 0})\lambda_2(t) + [1/(\beta^{1/2}T_{\Sigma 0}^2)]\lambda_3(t) -\\
&\qquad - [1/(\beta^{1/2}k_{P0}T_{\Sigma 0}^2)]x_{10}(t) + [1/(\beta^{1/2}k_{P0}T_{\Sigma 0}^2)]x_{30}(t) +\\
&\qquad + [1/(\beta^{1/2}k_{P0}T_{\Sigma 0}^2)]r_0(t),\\
&\dot{\lambda}_3(t) = -[1/(\beta T_{\Sigma 0})]\lambda_1(t),\\
&\sigma(t) = \lambda_1(t);
\end{aligned} \tag{24}$$

$$\begin{aligned}
&\dot{\lambda}_1(t) = \lambda_2(t),\\
&\dot{\lambda}_2(t) = -[1/(\beta^{1/2}T_{\Sigma 0}^2)]\lambda_1(t) - (1/T_{\Sigma 0})\lambda_2(t) + [1/(\beta^{1/2}T_{\Sigma 0}^2)]\lambda_3(t) +\\
&\qquad + [1/(\beta^{1/2}T_{\Sigma 0}^3)]x_{10}(t) + (1/T_{\Sigma 0}^2)x_{20}(t) - [1/(\beta^{1/2}T_{\Sigma 0}^3)]x_{30}(t) -\\
&\qquad - [1/(\beta^{1/2}T_{\Sigma 0}^3)]r_0(t),\\
&\dot{\lambda}_3(t) = -[1/(\beta T_{\Sigma 0})]\lambda_1(t),\\
&\sigma(t) = \lambda_1(t).
\end{aligned} \tag{25}$$

5 Application

To validate the development method and the sensitivity models it is considered a typical application for electrical drives with variable inertia in the field of rolling mills described in [3], for which the CP can be characterized in its simplified form by the t.f. (13) with the parameters $k_P = 1$, $0.8 \text{ s} \le T_\Sigma \le 1.2 \text{ s}$ and $T_1 = 10$ s. The development steps presented in Section 3 have been performed based on the design of three linear PI controllers. For $m = n = 1$ in Fig. 2, the values of the parameters of the TS PI-FC are $\Phi_1 = 0.5$ and $\Psi_2 = 0.0349$. The behaviour of the developed FCS is illustrated in Fig.4 in the simulation conditions characterized by the unit step modification of r followed by a −0.5 step modification of the disturbance input d_3 (after 75 s), where the continuous line is used for y and the dotted line for u. This behaviour proves that the FCS is globally stable.

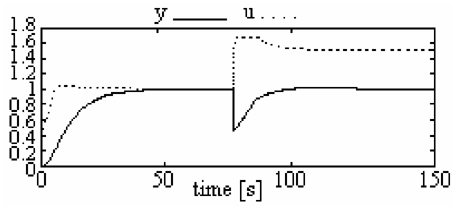

Figure 4
Fuzzy control system behaviour

The behaviors of the SMs (24) and (25), obtained for the simulation scenario that employs a unit step modification of r followed by a unit step modification of d3 (after 250 sec) in the initial conditions λ1(0)=2, λ2(0)=1, λ3(0)=0 are presented in Fig. 5 (a) and (b), respectively.

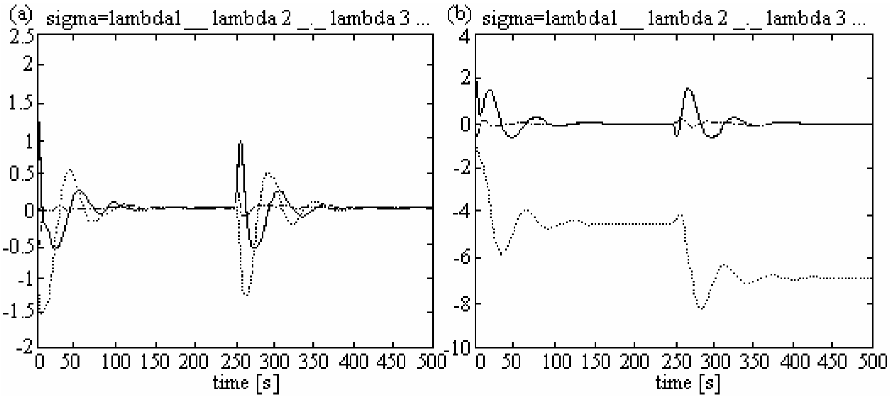

Figure 5
Sensitivity models (24) and (25) behaviour

Conclusions

The paper proposes a development method meant for a class of fuzzy control systems containing Takagi-Sugeno PI-fuzzy controllers with variable number of triangular input membership functions to control a class of second-order SISO linear / linearized plants having low-pass filter characteristics.

The method is expressed in terms of useful development steps focussed on the avoidance of the limit cycles in terms of the gain-phase margin analysis which permits the redesign of the fuzzy controller. The method is based also on the approximate equivalence in certain conditions between fuzzy control systems and the linear ones and on the application of the Extended Symmetrical Optimum method in the linear case.

The paper proposes also sensitivity models for the developed fuzzy control systems that enable the sensitivity analysis with respect to the parametric variations of the controlled plant.

The application presented briefly, dealing with the fuzzy control of electrical drives with variable inertia, validates both the development method and the sensitivity models.

Further research will concentrate on the computer-aided calculation of the describing function by means of the Equations (7) … (12).

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