Application of Shortest Path Algorithm to GIS using Fuzzy Logic

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Abstract: *Geographical Information Systems (GIS) and Image Processing Systems are usually based on classical logic and set theory. This fact pass in case of some GIS elements because of presence of many uncertainties in the nature. Hence, these GIS elements can not be based on classical model at all time. Constantly growing demands on utilization of GISs require extension of area of their usage in the way of time - dependent data processing, which form is uncertain in the time of their use. This article deals with representation of mentioned uncertainties with mathematical methods of Fuzzy Logic. Computation of vehicle travel time and vehicle capacity on a road network using GIS with implemented Fuzzy Shortest Path algorithm will be used as a subject of research in this article.*

Keywords: *GIS, Fuzzy Logic, Shortest Path Algorithm, Simulation*

1 Introduction

One of a frequently used tools for visualization of data acquired by the abstraction of real - world geographical elements in GISs are various kinds of graphs and their modifications tailored to their intended usage. Graph theory has many different applications in a system analysis, economy and transportation. In some specific cases we must use uncertain data, that we are not able to consider them in the calculations over usually used graphs. Fuzzy Logic and Fuzzy Graph Theory gives to us a proper tools to use in those cases.

GISs [5] usually use the “layer concept” [6] for modelling, visualization are representation of data based on geometrical basis. Other method for both geological objects modelling and geological data processing are closely described by [14]. Most of nowadays applications of layer concept suppose, that there are not uncertainties or ambiguities through the entire set of processed data. Ability to process those uncertain data can by achieved by implementation of various techniques of Fuzzy Logic. This promoted GISs to be useful as a decision-making system working over either time - dependent or uncertain data.

Solution for storage and representation of time – dependent geographical data is linkage of GIS and either active or temporal database systems (DBMS) [3]. A temporal DBMS usually has an extension to the basic relational operations to support queries that use the time dimension. Development of temporal DBMS began in the mid 80's, when data were marked with timestamps for the first time.

2 Fuzzy Logic in GIS

Next we show an example [1] in which GIS is used for making decision about the proper place to build a new house. Important criterions are placement of new house and character of landscape which surrounds selected locality. Exact position were not specified, but criterions have been taken as follows: "New house should be placed southerly and should be build on a less steep hillside as a former house." Given criterion can be easily understand by a human, but it is difficult to transform them to the mathematical form proper to use by the searching process performed by GIS [9].

Solution for this is to represent given data using fuzzy set theory, which will be closely described in the following equations 1 to 11 and used in an example in the chapter 3.

A *fuzzy set* [2] is a set where there is some measure of uncertainty of membership in the set. For a fuzzy set *S*, each element of a referential set Ω must be assigned a membership in *S* :

$$\mu_s : \Omega \text{ -> } M, \qquad (1)$$

where μ_s is the *membership function* for the set and *M* is the set of allowed measurements. Typically *M* is chosen to be the unit interval [0,1], so that:

$$\mu_s : \Omega \text{ -> } [0,1] \qquad (2)$$

The *support* of a fuzzy set *S*, written as supp(S), is the crisp subset of the referential set Ω defined by:

$$\text{supp}(S) = \{ x \in \Omega \mid \mu_s(x) > 0 \} \qquad (3)$$

The interpretation of this is that the support of a fuzzy set is the set of all objects that are possibly in the set.

The α *-cut* of a fuzzy set *S*, denoted by $S\alpha$, is the crisp subset of Ω that contains all of the elements of *S* with at least the given degree of membership α:

$$S_\alpha = \{ x \in \Omega \mid \mu_s(x) \geq \alpha \} \qquad (4)$$

Similarly, the α *-level cut* of a fuzzy set *S*, denoted by S^α, is the crisp subset of Ω that contains all of the elements of *S* with exactly the given degree of membership α:

$$S^\alpha = \{ x \in \Omega \mid \mu_s(x) = \alpha \} \qquad (5)$$

It is useful to define functions on fuzzy sets. Any unary function or operation:

