# Real-Time 3D Tool Path Generation for Numerical Control 

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#### Abstract

The conventional input of a CNC, a sequence of data points, describing the tool motion necessary for machining of sculptured surfaces, represent a huge amount of information. As free-form surfaces are usually described as a set of bicubic pathes, it is a strightforward solution to extend the capabilities of the Numerical Control unit with bicubic interpolation, offset generation and collision checking. The tool path is calculated during the cutting process.


Keywords:CNC, patch programming, motion planning, tool path calculation, sculptured surfaces, $21 / 2 D$ and $3 D$ machining

## 1 Introduction

In Computer Aided Geometric Design we wish to create, in the computer, a model which will adequately describe the particular class or classes of three dimensional shapes which are to be designed. The creation of the model is a mean to produce the necessary data to manufacture the physical object.
Unfortunatly numerical control was established as a discipline before CAGD and whereas it is possible to generate numerical control data directly from stored geometric information, conversation either manually, or by means of a computer takes place. The surface curves describing the tool path are in the majority of control units are interpolated by a sequence of short line segments. In the subsequent paragraphs it becomes apperent, that remarkable improvement in performance and memory utilisation can be gained, when the tasks are transferred in the vinicity of the manufturing process by increasing the CNC's capabilities.

This problem is especially important when we are machining free-form surfaces. Here the geometric model contains the information in condenced form, whereas the NC information is at least two orders of magnitude greater. The algorithms described in this paper enable the tool path generation for sculptured surfaces in the CNC unit directly form the surface model.

## 2 The Place of Motion Planning in the Design Process

Process planning can be broken down into three levels: process sequence, operation planning and motion planning level. This last step became in the last years more and more the integral part of CNC units. One can recognize the analogy with a conventional workshop, where the machining of small batches relies to a great extent on the skill of the worker.

In motion planning two different approaches can be distinguished: the so-called object surface driven approach, where the contact point between the tool and the final workpiece is determined first, together with the surface normal vector on appropriate points. After that offsetting of the cutter location is calculated. In this case modification of the tool correction can be easily determined. The second approach is the so-called offset surface driven approach, where the offset surface is generated first and than the reference point of the tool is calculated. The two approaches are compared qualitatively in Table 1.

| Point of view | Object surface driven | Offset surface driven |
| :---: | :---: | :---: |
| Deviation control | Direct | Indirect |
| Collision testing | Difficult | Simple |
| Maintaining constant <br> cutting speed | Simple | Simple |
| Modification of tool <br> correction | Simple | Complex |
| Exception handling | Simple | Simple |

Table 1. Comparison of object and offset surface driven approaches

Careful evaluation showed that the following steps can be effectively realized in a numerical control unit:
$>$ Determination of the path traced by the point of contact between the tool and the final surface, and the unit surface normal vector
$>$ Computation of the reference point's path according to the given tolerances
> Adjustment of feeds and speeds

## 3 3D Algorithms for Milling

First the form of the path traced by the contact point has to be decided. From the wide variety of surface curves available, based on computational and technological consideration, the subsquent four strategies are used:
> Machining along the cross section lines of the final surface and planes, parallel to one of the principal planes of the coordinate system
> Machining along isoparametric lines
> Tracing curves generated by intersecting the objects with concentric cylinders
> Polyhedral machining

The first two strategies are described in the subsequent paragraphs.

In the first algorithm, a piecewise linear approximation of the cross section line of the patch [3] with the tool driving plane is obtained, then a normal at each cross point is computed, thus the location of the center of a ball end tool is easily calculated by shifting the touching point's coordinates along the surfac normal with ball radius. This step is repeated at the adjecent point on the cross section until the boundary of the patch is reached.


Fig. 1. Cutter path generation using cross section lines

Heap [1] published an algorithm which produces contour lines from a triangular mesh approximation of the surface. The mesh consists of a set of planar triangular elements whose vertices are points on the surface. For each cross section the algorithm follows the contour line segment line by line. As each triangle is found, the edges re intersected by the tool drive plane. A modification of this algorithm forms the basis of the new approach. Since the B-spline and rational B-splines are controlled by parameter values, any subregion of the patch can be calculated if the corresponding parameter values are known. Using the methode of plates from finite element analysis an estimate of the parameter step in both direction can be given according to formula (2):

$$
\begin{equation*}
\Delta \mathrm{s}=\Delta \mathrm{t}=\frac{\mathrm{tol}}{4 \sqrt{ } 2 \mathrm{M}} \tag{2}
\end{equation*}
$$

