3-dimensional Representation of Residuum with Distance-based Uninorms

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Abstract:. By using special group of fuzzy operators like distance based operators or uninorms a novel reasoning method appears, which is based on residual operators of uninorms. The 3-dimensional representation of the compositional rule of inference will be given.[1]

Keywords: compositional rule of inference, fuzzy rule base system, distance based operators, residual operators

1 Introduction

In fuzzy control system the system state is described by a fuzzy rule base system, and the relationship between fuzzy rule base system, system output and system input is modelled by compositional rule of inference. The concept of approximate reasoning in the known framework of the linguistic information was introduced by Zadeh [13].

The first successful practical applications of fuzzy sets were realized by means of the Mamdani inference [10], but the Mamdani's approach is not fully coherent with the paradigm of approximate rasoning [3], [9].

In the fuzzy rule based control theory and usually in the approximate reasoning, as well as in the covering over of fuzzy rule base input and rule premise of a rule determine the importance of that fuzzy rule and the rule output, too. The practical realization of that notion usually depends on the application, see [18],

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[19] . A very thorough overview of mathematical background of that principle can be found in [4], [6].

The Mamdani type controller is based on Generalized Modus Ponens (GMP) inference rule, and the rule output is given with a fuzzy set, which is derived from rule consequence, as a cut of them. This cut is the generalized degree of firing level of the rule, considering actual rule base input, and usually it is the supremum of the minimum of the rule premise and rule input (calculating with their membership functions, of course). The firing level depends on the covering over of the rule base input and rule premise, but first of all it depends on the *height* of those covered membership functions. Engineering applications are satisfied with the minimum operator, but from a mathematical point of view it is interesting to study the behavior of other t-norms in inference mechanism. The using of distance based operators in fuzzy control theory (FLC) was described in [11],[12].

In fact the uninorms offer new possibilities in fuzzy approximate reasoning, because the low level of covering over of rule premise and rule input has measurable influence on rule output as well. In some applications the meaning of that novel t-norms, has practical importance. The modified Mamdani's approach , with similarity measures between rule premises and rule input, does not rely on the compositional rule inference any more, but still satisfies the basic conditions supposed for the approximate reasoning for a fuzzy rule base system [17].

From mathematical point of wiev, and having results from [14], we can introduce residuum-based inference mechanism using distance-based uninorms .

2 Modified distance-based operators

The distance-based operators can be expressed by means of the min and max operators as follows (the only modificaton on the original published distance based operators in [8] is the boundary condition for neutral element *e*):

th*e maximum distance minimum operator with respect* to $e \in]0,1]$ is defined as

$$max_e^{min} = \begin{cases} max(x,y), & \text{if } y > 2e - x \\ min(x,y), & \text{if } y < 2e - x \\ min(x,y), & \text{if } y = 2e\text{-}x \end{cases}$$

the *minimum distance minimum operator* with respect to $e \in [0,1[$ is defined as

$$min_e^{min} = \begin{cases} min(x,y), & \text{if } y > 2e - x \\ max(x,y), & \text{if } y < 2e - x \\ min(x,y), & \text{if } y = 2e\text{-}x \end{cases}$$

the *maximum distance maximum operator with respect to* $e \in]0,1]$ is defined as

$$max_e^{max} = \begin{cases} max(x,y), & \text{if } y > 2e - x \\ min(x,y), & \text{if } y < 2e - x \\ max(x,y), & \text{if } y = 2e\text{-}x \end{cases}$$

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The distance-based operators have the following properties

max_e^{min} *and* max_e^{max} are uninorms,

the dual operator of the uninorm max_e^{min} *is* max_{1-e}^{max} , *and*

the dual operator of the uninorm max_e^{max} *is* max_{1-e}^{min}.

Based on results from [14] and [15], we conclude:

Operator $max_{0.5}^{min}$ is a conjunctive left-continuous idempotent uninorm with neutral element $e \in]0,1]$ with the super-involutive decresing unary operator $g(x) = 2e - x = 2 \cdot 0.5 - x \Rightarrow g(x) = 1 - x$.

Operator $min_{0.5}^{max}$ is a disjunctive right-continuous idempotent uninorm with neutral element $e \in]0,1]$ with the sub-involutive decresing unary operator $g(x) = 2e - x = 2 \cdot 0.5 - x \Rightarrow g(x) = 1 - x$.

2.1 Idempotent uninorms

A binary operator V is called idempotent [15], if $V(x,x)=x,(\forall x \in X)$. It is well known, that the only idempotent t-norm is *min*, and the inly t-conorm is *max*.

In [15] and [14] has studied two important classes of uninorms: the class of left-continuos and the class of right-continuos ones.

If we suppose a unary operator g on set [0,1], then g is called

(i) sub-involutive if $g(g(x)) \leq x$ for $(\forall x \in [0,1])$, and

(ii) super-involutive if $g(g(x)) \geq x$ for $(\forall x \in [0,1])$.