$$f : D \text{ -> } R \qquad (6)$$

can be generalized to apply to fuzzy sets. Consider a fuzzy set *A* with measure $\mu A : D \rightarrow [0,1]$; we define the measure for *f(A)* as:

$$\mu_{f(A)} : \sup_{y=f(x)} \{ \mu_A (x) \} \qquad (7)$$

Likewise, one can generalize any binary function or operation as:

$$\mu_{A \otimes B}(z) = \sup_{x \otimes y = z} \{ \min \{ \mu_A (x), \mu_B (y) \} \} \qquad (8)$$

Here the function $\otimes$ could be, for example, an operation from set theory (e.g., $\otimes \in \{\cup, \cap, -\}$) or - in the case of fuzzy numbers - an arithmetical operation (e.g. $\otimes \in \{+, -, \times, \div, \min, \max\}$), etc.

We can also generalize the comparison operators in the same way as other arithmetic operations if we interpret them as Boolean-valued functions:

$$\otimes : R \times R \rightarrow \{ \text{true}, \text{false} \} \qquad (9)$$

where $\otimes \in \{\leq, \geq, \neq, >, <, =\}$. If we use the convenient shorthand notation [2]:

$$\mu_{A\otimes B} \equiv \mu_{A\otimes B}(\text{true}) \qquad (10)$$

$$\overline{\mu}_{A\otimes B} \equiv \mu_{A\otimes B}(\text{false}) \qquad (11)$$

Next important term is Fuzzy Graph and his elements. So it will be closely described in following equations 12 to 17.

A *graph G* consists of a set of *vertices V* and a set of *edges E*:

$$G = (V, E) \qquad (12)$$

We label the vertices and edges with indices:

$$V = \{ v_1, v_2, \ldots , , v_{nV}\}, \qquad (13)$$

$$E = \{ e_1, e_2, \ldots , , e_{nE} \}, \qquad (14)$$

where *nV* is the number of vertices and *nE* is the number of edges.

In a *weighted graph*, each edge also has a *weight* (sometimes called its *length* or *capacity*):

$$w_i = W (e_i) \qquad (15)$$

specified by a weight function *W* that maps edges to numbers (which may be crisp or fuzzy).

A *path P* is a sequences of edges:

$$P = (e_{i1}, e_{i2}, \ldots , e_{in}) \qquad (16)$$

where the head of one edge is the same as the tail of the following edge.

If the graph is weighted, the path has a *length* given by the sum of the weights for the edges in the path:

$$l_p = \text{length}(P) = w_1 + w_2 + \ldots + w_i \tag{17}$$

In the *triangular representation* a fuzzy number A = [M,L,R] has a membership function (as shows fig. 1):

$$\mu_A(x) = \begin{cases} \dfrac{x-L}{M-L}, & x \in [L,M] \\ \dfrac{R-x}{R-M}, & x \in (M,R] \\ 0, & x \notin [L,R] \end{cases} \tag{18}$$

Here also there are simple formulas for the basic arithmetic operations:

$$\begin{aligned} [A_0, A_1, A_2] + [B_0, B_1, B_2] &= [A_0 + B_0, A_1 + B_1, A_2 + B_2] \\ [A_0, A_1, A_2] - [B_0, B_1, B_2] &= [A_0 - B_2, A_1 - B_1, A_2 - B_0] \\ [A_0, A_1, A_2] \times [B_0, B_1, B_2] &= [A_0 \times B_0, A_1 \times B_1, A_2 \times B_2] \\ [A_0, A_1, A_2] \div [B_0, B_1, B_2] &= [A_0 \div B_2, A_1 \div B_1, A_2 \div B_0] \end{aligned} \tag{19}$$