where

$$
\begin{aligned}
\mathrm{M}= & \max \left(\left\|\mathrm{D}_{\mathrm{s}}^{2} \mathrm{~S}_{\mathrm{x}}\right\|,\left\|\mathrm{D}_{\mathrm{st}}^{2} \mathrm{~S}_{\mathrm{x}}\right\|,\left\|\mathrm{D}_{\mathrm{t}}^{2} \mathrm{~S}_{\mathrm{x}}\right\|\right)+ \\
& \max \left(\left\|\mathrm{D}_{\mathrm{s}}^{2} \mathrm{~S}_{\mathrm{y}}\right\|,\left\|\mathrm{D}_{\mathrm{st}}^{2} \mathrm{~S}_{\mathrm{y}}\right\|,\left\|D_{\mathrm{t}}^{2} \mathrm{~S}_{\mathrm{y}}\right\|\right)+ \\
& \max \left(\left\|\mathrm{D}_{\mathrm{s}}^{2} \mathrm{~S}_{\mathrm{z}}\right\|,\left\|\mathrm{D}_{\mathrm{st}}^{2} \mathrm{~S}_{\mathrm{z}}\right\|,\left\|\mathrm{D}_{\mathrm{t}}^{2} \mathrm{~S}_{\mathrm{z}}\right\|\right)
\end{aligned}
$$

Using this estimates a B-spline or rational B-spline surface can be triangulated. It is obvious that only those plane triangles are of interest which may intersect with the tool drive plane. It is practical to generate and intersect them one by one as it is required when traversing the cross section line. At the boundary of each triangle the surface normal is determined and the tool reference point offsetted accordingly. The diadvantage of this procedure is that the end points of the cross section line's piecewise approximation do not lay on the surface. This can be improved by subdividing the edges of the curved triangle and getting a more accurate approximation.


Fig. 2 The original and the improved cross section line

The second algorithm moves the tool along isoparametreic lines. Hereby the proble is reduced to one with a single degree of freedom. The surface curve is approximated by short line segments and the surface normal is calculated in the endpoints. The tools's reference point is computed again by moving the contact point on the surface normal by the tool radius.


Fig 3. Machining along isoparametric lines

The parameter step $\Delta \mathrm{s}$ was estimated according to formula (1) and the endpoints of the line segment is calculated again using Horner's scheme. In order to avoid undercut as a result of twisting surface, the distance between surface curve and the reference point's path is checked at the midpoint. If it is out of tolerance the parameter step is decreased and the calculation is repeated with the new value.
The segments of the tool path machining one patch can be connected to each other in three different way: one way, zig-zag and spiral cutting.

The algorithm ensures that the required tolerance is met in the direction of the tool movement. However, we have to take care of the distance between the adjacent tool paths. The surface roughness can deterogate due to the increaseíof the Euclidean distance between two neighbouring paths, due to change of the surface curvature.

In order to avoid these unwanted situations the following actions will be taken:
$>$ If the distance between adjacent tool paths becomes unacceptably big, the distance will be changed to either $1 / 2$ or $1 / 3$ of the original according to the selected strategy. The beginpoint of the modified tool path segment is pushed on a stack. The new path will be followed until the boundary or to
the point whre path became close enough again. At this point the coordinate values are taken from the stack and the machining will be resumed from that point
$>$ If the distance became to small the path will be covered by high speed motion until the distance reaches the required level again

## 4 Collision Checking

Typically a surface to machined consists of a number of patches. The adjacent patches have up to a certain tolerance a common boundary. When the tangent's direction changes at the patch boundary, the tool may interfere with the adjacent patch and may damage the surface.
In CAD systems this problem is solved by computing the intersection curve of the offset surfaces and restrict the parameter domain to the area between them. This solution is time consuming and can't be afforded in real-time environment.

However by approximating the offset of bicubic patches by fifth order surfaces the computation can be reduced significantly. The points of the intersection curves are calculated by subdivision using the minimax box technique. This process is done for each pair of adjacent patches before the machining of the patch starts. The points of the boundary are then stored and used during the tool patch generation to check whether the next increment crosses the boundary or not.

## 5 Realization in Firmware

To implement the previously developed algorithms two ways have been used:
$>$ The interpolation of the space curves is computed in the interpolator separately for the three axis. The interpolation is implemented in software on digital signal processor
> The restriction of the area for avoiding collision is calculated by a separeted processor working in a pipe line with the interpolator

With this structure the time needed for the calculation of the endpoints of the next line segment is less then 1 msec , while the calculation of the restricted area for one patch takes about 10 msec .

## 6 Experimental results

In order to evaluate the performance of the processor parabolic and elliptic surface have been described by spline surfaces and machined. For the purpose of reference the surface was machined by approximating the tool path with short line segments generated by the technological procesor. The machnined workpiece was used as a reference with which the results of the built in processor were compared.

| Workpiece | Version | Deviation (mm) | Rz( $\mu$ ) |
| :---: | :---: | :---: | :---: |
| Elliptic | Reference | 0.025 | 22.0 |
| Elliptic | Option | 0.018 | 13.5 |
| Parabolic | Reference | 0.028 | 23.5 |
| Parabolic | Option | 0.022 | 20.5 |

## Conclusions

The presented algorithms for a natural extension of the so-called contour programming and proved to be very advantageous both in terms of NC data and memory utilization. The part programmer describes the surfaces and curves in a compact form. This leads also to improved man -machine interface.

## References

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