A binary operator U is a conjunctive left-continuous idempotent uninorm with neutral element $e \in]0,1]$ if and only if there exist a super-involutive decresing unary operator g with fixpoint e and $g(0)=1$ such that U for any $\forall(x,y) \in [0,1]^2$ is given by

$$U(x,y)=\begin{cases} min(x,y) & if \quad y \leq g(x) \\ max(x,y) & elsewhere \end{cases}.$$

A binary operator U is a dijunctive right-continuous idempotent uninorm with neutral element $e \in [0,1[$ if and only if there exist a sub-involutive decresing unary operator g with fixpoint e and $g(1)=0$ such that U for any $\forall(x,y) \in [0,1]^2$ is given by

$$U(x,y)=\begin{cases} max(x,y) & if \quad y \geq g(x) \\ min(x,y) & elsewhere \end{cases}.$$

2.2 Residual implicators for uninorms

In [14] we find general theoretical results related to residual implicators of uninorms, based on residual implicators of t-norms and t-conorms.

Residual operator R_U, considering the uninorm U , can be represented in the following form:

$$R_U(x,y)=sup\{z \mid z \in [0,1] \wedge U(x,z) \leq y\}.$$

Uninorms with neutral elements $e=0$ and $e=1$ are t-norms and t-conorms, respectivly, and related residual operators are weidly discussed [5],[6][14]. In [14] we also find suitable definitions for uninorms with neutral elements $e \in]0,1[$.

If we consider a uninorm U with neutral element $e \in]0,1[$, then the binary operator R_U is an implicator if and only if $(\forall z \in]e,1[)(U(0,z)=0)$. Furthermore R_U is an implicator if U is a disjunctive right-continuous idempotent uninorm with unary operator g satisfying $(\forall z \in [0,1])(g(z)=0 \Leftrightarrow z=1)$.

The residual implicator R_U of uninorm U can be denoted by Imp_U.

2.3 Residual implicators of distance based operators

According to Theorem 8. in [14] we introduce implicator of distance based operator $max_{0.5}^{min}$.

Consider the conjunctive left-continuous idempotent uninorm $max_{0.5}^{min}$ with the unary operator $g(x)=1-x$, then its residual implicator $Imp_{max_{0.5}^{min}}$ is given by

$$Imp_{max_{0.5}^{min}} = \begin{cases} max(1-x,y) & if \quad x \le y \\ min(1-x,y) & elsewhere \end{cases}.$$

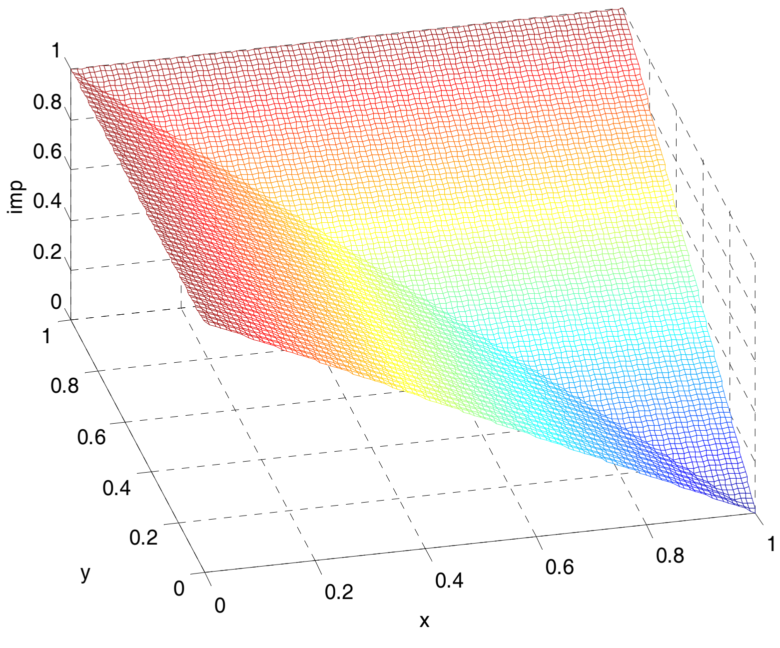

Figure 1. The 3-dimensional representation of $Imp_{max_{0.5}^{min}}$ implicator

4 Conclusions

Despite the fact, that Mamdani's approach is not entirely based on compositional rule of inference, it is nevertheless very effective in fuzzy approximate reasonng. Because of this it is poosible to apply several t-norms, or, as in this case, uninorms. In this paper a residual-based approach and its 3-dimensional representation was presented usind general results for residual implicators of uninorms. This leads to further tasks and problems, because in any case there must be a system of conditions that is to be satisfied by the new model of inference mechanism in fuzzy systems.

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