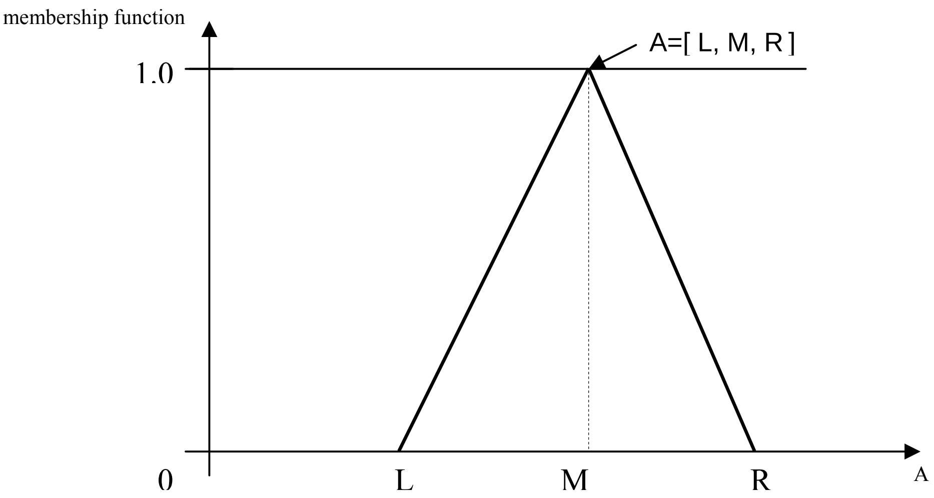

Fig. 1 Membership function for A=[L, M, R]

3 Fuzzy Shortest Path Algorithm (FSA)

Consider a fuzzy graph *G* with pure Type V fuzziness [2]. Let Π be the set of all paths from vertex *va* to vertex *vb* (see fig.2) and let the fuzzy length of a path be

$$l_p = \text{length}(P) = \sum_{e_k \in p} w_k, \text{ where } P \in \Pi, \quad (20)$$

where e_k are edges of G.

The *fuzzy set of shortest paths* is a fuzzy set *S* on Π with memberships πS given by:

$$\pi_S(P) = \min_{Q \in \Pi} \{ \mu_{lp \leq lQ} \}, \text{ where } P \in \Pi, \quad (21)$$

The support consists of all of the paths which potentially could have the minimum length:

$$\text{supp}(S) = \{ P \in \Pi \mid \mu_{lp \leq lQ} > 0, \forall Q \in \Pi \} \quad (22)$$

The fuzzy set of shortest paths defined above can be collapsed into a *fuzzy shortest path*, where each edge *ei* has a membership in the fuzzy set *S'*:

$$\mu_{S'}(i) = \max_{e_i \in P,\, P \in \Pi} \{ \pi_S(P) \}, \text{ for } i = 1, \dots, n_E \quad (23)$$

FSA algorithm :

Step 1:

Construct graphs $\overline{G}$ and $\underline{G}$, which are identical to G, and weight on the edges of $\overline{G}$ and $\underline{G}$ can be computed as follows:

$$\text{for } \overline{G}: \qquad \overline{\tau}_a = \sup\{\text{supp}(\tau_a)\} \quad (23)$$

$$\text{for } \underline{G}: \qquad \underline{\tau}_a = \inf\{\text{supp}(\tau_a)\} \quad (24)$$

Step 2:

Find the shortest path *p* from v_a to v_b in $\overline{G}$. This is classical shortest path problem and many well-known algorithms [4] can be used to solve it. Denote *k* the length of path p.

$$\mu_{S'}(i) = \min_{P \in \Pi} \{ l_p \} \qquad (25)$$

Step 3 :

Let $\underline{S}$ be the set of all paths from v_a to v_b in $\underline{G}$, which length is less than *k*. Let S be the set of all path in G. Shapes of paths in both S and $\underline{S}$ are identical. So S, is the set of all fuzzy shortest path. Finally calculate membership for each fuzzy path from S in consideration of *k*.

Figure 2 shows weighted type V fuzzy graph. Vertex *a* is the source of path and *f* is the destination of path. Weights can be either crisp numbers or fuzzy triangular numbers.

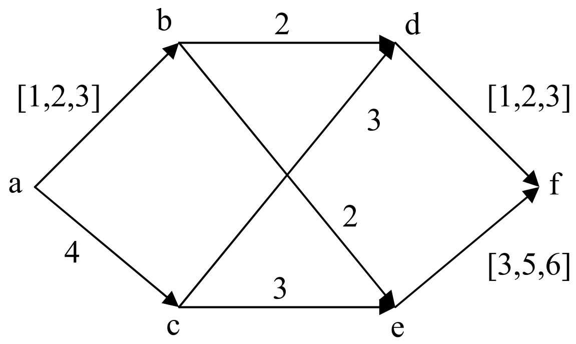

Fig. 2 Oriented type V fuzzy graph G

The fuzzy lengths for the four paths from vertex *a* to vertex *f* are listed in Figure 2 - from this we see that κ=8 and that path *abdf* has membership πS (*abdf*)=1, path *abef* has membership πS (*abef*)=2/5, and the other paths have membership πS(*acdf*) = πS(*acef*) =0 in the fuzzy set of shortest paths. Figure 3 illustrates the fuzzy shortest path.

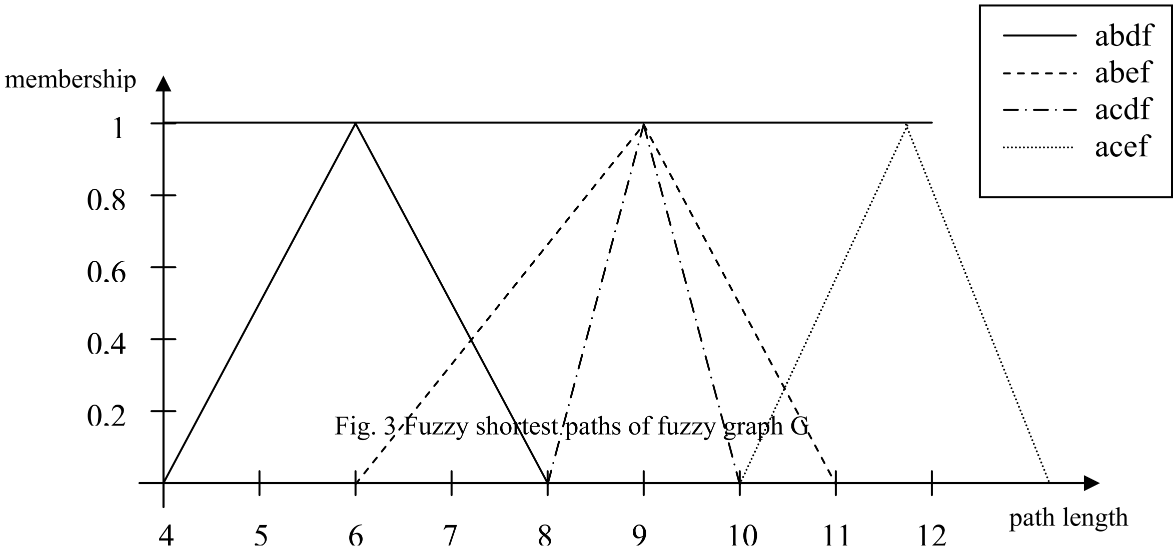

Fig. 3 Fuzzy shortest paths of fuzzy graph G

4 Conclusion

An application of FSA (chapter 3) in this article is to plan path of vehicle which moved along the edges of supposed graph G, acquired by an abstraction of the real road network. Shape of this graph is obvious. But there are uncertainties about weights on its edges according to actual situation on the road (e.g. weather conditions, road capacity, surface quality at the specified time).

All mentioned facts will be considered as a time-dependent variables, which affect the ideal path selected as best one at the specified time. Vehicle travel time is expressed as a fuzzy triangular number assigned to specified edge of graph G.

Product of this research is application *GeoFind*. Simulations in *GeoFind* use aforementioned FSA algorithm (chapter 3) and Fuzzy Graph theory. Application is being developed by the Department of Computers and Informatics, Faculty of Electrical Engineering and Informatics, Technical University of Košice, Slovakia. Visual C++ 6.0 environment have been selected. Program of application is based on object – oriented basis, which offers great environment for description all elements of simulated system in their abstract form.

See [5,15] to find more details about each phase of GIS development and geological data processing. Application is designed as an independent element using developed interface to communicate with GIS database. Acquired data are to be analyzed and processed using FSA (see chapter 3 of this article).

There is a test database developed for test purposes and an interface to transform supposed data format to the format used by *GeoFind*.

Figure 4 shows the objects hierarchy used in *GeoFind* application. Figure 4 also include mentioned interface which provides functions for GIS database – *GeoFind* communication.

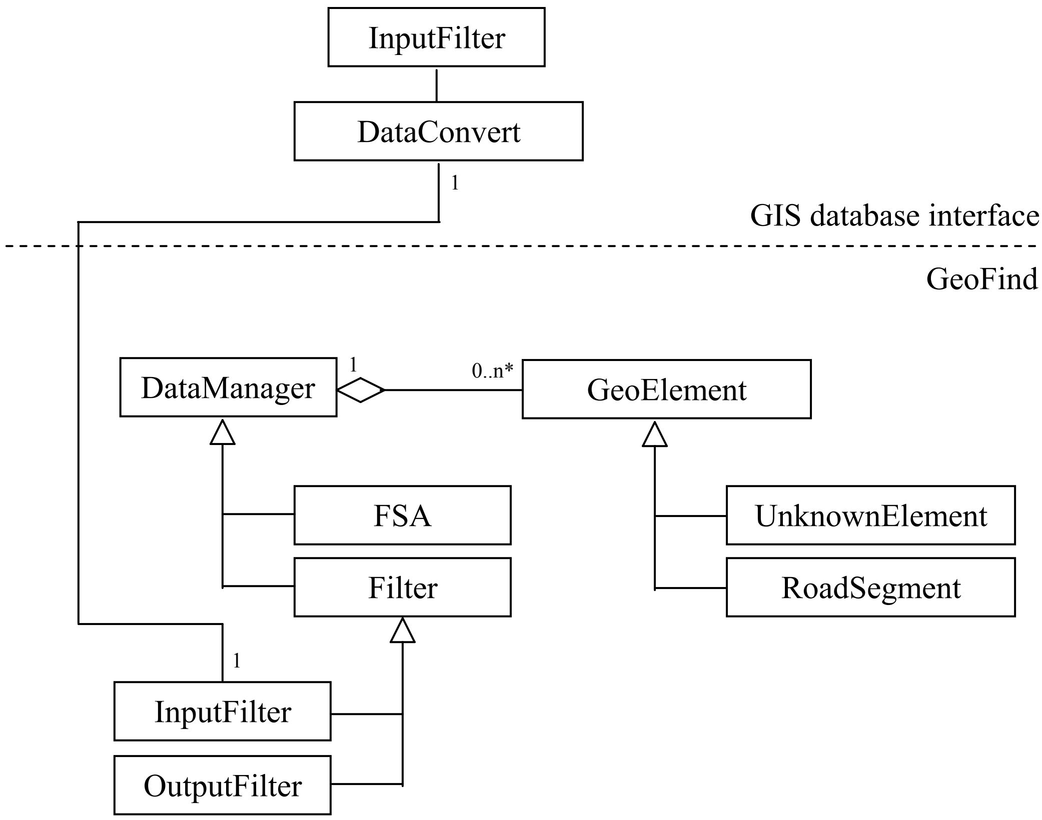

Fig. 4 Class diagram of GeoFind application

GeoFind application development is in its advanced phase. There are also parallel development of interfaces for various kinds of GIS.

Utilization of *GeoFind* application will be oriented mostly for learning purposes. Beside this, development team intends to extend functionality of *GeoFind* in the future by adding different program modules to provide ability to process various types of geographical data.

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